Splitting of Sodium D-lines

using diffraction grating

MAITREY SHARMA
(1911093)

Third Year Integrated M.Sc.
School of Physical Sciences
National Institute of Science Education and Research, Bhubaneshwar

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1 Aim

Measurement of the wavelength separation of sodium D-lines using a diffraction grating and to calculate the angular dispersive power of the grating.

2 Apparatus

1. Spectrometer,
2. A prism,
3. Diffraction grating,
4. A sodium lamp with power supply.

3 Introduction

The *emission spectrum* of a chemical element or chemical compound is the spectrum of frequencies of electromagnetic radiation emitted due to an atom or molecule making a transition from a high energy state to a lower energy state. The photon energy of the emitted photon is equal to the energy difference between the two states. There are many possible electron transitions for each atom, and each transition has a specific energy difference. This collection of different transitions, leading to different radiated wavelengths, make up an emission spectrum.
The observations of emission spectra began in 1756 when Thomas Melvill observed the emission of distinct patterns of colour when salts were added to alcohol flames. By 1785, James Gregory discovered the principles of diffraction grating and American astronomer David Rittenhouse made the first engineered diffraction grating. A ***diffraction grating*** is an optical component with a periodic structure that splits and diffracts light into several beams travelling in different directions. The sodium spectrum is dominated by the bright doublet known as the sodium D-lines at 589.0 nm and 589.6 nm as shown in figure (1). This appears as an absorption line in many types of stars,

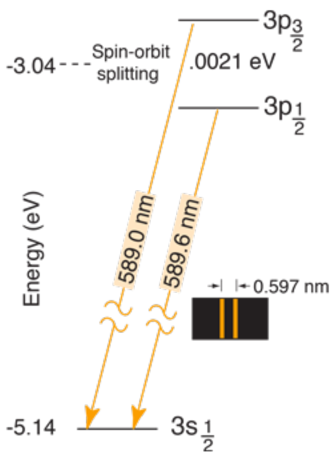

Figure 1: Sodium D-lines

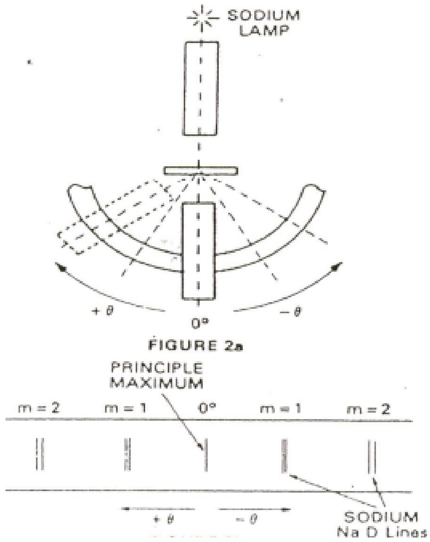

Figure 2: Schematic for diffraction of sodium Na-D lines

including the Sun. The line was first studied in 1814 by Joseph von Fraunhofer during his investigation of the lines in the solar spectrum, now known as the Fraunhofer lines. Fraunhofer named it the 'D' line, although it is now known to actually be a group of closely spaced lines split by a fine and hyperfine structure.
Using an appropriate diffraction grating the wavelength separation of these two lines can be determined. A schematic for diffraction of sodium light (Na-D lines) with a plane transmission grating is shown in figure (2).

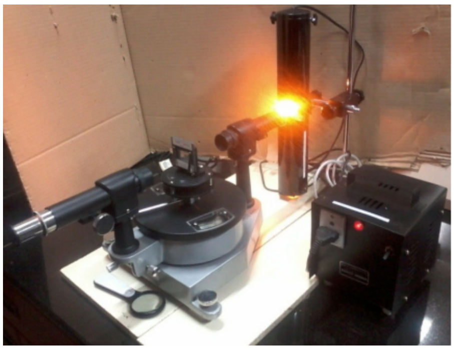

Figure 3: Experimental set up

4 Setup

An arrangement consisting of a large number of parallel slits of the same width and separated by equal opaque spaces is known as diffraction grating. It is usually made by ruling equidistant, extremely close tine grooves with a diamond point on an optically plane glass plate. A photographic replica of a plate made in this way is often used as a commercial transmission grating. The actual experimental set up is shown in figure (3).

5 Theory

For N parallel slits, each with a width e and separated by an opaque space of width b, the diffraction pattern consists of diffraction modulated interference fringes. The quantity $(e+b)$ is called the *grating element* and $N(= 1/(e+b))$ is the number of slits per unit length, which could typically be 300 to 12000 lines per inch. For a large number of slits, the diffraction pattern consists of extremely sharp (practically narrow lines) principal maxima, together with weak secondary maxima in between the principal maxima. The various principal maxima are called *orders*.
For polychromatic incident light falling normally on a plane transmission grating the principal maxima for each spectral wavelength are given by

$$(e+b)\sin\theta = \pm m\lambda \tag{1}$$

where m is the order of principal maximum and θ is the angle of diffraction.
The *angular dispersive power* of the grating is defined as the rate of change of angle of diffraction with the change in wavelength. It is obtained by differentiating equation (1) and is given by

$$\omega = \frac{d\theta}{d\lambda} = \frac{m}{(e+b)\cos\theta} \tag{2}$$

6 Observations and Results

1. Number of lines on grating, $N = 1200\,\mathrm{mm}^{-1}$.
2. Order of the principal maximum, $m = 1$.
3. From the table, $\lambda_1 = 581.414\,\mathrm{nm}$.

4. From the table, $\lambda_2 = 581.336\,\text{nm}$.
5. Thus, $\lambda_1 - \lambda_2 = 0.078\,\text{nm}$.
6. Angular dispersive power, $\dfrac{d\theta}{d\lambda} = \dfrac{\theta_1 - \theta_2}{\lambda_1 - \lambda_2} = 1.678\,205\,1 \times 10^{6}\,\text{rad}\,\text{m}^{-1}$.

Sodium Doublet	LHS Spectrometer Readings						RHS Spectrometer Readings						$2\theta_A$ (LHS Ver-I ~ RHS Ver-I) (°)	$2\theta_B$ (LHS Ver-II ~ RHS Ver-II) (°)	Average θ (°)	λ (nm)
	Vernier I			Vernier II			Vernier I			Vernier II						
	MSR (°)	VSR	Total (°)	MSR (°)	VSR	Total (°)	MSR (°)	VSR	Total (°)	MSR (°)	VSR	Total (°)				
D1	318.3	25	318.369	134.56	6	134.577	46.8	19	46.853	223.06	3	223.068	88.483	88.492	44.243	581.414
	318.3	29	318.380	134.56	13	134.596	46.8	22	46.861	223.06	8	223.082	88.481	88.486		
D2	318.3	52	318.444	134.56	29	134.640	46.8	43	46.919	223.06	22	223.121	88.475	88.481	44.235	581.336
	318.3	58	318.461	134.56	38	134.665	46.8	45	46.925	223.06	23	223.124	88.464	88.458		

Figure 4: The spectrometer readings for the experiment

7 Error Analysis

The values tabulates are in degrees so we need to first convert them into radians. From here, we have

- $\theta_1 = 0.772\,177\,29\,\text{rad}$
- $\theta_2 = 0.772\,046\,39\,\text{rad}$

The error analysis would be limited to the propagational errors. Now we have to find the error in λ_i's, for that, using equation (1), we have

$$\frac{\delta\lambda}{\lambda} = \sqrt{\left(\frac{\delta \sin\theta}{\sin\theta}\right)^2} \tag{3}$$

For this, first we need the error in $\sin\theta$, that is $\delta\sin\theta$. We know,

$$\delta \sin\theta_i = (\cos\theta_i) \cdot \delta\theta_i \tag{4}$$

Now $\delta\theta$ in (degrees) is provided by the vernier constants, that is, $\left(\dfrac{1}{360}\right)^{\circ}$. We convert this into radians to get

$$\delta\theta = 4.848\,136\,8 \times 10^{-5}\,\text{rad} \tag{5}$$

Therefore

$$\delta\lambda_1 = 581.414 \times 10^{-9} \times \sqrt{\left(\frac{\cos(0.77217729) \cdot (4.8481368 \times 10^{-5})}{\sin(0.77217729)}\right)^2} = 0.029\,\text{nm} \tag{6}$$

and

$$\delta\lambda_2 = 581.336 \times 10^{-9} \times \sqrt{\left(\frac{\cos(0.77204639) \cdot (4.8481368 \times 10^{-5})}{\sin(0.77204639)}\right)^2} = 0.029\,\mathrm{nm} \tag{7}$$

And, the uncertainty in $\lambda_1 - \lambda_2$ will be consequently the summation of uncertainty in λ_1 and λ_2, that is, 0.058 nm.
Now, for the error in angular dispersive power ω, we have from equation (2)

$$\delta\omega = \omega \times \sqrt{\left(\frac{\delta(\cos\theta)}{\cos\theta}\right)^2} \tag{8}$$

Note that here $\delta\cos\theta = -(\sin\theta)\cdot\delta\theta$. We will be taking average value of θ_1 and θ_2 for calculations. Putting in the values, we get

$$\delta\omega = 1.6782051 \times 10^6 \times \sqrt{\left(\frac{-0.69769980 \times 4.848136 \times 10^{-5}}{0.71643894}\right)^2} = 79.233\,574\,\mathrm{rad\,m^{-1}} \tag{9}$$

8 Results and Discussions

1. Using diffraction grating, the value of the first sodium D-line was found to be $\lambda_1 = (581.41 \pm 0.03)$ nm after taking care of significant figures.
2. Using diffraction grating, the value of the second sodium D-line was found to be $\lambda_2 = (581.34 \pm 0.03)$ nm after taking care of significant figures.
3. The difference in wavelengths of the sodium D-lines was found to be $\Delta\lambda = \lambda_1 - \lambda_2 = (78 \pm 58)$ pm, which is close to that of the literature value of 6 pm after taking care of significant figures.
4. The angular dispersive power was found to be $\omega = (1\,678\,200 \pm 80)\,\mathrm{rad\,m^{-1}}$ after taking care of significant figures.
5. One of the source of this error/ambiguity could be the way readings have been noted.
6. Another source of error for the obtained results could be the way observations have been made since we can not definitely say when a fringe has vanished.

9 Precautions

1. Once the collimator and the telescope are adjusted for parallel rays, their focusing should not be disturbed throughout the experiment.
2. Once the grating is properly adjusted on the turntable it should be locked.
3. While taking measurements at different positions of the telescope. It must always be in locked condition.
4. While rotating the telescope arm if the vernier crosses over 0° (360°) on the circular main scale take the angular difference appropriately.