Diffraction of Light

using Single and Double Slit

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1 Aim

- To determine the wavelength of laser light from single-slit diffraction pattern.
- To determine the thickness of a fine wire from its diffraction pattern.
- Compare the thickness of the wire with the single-slit width that form the same diffraction pattern as wire and hence verify the Babinet's principle.
- To understand the diffraction pattern due to double slit and determine the slit width and the opaque gap between the two slits.

2 Apparatus

1. Laser source (and safety goggles),
2. Screen and a ruled-paper for measurement,
3. A thin-wire,
4. Variable single-slit and double-slit,
5. Measuring tape,
6. A travelling microscope and a digital camera.

3 Introduction

Diffraction refers to various phenomena that occur when a wave encounters an obstacle or opening. It is defined as the bending of waves around the corners of an obstacle or through an aperture into the region of geometrical shadow of the obstacle/aperture. The diffracting object or aperture effectively becomes a secondary source of the propagating wave. When light passes through a small opening or around an edge, secondary waves from different portions of the emerging wavefront will, in general, travel different distances before reaching a screen. Although the waves from secondary sources are all in phase to start with, they will be out of phase by the time they reach the screen. The interference of these radiation emitted by secondary wavefront leads to the phenomenon of diffraction.
The diffraction phenomena are usually divided into two categories: ***Fresnel diffraction*** and ***Fraunhofer diffraction***. In the Fresnel class of diffraction the source of light and the screen are, in general, at a finite distance from the diffracting aperture (see figure 1 (a)). In the Fraunhofer class of diffraction, the source and the screen are at infinite distances from the aperture; this is easily achieved by placing the source on the focal plane of a convex lens and placing the screen on the focal plane of another convex lens (see figure 1 (b)).

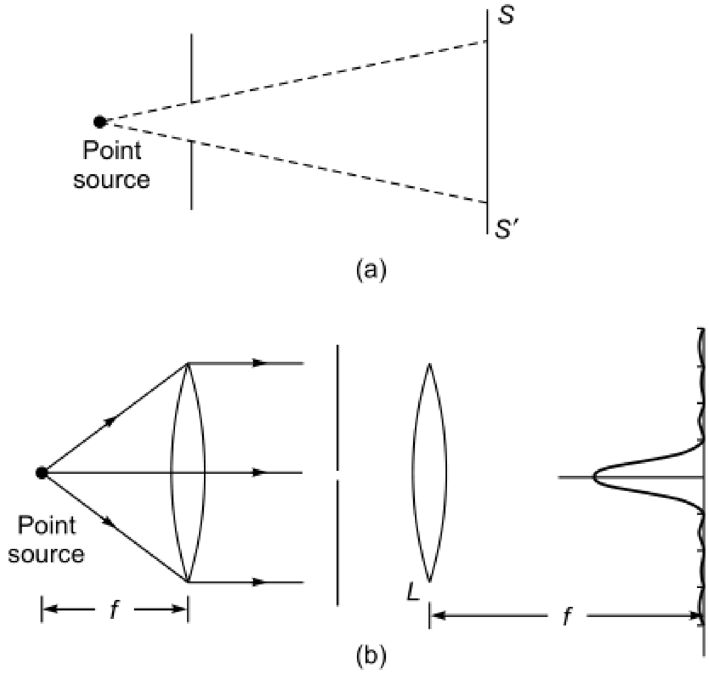

Figure 1: (a) When either the source or the screen (or both) is at a finite distance from the aperture, the diffraction pattern corresponds to the Fresnel class. (b) In the Fraunhofer class both the source and the screen are at infinity.

In this experiment, our study will be limited to the Fraunhofer class of diffraction.
The ***Babinet's principle*** states that the diffraction pattern from an opaque body is identical to that from a hole of the same size and shape except for the overall forward beam intensity. The fact that Fraunhofer diffraction pattern due to an obstacle is virtually identical to that of an opening of same dimension is an example of a general rule called Babinet's principle. This principle can be verified by replacing once again the wire with a single- slit and varying the slit-width until the pattern matches exactly. The slit width can then be compared with the wire thickness.

4 Schematics and Setup

The following figures present the outline diagrams and schematics of the the two modes of Fraunhofer diffraction we will be studying in this experiment, along with the experimental setup of the apparatus.

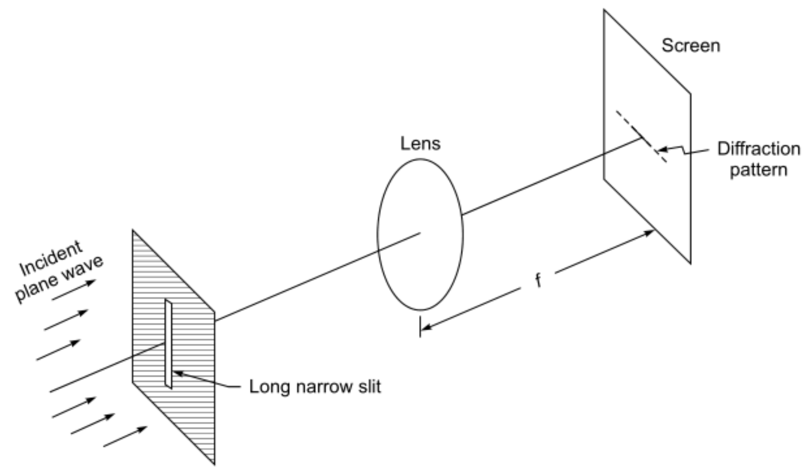

Figure 2: Diffraction of a plane wave incident normally on a long narrow slit of width b. Notice that the spreading occurs along the width of the slit.

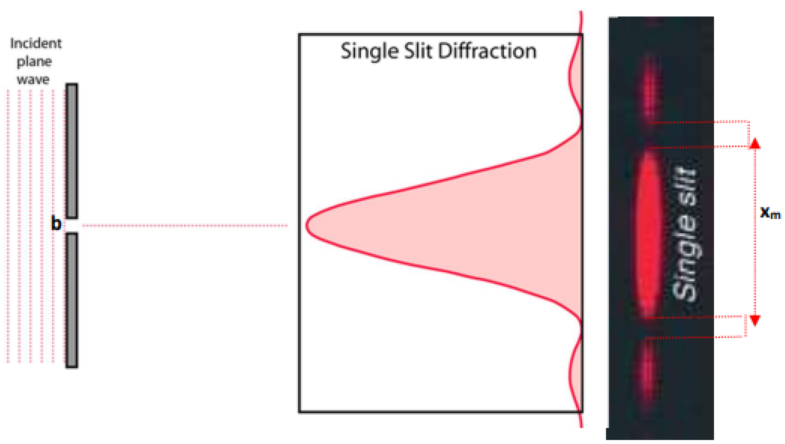

Figure 3: Schematics for single-slit diffraction. Distance between minima x_m is calculated from the average minima position on either side of principal maxima.

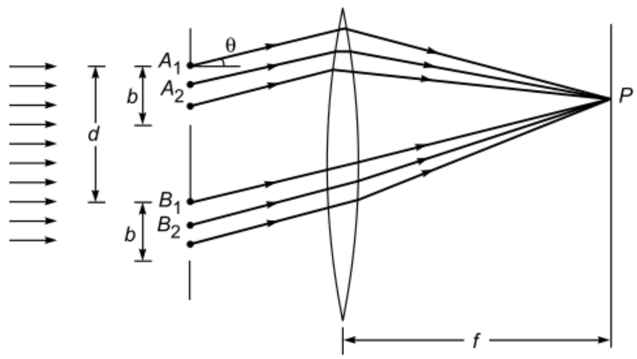

Figure 4: Fraunhofer diffraction of a plane wave incident normally on a double slit.

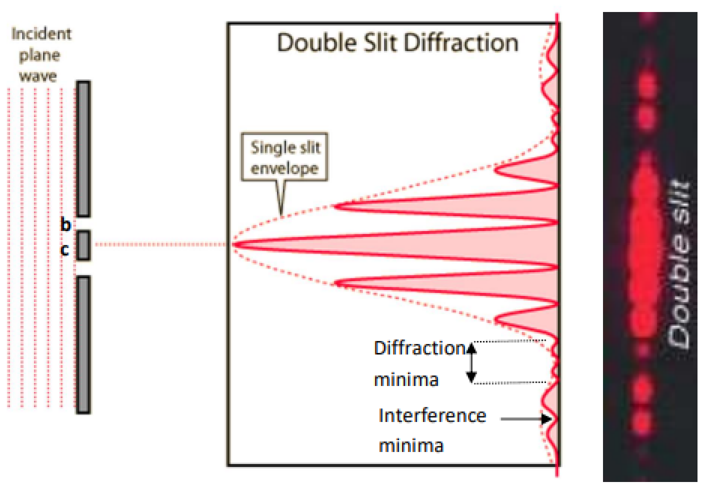

Figure 5: Schematics for Double-slit diffraction

Figure 6: Setup with Laser as the source

5 Theory

When a light of wavelength λ is incident normally on a narrow slit of width b as shown in figure (3), the resultant intensity of the transmitted light is given by,

$$I = I_0 \frac{\sin^2 \beta}{\beta^2} \tag{1}$$

with β,

$$\beta = \frac{\pi b \sin \theta}{\lambda}$$

Here, θ is the angle of diffraction. The diffraction pattern consists of a principal maximum for $\beta = 0$, where all the secondary wavelets arrive in phase, and several secondary maxima of diminishing intensity with equally spaced points of zero intensity at $\beta = m\pi$. The positions of the minima of a single-slit diffraction pattern are given by,

$$m\lambda = b \sin \theta \tag{2}$$

for $m = \pm 1, \pm 2, \pm 3 \ldots$.
If θ is small, i.e. the slit to screen distance D is large compared to the distance x_m between two m^{th} order minima (on either side of principal maximum), then

$$\sin \theta \approx \theta = \frac{x_m}{2D} \implies m\lambda = \frac{b x_m}{2D} \tag{3}$$

The above equation (3) can be used to determine the wavelength of the monochromatic light source, laser in this case, by measuring b, D and x_m for various m. The positions of the minima can be obtained by averaging the two extremities of the zero intensity region, as shown in figure (3). A real photographic image of the pattern is shown in figure (7).

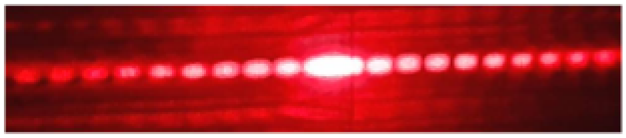

Figure 7: Photographic image of a single-slit diffraction pattern

If the single-slit is replaced by a thin wire obstacle, which blocks as much laser light as a single-slit will allow to pass, the resulting diffraction pattern will be identical to that of a single-slit. Knowing the wavelength λ of the laser light, the equation (3) can be used to determine the thickness of the wire b as,

$$b = \frac{2m\lambda D}{x_m} \tag{4}$$

A typical diffraction pattern of a wire obstacle is shown in figure (8). Here too, the positions of the minima are calculated by averaging the two ends of the spread of zero intensity regions as shown in figure (3).

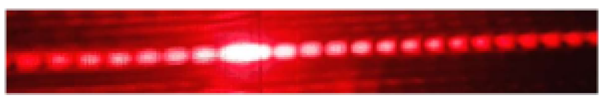

Figure 8: Photographic image of diffraction pattern from a thin wire–similarity with single-slit pattern is what Babinet's principle asserts.

If instead of single-slit, we have two parallel slits each of width b separated by an opaque space of width c (figure (5)), the corresponding intensity distribution of the Fraunhofer pattern formed is given by,

$$I - I_0 \frac{\sin^2 \beta^2}{\beta^2} \cos^2 \gamma \tag{5}$$

θ being the angle of diffraction. Also,

$$\beta = \frac{\pi b \sin\theta}{\lambda},\ \gamma = \frac{\pi d \sin\theta}{\lambda},\ d = b + c$$

The intensity distribution is a product of two terms: the first term $(\frac{\sin^2 \beta^2}{\beta^2})$ represents diffraction pattern produced by single-slit (1) and the second term $(\cos^2 \gamma)$ is the characteristic of interference produced by two beams of equal intensity and phase difference γ. The overall pattern, therefore, consists of single-slit diffraction fringes each broken into narrow maxima and minima of interference fringe.

The minima for the interference fringes are at $\gamma = (2p+1)\pi/2$ with $p = 0, 1, 2 \ldots$ and those for diffraction fringes are at $\beta = m\pi$ where $m = 1, 2, 3 \ldots$

The conditions for minima are,

$$d \sin\theta = (p + \frac{1}{2})\lambda \tag{6}$$

$$b \sin\theta = m\lambda \tag{7}$$

A photographic image of the double-slit Fraunhofer pattern obtained with laser beam is shown in figure (9).

Figure 9: Photographic image of double-slit diffraction pattern – each diffraction maxima is broken up into interference fringes

As it is evident from the figure (9) that the positions of interference and diffraction minima hardly show any spread at all, it is better to consider differences in positions between n consecutive minima, that is,

$$\Delta x_p = x_{p+n} - x_p \text{ and } \Delta x_m = x_{m+n} - x_m$$

Assuming as before, the distance D of the screen from the double-slit is large compared to x_p and x_m, we have,

$$\sin\theta \approx \theta = \frac{x_p}{D} \implies \boxed{d = \frac{n\lambda D}{\Delta x_p}} \tag{8}$$

$$\sin\theta \approx \theta = \frac{x_m}{D} \implies \boxed{b = \frac{n\lambda D}{\Delta x_m}} \tag{9}$$

6 Observations

1. Least count of vernier callipers = 0.001 cm.
2. Wavelength, $\lambda = 632.816\,\text{nm}$.
3. Slit screen distance, $D = 3.42\,\text{m}$.

Table 1: **Determination of b and c using travelling microscope**

Object	Left Edge M (cm)	Left Edge V	Left Edge T (cm)	Left Edge <T> α_l (cm)	Right Edge M (cm)	Right Edge V	Right Edge T (cm)	Right Edge <T> α_r (cm)	Width $(\alpha_l - \alpha_r)$ (cm)	<Width> (cm)
Slit - 1	6.10	8	6.108	6.109	6.05	9	6.059	6.062	0.047	$b = 0.048$
	6.10	10	6.110		6.05	15	6.065			
Slit - 2	6.05	5	6.055	6.057	6.00	2	6.002	6.009	0.048	
	6.05	9	6.059		6.00	15	6.015			
Slit - 1 + wire	6.10	8	6.108	6.109	6.05	5	6.055	6.057	0.052	$d = 0.050$
	6.10	10	6.110		6.05	9	6.059			
Slit - 2 + wire	6.05	5	6.055	6.057	6.00	2	6.002	6.009	0.048	
	6.05	9	6.059		6.00	15	6.015			

$$\boxed{\boldsymbol{c = d - b =} \; 0.002\,\mathrm{cm}}$$

Table 2: **Determination of d using diffraction pattern**

Interference Order $p+n$	**Left fringes** Δx_p (cm)	**Left fringes** d $(10^{-4}\ m)$	**Right fringes** Δx_p (cm)	**Right fringes** d $(10^{-4}\ m)$	$<d>$ $(10^{-4}\ m)$
$p+4$	2.1	4.12	2.1	4.12	4.15
$p+5$	2.7	4.01	2.6	4.16	
$p+6$	3.2	4.06	3.1	4.19	
$p+7$	3.7	4.09	3.6	4.21	
$p+8$	4.1	4.22	4.0	4.33	

Table 3: **Determination of b using diffraction pattern**

Diffraction Order $m+n$	**Left fringes** Δx_m (cm)	**Left fringes** b $(10^{-4}\ m)$	**Right fringes** Δx_m (cm)	**Right fringes** b $(10^{-4}\ m)$	$<b>$ $(10^{-4}\ m)$
$m+1$	0.6	3.61	0.6	3.61	3.47
$m+2$	1.3	3.33	1.2	3.61	
$m+3$	2.0	3.25	1.9	3.42	
$m+4$	2.6	3.33	2.5	3.46	
$m+5$	3.1	3.49	3.0	3.61	

$$\boxed{\boldsymbol{c = d - b =} \; 0.007\,\mathrm{cm}}$$

7 Error Analysis

The propagation error in a quantity $Q = a + b$ is given by

$$\delta Q = \sqrt{(\delta a)^2 + (\delta b)^2} \tag{10}$$

The standard deviation is given by,

$$\sigma = \sqrt{\frac{\Sigma(x_i - \mu)^2}{N - 1}} \tag{11}$$

7.1 Through travelling microscope

We have, error in average width (b and d) $\alpha_l - \alpha_r = 0.001\,\text{cm}$ using equation (10) twice. Again applying equation (10), gives error in c as 0.001 cm after rounding off to most significant digit.

7.2 Through diffraction pattern

Here, to calculate error, we will use standard deviation of the values of d and b. Using equation (10) and equation (11), error in c is 0.001 cm after rounding off to most significant digit.

8 Results and Discussions

1. Using the travelling microscope, the value of c is given by
   $\mathbf{0.002 \pm 0.001cm}$.

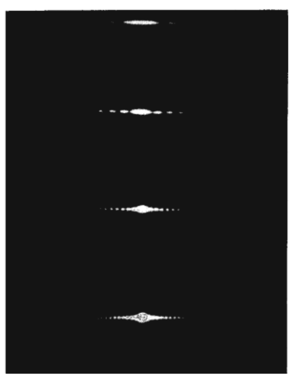

Figure 10: The single-slit diffraction patterns corresponding to $b = 0.0088$, 0.0176, 0.035, and 0.070 cm, respectively. The wavelength of the light used is 6.328×10^{-5} cm.

2. Using the diffraction pattern, the value of c is given by $\mathbf{0.007 \pm 0.001cm}$.

3. The value of c in case of travelling microscope is found to be close to value obtained through diffraction pattern.

4. The concept of diffraction is clearly understood through this experiment.

5. One of the source of this error/ambiguity could be the way readings have been noted.

6. Some images of diffraction patterns are shown in figures (10) and (11).

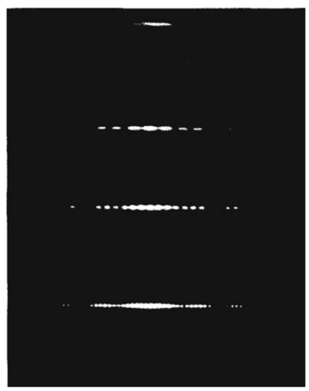

Figure 11: The double-slit Fraunhofer diffraction pattern corresponding to $b = 0.0088$ cm and $l = 6.328 \times 10^{-5}$ cm. The values of d are 0, 0.0176, 0.035, and 0.070 cm, respectively.

9 Precautions

1. The laser beam should not penetrate into eyes as this may damage the eyes permanently.

2. The laser should be operated at a constant voltage 220V obtainable from a stabilizer. This avoids the flickering of the laser beam.

3. Keep the laser turned off when not in use.

4. Do not move the laser around when it is on.

5. Never aim a laser at another person.

6. Avoid backlash errors.