The Michelson Interferometer

MAITREY SHARMA
(1911093)

Third Year Integrated M.Sc.
School of Physical Sciences
National Institute of Science Education and Research, Bhubaneshwar
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1 Aim

- Alignment of Michelson's Interferometer using He - Ne laser to observe concentric circular fringes.
- Measurement of the wavelength of He-Ne Laser and Na lamp using circular fringes.
- Study of fringes of equal inclination and equal thickness using Na lamp.

2 Apparatus

1. Michelson's Interferometer
2. He-Ne Laser
3. Na lamp
4. Screen.

3 Introduction

Interference is a phenomenon in which two waves superpose to form a resultant wave of greater, lower, or the same amplitude. **Interferometers** are devices that extract information from interference. These are basic optical tools used to precisely measure wavelength, distance, index of refraction, and temporal coherence of optical beams. They are also widely used in science and industry for the measurement of microscopic displacements, refractive index changes and surface irregularities. In the case with most interferometers, light from a single source is split into two beams that travel in different optical paths, which are then combined again to produce interference.
The **Michelson interferometer** is an amplitude-splitting interferometer devised by Albert Michelson. It became well-known for its use by Michelson and Edward Morley in the famous Michelson–Morley experiments in 1887 in a configuration which would have detected the Earth's motion through the supposed luminiferous ether that most physicists at the time believed was the medium in which light waves propagated. The null result of that experiment essentially disproved the existence of such an ether, leading eventually to the special theory of relativity and the revolution in physics at the beginning of the twentieth century. The Laser Interferometer Gravitational-Wave Observatory (LIGO) is also an application of the Michelson interferometer which made the first direct observation of gravitational waves. It is still an important instrument in today's laboratories and it is being widely used as an instrument for measuring the wavelength of an unknown light source, to measure extremely small distance and for investigating optical media.

4 Construction

The Michelson interferometer consists of two highly polished mirrors M_1 and M_2. Two glass plates beam splitter (BS) and compensatory glass plate (CP), are placed parallel to each other between the mirrors at an angle of 45°. The rear side of glass plate BS is semi-silvered such that the light from a source is equally reflected and transmitted by it. In this way division of amplitude takes place.

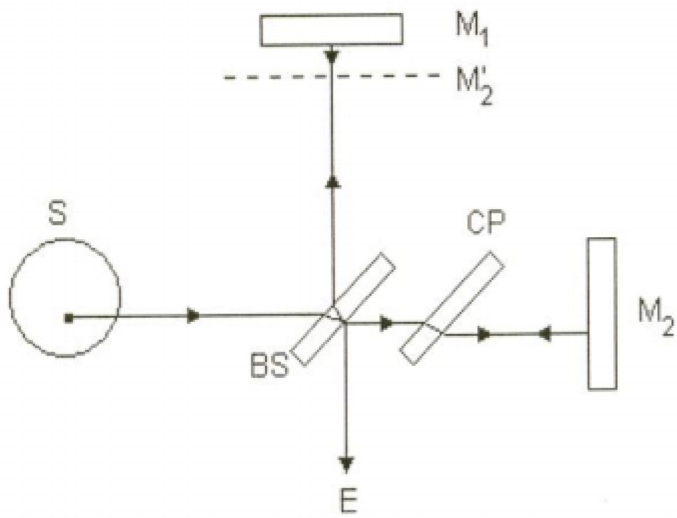

Figure 1: Construction of Michelson's interferometer

A monochromatic light of wavelength λ falling on BS is half reflected towards the mirror M_1 and the other half is transmitted towards mirror M_2. After splitting, the two rays are reflected back by the mirrors M_1 and M_2 and return to the plate BS. The ray reflected from M_1 is transmitted through BS and the ray reflected from M_2 is reflected again by BS. The two rays coming from the two mirrors interfere and fringes are observed on a screen (for laser) or by naked eye (Na lamp) at E.

5 Theory

The wave reflected from M_1 and entering the eye crosses BS twice. However the path of the other wave falling on the mirror M_2, in the absence of compensating plate CP, travels totally in air. Thus an extra **optical path** $2(\mu - 1)t$ is introduced where, t is the thickness of the plate and μ is the refractive index of the BS plate for the monochromatic light used. Thus, the two waves will interfere constructively or destructively as per the following conditions of path difference, Δ:

$$\Delta = 2n\lambda/2 = n\lambda \qquad \text{(for maxima, } n \text{ is an integer)}$$
$$\Delta = (2n+1)\lambda/2 \qquad \text{(for minima, } n \text{ is an integer)}$$

Depending upon the nature of the air film, the fringes formed in the interferometer can be of various geometries. **Concentric circular fringes** or **fringes of equal thickness** are obtained when the air film is parallel as shown in figure (2). M_2' is the virtual image of M_2 and it is parallel to M_1. L_1 and L_2 are the virtual images of L formed by M_1 and M_2' and are coherent. Let d be the distance between M_1 and M_2', therefore the distance between L_1 and L_2 is $2d$. Let θ be the angle between the incident beam originated at P and the reflected beams from M_1 and M_2'. Then path difference between light beams from points P' and P'' is $2d\cos\theta$. A maximum (bright fringe) will be formed when $2d\cos\theta = n\lambda$. Each circular ring corresponds to a particular value of θ.

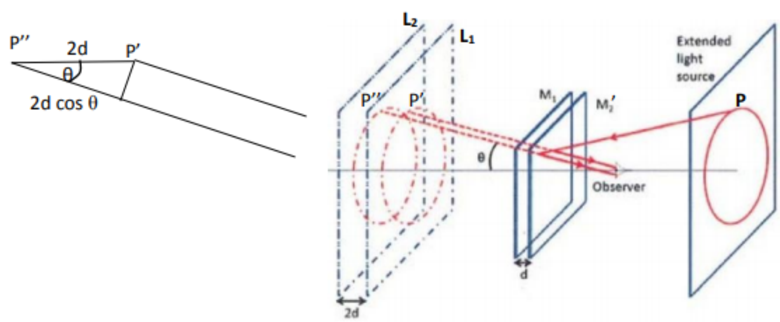

Figure 2: Formation of circular fringes

When M_1 and virtual image M_2' are inclined to each other, the film enclosed is wedge shaped. Then **curved fringes** can be observed as shown in figure (3). These are also known as **fringes of equal thickness**.

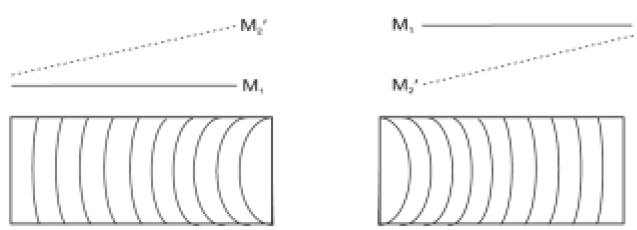

Figure 3: Formation of curved fringes

When M_1 and virtual image M_2' intersect, straight line fringes are obtained around the point of intersection.

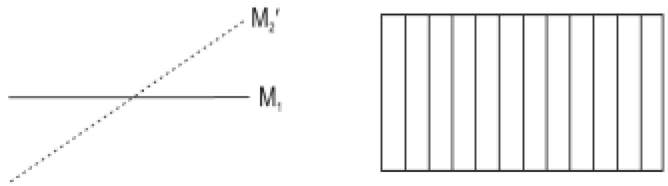

Figure 4: Formation of straight line fringes

Circular fringes are used to determine the wavelength of the source of light. For a given separation of d between the mirrors M_1 and M_2 and normal incidence ($\theta = 0$), the path difference is given as $2d = n\lambda$. If one mirror is moved by a distance Δd and N number of rings appear/disappear at the center, then the path difference after moving the mirror is given as

$$2(d + \Delta d) = (n + N)\lambda \tag{1}$$

Hence,

$$\boxed{\lambda = \mathbf{2(\Delta d)/N}} \tag{2}$$

6 Experimental Setup

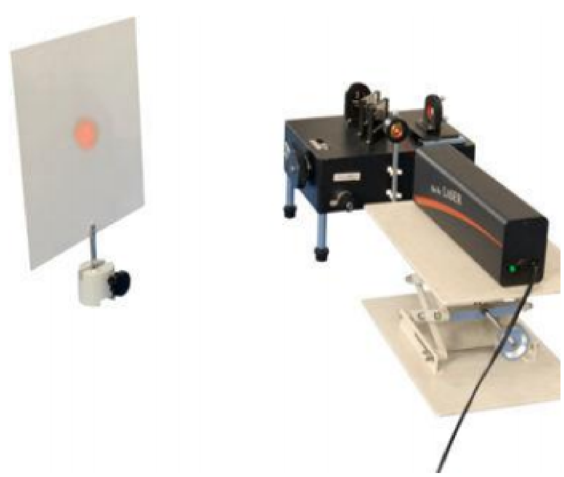

Figure 5: Setup with Laser as the source

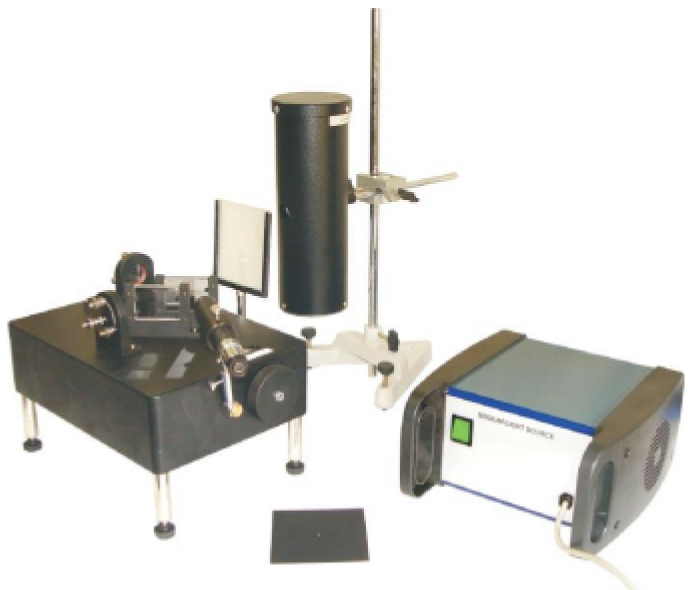

Figure 6: Setup with Na lamp as source

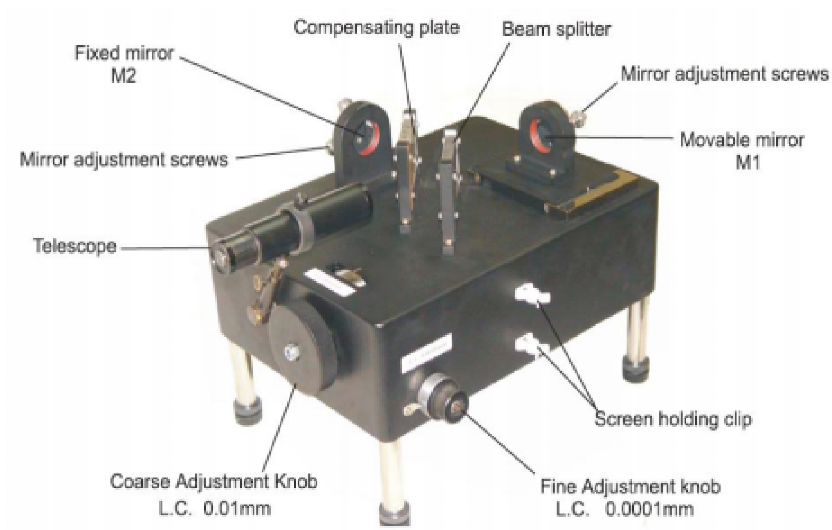

Figure 7: Schematics of Michelson interferometer

7 Observations

1. Least count of coarse adjustment knob = 0.01 mm
2. Least count of fine adjustment knob = 0.0001 mm

Table 1: **Data for He-Ne Laser**
Initial position of gauge, $d_1 = 0.0341$ mm

#	**No. of rings** N	**Coarse adjustment reading,** m	**Fine adjustment reading,** n	$\boldsymbol{d_2}$ (mm)	$\boldsymbol{\Delta d}$ $(\boldsymbol{d_2 - d_1})$ (mm)
1	10	3	73	0.0373	0.0032
2	20	4	8	0.0408	0.0067
3	30	4	42	0.0442	0.0101
4	40	4	68	0.0468	0.0127
5	50	5	6	0.0506	0.0165
6	60	5	38	0.0538	0.0197
7	70	5	63	0.0563	0.0222
8	80	5	96	0.0596	0.0255
9	90	6	29	0.0629	0.0288
10	100	6	56	0.0656	0.0315
11	110	6	86	0.0686	0.0345
12	120	7	19	0.0719	0.0378

Table 2: **Data for Na Lamp**
Initial position of gauge, $d_1 = 0.0165$ mm

#	**No. of rings** N	**Coarse adjustment reading,** m	**Fine adjustment reading,** n	$\boldsymbol{d_2}$ (mm)	$\boldsymbol{\Delta d}$ $(\boldsymbol{d_2 - d_1})$ (mm)
1	10	1	90	0.0190	0.0025
2	20	2	12	0.0212	0.0047
3	30	2	37	0.0237	0.0072
4	40	2	66	0.0266	0.0101
5	50	2	94	0.0294	0.0129
6	60	3	20	0.0320	0.0155
7	70	3	46	0.0346	0.0181
8	80	3	73	0.0373	0.0208

8 Graphs

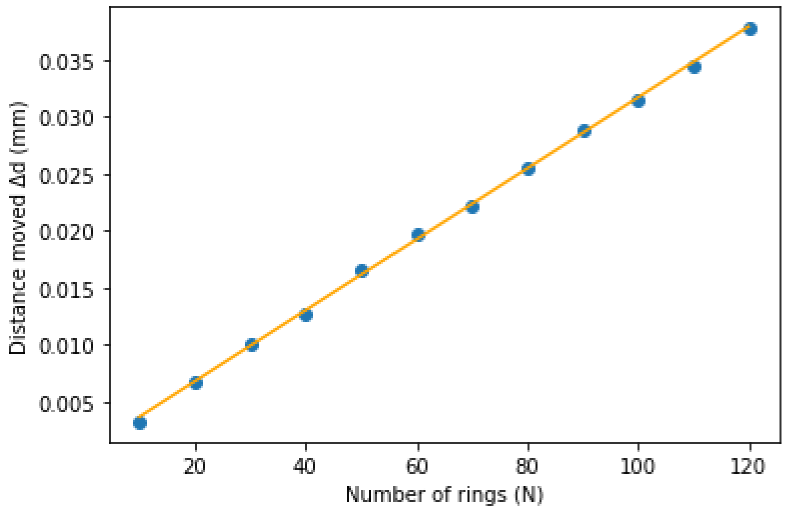

Figure 8: Δd vs N plot for the He-Ne Laser

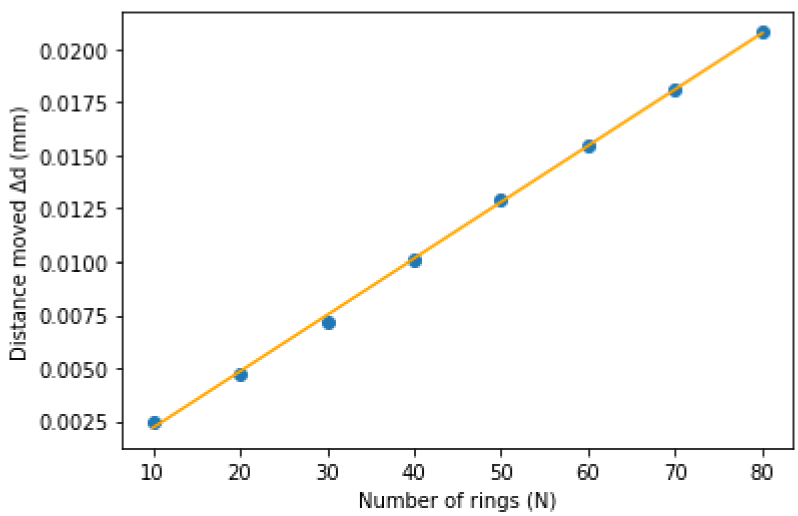

Figure 9: Δd vs N plot for the Na lamp

9 Calculations

9.1 He-Ne Laser

We first analyse the plot (figure (8)) obtained from the table (1). The five summations for this data-set are as follows:

$$S_x = \sum x_i = 780, \qquad S_y = \sum y_i = 2.492 \times 10^{-4}\,\mathrm{m},$$

$$S_{xx} = \sum x_i^2 = 65000, \qquad S_{yy} = \sum y_i^2 = 6.563 \times 10^{-9}\,\mathrm{m}^2,$$

$$S_{xy} = \sum x_i y_i = 2.065 \times 10^{-2}\,\mathrm{m}$$

Now the slope is given by

$$m = \frac{SS_{xy} - S_x S_y}{SS_{xx} - S_x^2} = 3.113 \times 10^{-7}\,\mathrm{m} = 311.3\,\mathrm{nm} \tag{3}$$

$\therefore \boxed{\lambda = 2 \times 311.3\,\mathrm{nm} = 622.6\,\mathrm{nm}}$

9.2 Na Lamp

We first analyse the plot (figure (9)) obtained from the table (2). The five summations for this data-set are as follows:

$$S_x = \sum x_i = 360, \qquad S_y = \sum y_i = 9.180 \times 10^{-5}\,\mathrm{m},$$

$$S_{xx} = \sum x_i^2 = 20400, \qquad S_{yy} = \sum y_i^2 = 1.349 \times 10^{-9}\,\mathrm{m}^2,$$

$$S_{xy} = \sum x_i y_i = 5.245 \times 10^{-3}\,\mathrm{m}$$

Now the slope is given by

$$m = \frac{SS_{xy} - S_x S_y}{SS_{xx} - S_x^2} = 2.652 \times 10^{-7}\,\mathrm{m} = 265.2\,\mathrm{nm} \tag{4}$$

$\therefore \boxed{\lambda = 2 \times 265.2\,\mathrm{nm} = 530.4\,\mathrm{nm}}$

10 Error Analysis

The error in the slope is given by

$$\sigma_m = \sigma_y \sqrt{\frac{S}{\Delta}} \tag{5}$$

Here σ_y represents the least count of the micrometre and is equalt to 0.0001 mm.

10.1 He-Ne Lamp

We have,

$$\Delta = SS_{xx} - S_x^2 = 1.716 \times 10^5 \tag{6}$$

Using (3), (5) and (6),

$$\sigma_m = 0.0001 \times \sqrt{\frac{12}{1.716 \times 10^5}} = 8.362 \times 10^{-10}\,\text{m} = 0.8\,\text{nm} \tag{7}$$

Now, error in λ

$$\boxed{\sigma_\lambda = 2 \times \sigma_m = 1.6\,\text{nm}} \tag{8}$$

10.2 Na Lamp

We have,

$$\Delta = SS_{xx} - S_x^2 = 3.360 \times 10^4 \tag{9}$$

Using (4), (5) and (9),

$$\sigma_m = 0.0001 \times \sqrt{\frac{8}{3.360 \times 10^4}} = 1.543 \times 10^{-9}\,\text{m} = 1.5\,\text{nm} \tag{10}$$

Now error in λ

$$\boxed{\sigma_\lambda = 2 \times \sigma_m = 3.0\,\text{nm}} \tag{11}$$

11 Results and Discussions

1. From (3) and (8), the wavelength of the He-Ne laser is given by $\lambda = 622.6 \pm 1.6$ nm.
2. From (4) and (11), the wavelength of the Na Lamp is given by $\lambda = 530.4 \pm 3.0$ nm.
3. The obtained value are in reasonable range in case of the He-Ne laser whereas in the case of the Na lamp, the obtained value is considerably off from the literature value.
4. One of the source of this error/ambiguity could be the way readings have been noted as we can not definitely say when a circular fringe has vanished.

5. Fringes of equal inclination were observed using He-Ne laser as shown below

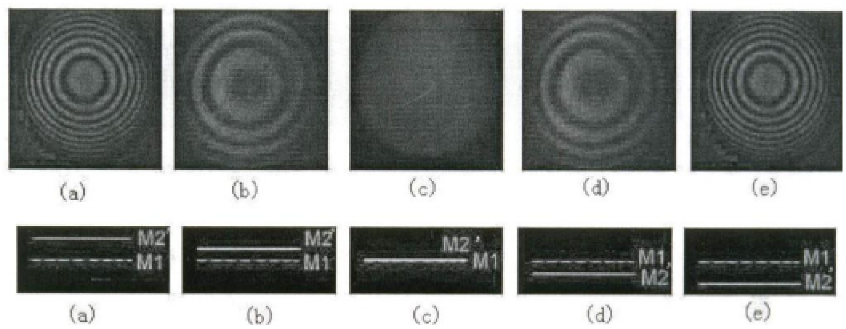

Figure 10: Fringes of equal inclination

6. Fringes of equal thickness were observed using He-Ne laser as shown below

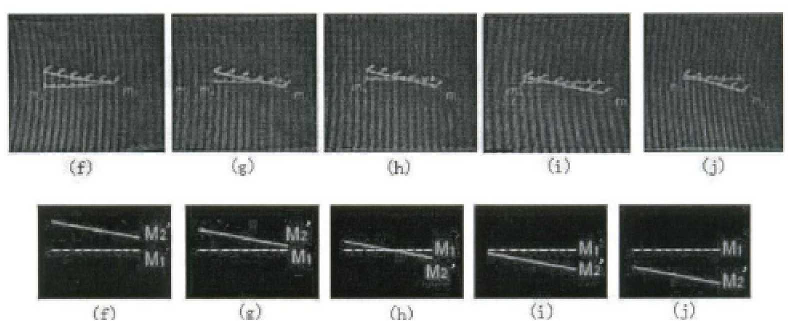

Figure 11: Fringes of equal thickness

12 Precautions

1. Avoid backlash errors.
2. Always turn the fine adjustment knob in one direction either clockwise or anti-clockwise.
3. Direct eye exposure to laser should be avoided.
4. Observing laser interference fringes by reflecting mirror is prohibited.
5. Avoid touching any of the optics with bare hand.