Lecture 1

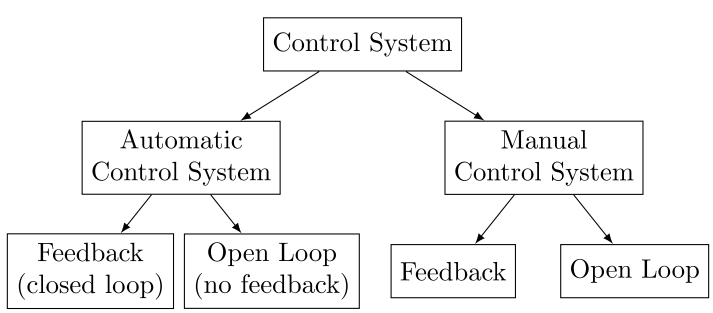
Introduction

The title of this course is Automatic Control of Engineering Systems. The key work here is "control". In general, to control something means to get the thing to behave in a certain fashion. From the engineering point of view, "control" is the process of causing one or more system variable(s) to conform to some desired value(s) called reference value(s).

Here, a system is a collection of things (by man or nature) that form an integral whole. For example, an automobile, building, airplane, dog, human being, plant, etc... The "variable" of interest would depend on the system.

- For a building:
  - temperature
  - humidity
  - etc...
- For an airplane:
  - temperature inside the cabin
  - speed of the plane
  - heading
  - etc...
- For an automobile:
  - speed control
  - yaw control
  - etc...

Control systems can be classified as follows



Definitions:

- **Automatic Control System:** completes desired tasks with minimal human intervention

- **Manual Control System:** Needs frequent intervention by humans

Examples: Automatic Control Systems:

- Washing machine
- Automatic lawn sprinkler
- Central heating/air conditioning
- Elevators
- etc...

Manual Control Systems:

- Driving a car
- Manual lawn sprinkler
- Lighting in a house
- Space heaters
- etc...

More definitions:

- **Feedback Control System:** Contains a measurement device (called a sensor) that monitors the variable(s) of interest. The value(s) of these variables are compared to desired values and the difference (if any) is used to “move” the control system in one direction or the other.
- **Open-Loop System:** Not equipped with any sensor, so, control action is not based on actual value of output.

Examples: Feedback Control Systems:

- Car/Airplane speed control
- Central heating/air conditioning
- etc...

Open-Loop Control Systems:

- Automatic lawn sprinkler
- Washing machine
- Clothes dryer
- etc...

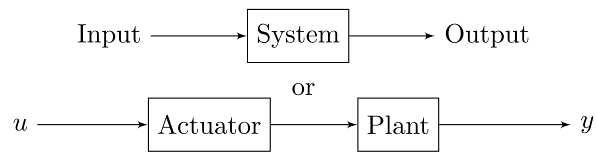
In summary, we have 4 types of control system:

1. Automatic Open-Loop Control System
2. Manual Open-Loop Control System
3. **Automatic Feedback Control System**
4. Manual Feedback Control System

Item 3 will be the focus of this course.

Representation of Control Systems

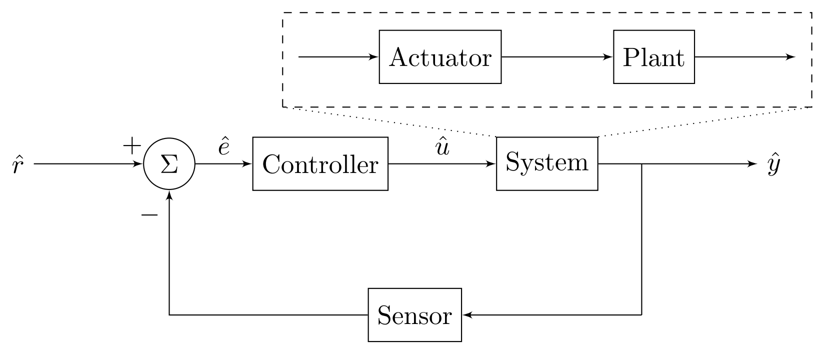
In the study of control systems, "block diagrams" are used a lot.



For example, the "plant" could be something like a car, while the actuator would be the engine or motor driving the car.

A simplified block diagram of a feedback (closed-loop) system might look as follows, where:

- r is the reference input
- e is the error signal
- u is the controlled input (control signal)
- y is the output



- A control system can have a single input and a single output (SISO)
- Or, it can have multiple inputs and outputs (MIMO)

Very often, it is possible to choose between open-looped and closed-loop design for a control system. In general,

- Open-loop systems are simple and low-cost
- Closed-loop systems are more complex and cost more (they involve measurements/sensors and correction/compensators)

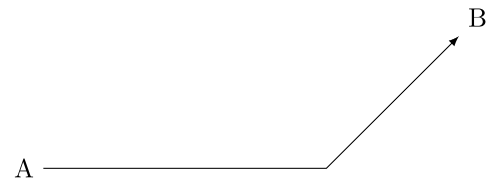
Why is feedback needed in some control systems?

The principal reason is **uncertainty**.

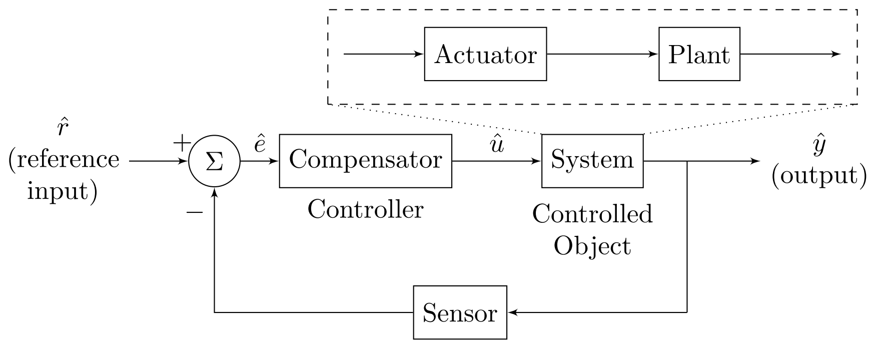
- Disturbances
- Uncertainty in the model of the controlled object

Example

Speed control for a vehicle that has to go from Point A to Point B, following the path shown



Open loop design would be easy (and sufficient) of there were not disturbances, or all disturbances were known exactly. In reality, this is not possible. Feedback control would be needed if we want to be accurate.

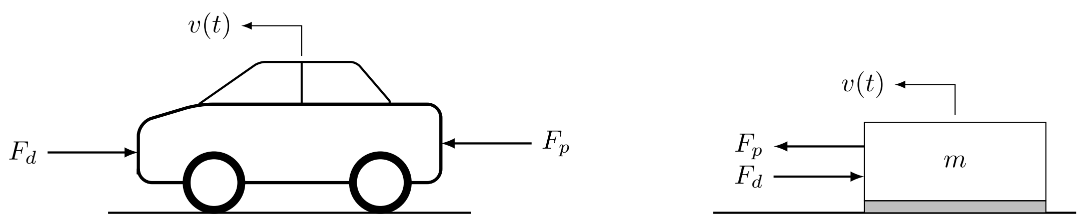


- The compensator/controller takes the place of the human — compensates for error.
- **The main objective of this course is the design of the compensator**.

Example

Cruise Control for a Car: Goals - maintain speed of a car at a prescribed value in the presence of external disturbances (**external forces such as wind gusts, gravitational forces on an incline, etc...**). Also - improve the dynamic response of the car as the driver "steps on the gas".

(a) Form a dynamic model of the car:



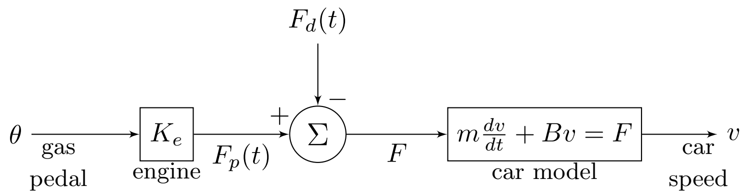
- We are going to model the car as a simple lumped mass element m, sliding on a viscous friction element $F_B = Bv$ as a simplification of rolling resistance forces.
- Next, we assume two external forces acting on the car:
  - $F_p(t)$ - the propulsive force from the engine
  - $F_d(t)$ - the disturbance force from the environment

  Also, assume $F_p(t) = K_e\theta(t)$ where $\theta(t)$ is the gas-pedal depression and K_e is a constant. Then from a simple force-balance:

$$m\frac{dv}{dt} + Bv = F_p(t) - F_d(t)$$

$$m\frac{dv}{dt} + Bv = K_e\theta(t) - F_d(t)$$

This has the block diagram:



(b) Closed-loop Control: Now design a controller. Assume we will use error-based control. In other words, given a desired speed v_d, and the measured car speed $v(t)$, we define the error $e(t)$ as

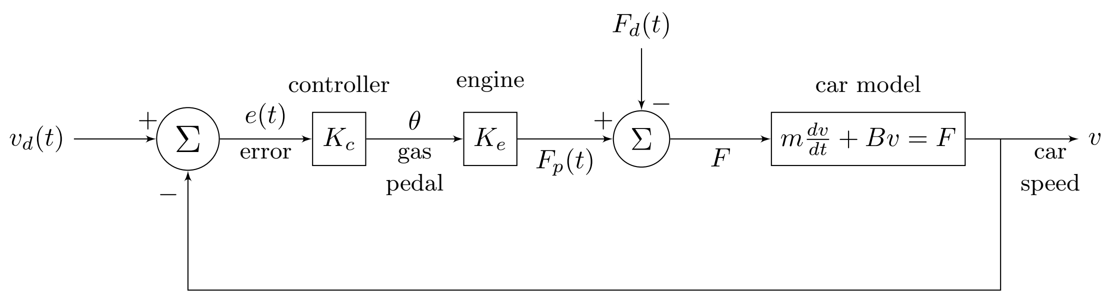
$$e(t) = v_d(t) - v(t)$$

and choose a control law that tells us to depress the gas pedal by an amount proportional to the error:

$$\theta(t) = K_c e(t) = K_c(v_d(t) - v(t)),$$

so that the propulsive force acting on the car is

$$F_p(t) = K_e\theta(t) = K_c K_e(v_d(t) - v(t))$$



This is known as ***proportional control***.

Compensator Design

The Big Picture:

- We have an object (or system/plant) to control, and we are told what the system must be able to do.
- Principal steps in the controller design process:
  1. We obtain a math model for the object or system to be controlled. In other words, a mathematical relationship between the input to the system and the output from the system. We need to understand the "dynamics" of the system.
  2. Use knowledge of control theory to come up with a mathematical representation of the controller or compensator (i.e. math relationship between the input to the controller and its output).
  3. Build a physical device that has the input-output relationship developed in (2) above. For example, a microcontroller to implement our control system.
- In this course, we concern ourselves with Step 2. Pencil and paper design, using mostly mathematics.

Once again, the goal for this course is to learn how to design compenators. This is done in 3 phases:

1. Learn how to characterize dynamic systems (review ENG 102, EME 171, etc...)
2. Learn feedback principles (principles that govern feedback systems)
3. Design methods

Characterization of Dynamic Systems

The input and output of a dynamic system are usually related by one or more differential equations. These equations can be obtained in a variety of ways.

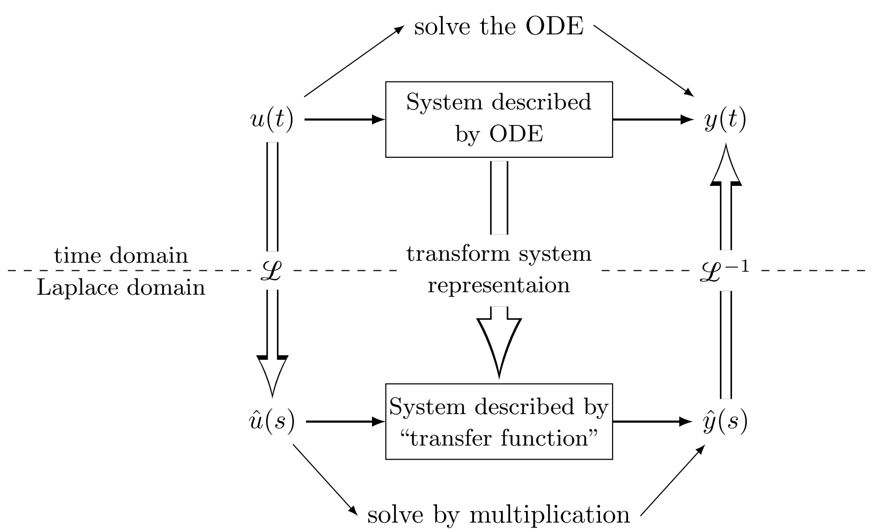
- Newton's laws ($F = ma$) (mechanical systems)
- Lagrange Method
- Kirchoff's voltage/current laws (electrical systems)
- Bond graphs
- etc...

To do control work, we can (a) choose to work directly with such differential equations or (b) we can transform them first into algebraic equations using the Laplace Transform before proceeding with control work. In Case (a) we speak of the time-domain analysis (state-space approach), called "modern control". In Case (b) we speak of frequency-domain analysis (such as Bode or Nyquist plots), called "classical control". The main mathematical tool used in the second approach is the Laplace transform. We will thus start with a review of the Laplace transform.

Review of the Laplace Transform

The Laplace transform converts a function of time $f(t)$ into a function of a different variable s, referred to as the Laplace variable. The reason this is useful is that it allows manipulation of ordinary differential equations (ODEs).

1. The solution to ODEs is "difficult", so
2. Transform the problem into a "domain" where the solution is easier.
3. Solve the problem in the new domain.
4. Perform the "inverse" transform to move the solution back to the original domain.



- The main merit of the Laplace transform is that it converts differential equations to algebraic problems of polynomials and rational functions (ratios of polynomials).
- The main drawback is that the Laplace transform only works with linear, time-invariant (LTI) ODEs.

We are going to have to limit ourselves to the study of control systems that can be described by LTI ODEs. For example:

$$a\ddot{x} + b\dot{x} + cx = 0$$

$$\alpha\dot{y} + \beta y = \delta \sin \omega t$$

where a, b, c, α, β, δ, ω are all constants.

Definition

Let $f(t)$ be a function of time. The Laplace transform of $f(t)$ is defined as

$$\mathscr{L}[f(t)] \triangleq \int_{0^-}^{\infty} e^{-st} f(t) dt = F(s)$$

where the integral exists (that is, has some finite value) and where 0^- is slightly before 0. The Laplace transform converts known functions of time to functions of s. It turns out that it makes things easier to take the variable s to be complex:

$$s = \sigma + j\omega$$

where $j = \sqrt{-1}$. This is because the roots of a polynomial may be complex, and because we will later be interested in the frequency response of systems.

Let's clarify our terminology:

- $f(0^-)$: initial conditions
- $f(0^+)$: initial value

Sometimes $f(0^-) \neq f(0^+)$. For example, the unit step function:

$$1(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

The step function if a mathematical idealization of a true physical signal, i.e. switches.

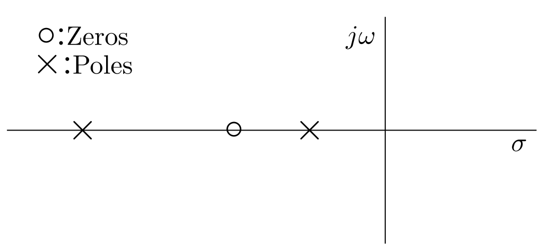
Example

The unit step.

$$1(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$\mathscr{L}[1(t)] = \int_{0^-}^{\infty} e^{-st} 1(t) dt = \int_{0^-}^{\infty} e^{-st} dt = -\frac{1}{s} e^{-st}|_0^{\infty} = -\frac{1}{s}(0-1) = \frac{1}{s}$$

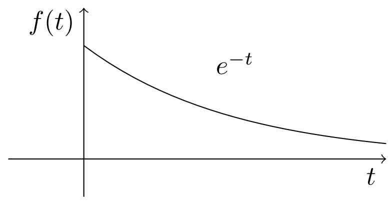
We would like to be able to plot $F(s)$ just as we plot $f(t)$. But, s is complex! So, we have to think of a way of drawing a picture of $F(s)$ that is different from the usual way of plotting $f(t)$. What is usually done is to plot where $F(s)$ goes to infinity (poles) and where it goes to zero (zeros). This is called a pole-zero diagram. For example, a system with two poles and one zero might look like this:



When does $F(s) = 1/s$ for to zero? There is no **finite** zero. Obviously, when $s \to \infty$, $F(s) \to 0$. So, there is a zero at infinity. But we don't worry about that in this course.

Next, let's clarify some issues concerning the Laplace Transform in general.

- Recall the definition of $F(s) = \int_{0^-}^{\infty} e^{-st} f(t) dt$
- $F(s)$ has no information about $f(t)$ before $t = 0$. Only $f(t)$ for $t \geq 0$ is encoded in $F(s)$. This is because of how $F(s)$ is defined.
- Functions with such restrictions are often represented as “causal” functions. For example:



We can write such a function in one of 2 ways:

$$f(t) = \begin{cases} e^{-t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

or more simply

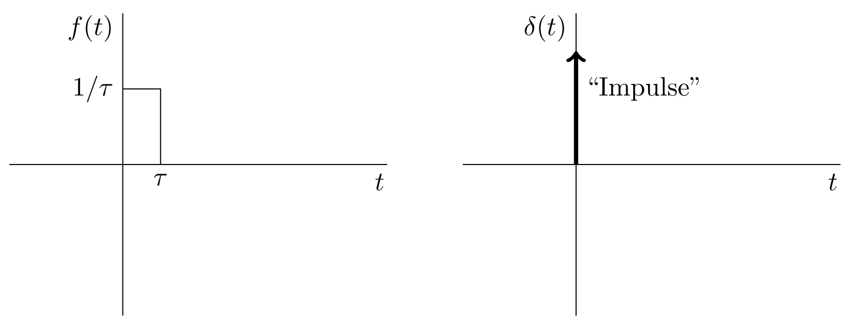
$$f(t) = e^{-t} 1(t)$$

The initial condition is always zero for a causal function. So, we can talk of

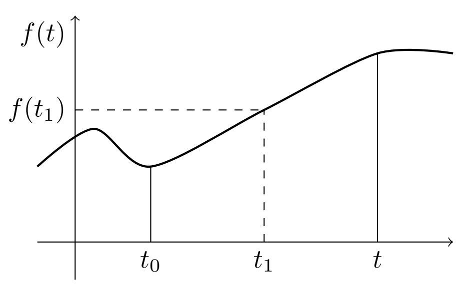
- causal exponential (drawn above)
- causal ramp: $f(t) = at \cdot 1(t)$ where a is a constant
- causal sine: $f(t) = a \sin(\omega t) 1(t)$ where a is a constant
- causal cosine: $f(t) = a \cos(\omega t) 1(t)$ where a is a constant
- etc...

From now on, we will only take the LT of causal functions.

One other item if interest is the **Dirac Delta Function** $\delta(t)$. Consider the rectangle shown.



It’s area is one. The delta function $\delta(t)$ can be viewed as the area of this rectangle as $\tau \to 0$. For a function $f(t)$ as shown



It can be shown that

$$\int_{t_0}^{t} f(t)\delta(t-t_1)dt = f(t_1)$$

This is called the shifting property of the delta function (from calculus). Then,

$$\mathscr{L}[\delta(t)] = \int_{0^-}^{\infty} e^{-st}\delta(t)dt = e^{-s\cdot 0} = 1$$

Thus,

$$\Rightarrow \quad \mathscr{L}[\delta(t)] = 1$$

Some properties of the Laplace Transformation:

If $\mathscr{L}[f(t)] = F(s)$ and $\mathscr{L}[g(t)] = G(s)$ then

1. $\mathscr{L}[e^{-at}f(t)] = F(s+a)$ where a is constant (frequency shifting)
2. $\mathscr{L}[tf(t)] = -\frac{d}{ds}F(s)$
3. $\mathscr{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}F(s)$
4. $\mathscr{L}[af(t) + bg(t)] = aF(s) + bG(s)$ where a and b are constant (linearity)
5. $\mathscr{L}[\frac{d}{dt}f(t)] = sF(s) - f(0^-)$
6. $\mathscr{L}[\frac{d^2}{dt^2}f(t)] = s^2F(s) - sf(0^-) - \frac{df}{dt}(0^-)$
7. $\mathscr{L}[\frac{d^n}{dt^n}f(t)] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}\frac{d^{k-1}f}{dt^{k-1}}(0^-)$
8. $\mathscr{L}[\int_{0^-}^{t} f(\tau)d\tau] = \frac{F(s)}{s}$
9. $\mathscr{L}[f(t-\tau)] = e^{-\tau s}F(s)$ (time shifting)

Proof of the theorems

Show that $\mathscr{L}[\frac{d}{dt}f(t)] = sF(s) - f(0^-)$:

$$\mathscr{L}[\frac{d}{dt}f(t)] = \int_{0^-}^{\infty} e^{-st}\frac{d}{dt}f(t)dt$$

We use integration by parts: $u = e^{-st}$, $du = -se^{-st}$, $v = f(t)$, $dv = \frac{df}{dt}(t)dt$. Recall that for integration by parts,

$$d(uv) = du \cdot v + u \cdot dv$$
$$udv = d(uv) - vdu$$
$$\int_{c_1}^{c_2} udv = uv|_{c_1}^{c_2} - \int_{c_1}^{c_2} vdu$$

So,

$$\begin{aligned}
\int_{0^-}^{\infty} e^{-st}\frac{d}{dt}f(t)dt &= e^{-st}f(t)\Big|_{0^-}^{\infty} - \int_{0^-}^{\infty} -se^{-st}f(t)dt \\
&= 0 - f(0^-) + sF(s) \\
&= sF(s) - f(0^-)
\end{aligned}$$

Armed with the result $\mathscr{L}[1(t)] = 1/s$ and together with these theorems, we are able to determine the Laplace transform of practically all the functions encountered in the study of LTI systems.

Example

$$f(t) = 1 - 2t$$

Or more "formally":

$$f(t) = 1(t) - 2t \cdot 1(t)$$

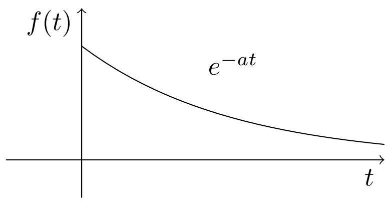
Find $F(s)$:

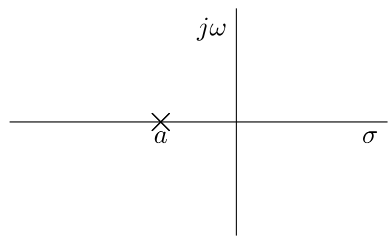
$$F(s) = \frac{1}{s} - 2\left[-\frac{d}{ds}\frac{1}{s}\right] = \frac{1}{s} - 2\frac{1}{s^2}$$

$$F(s) = \frac{s-2}{s^2}$$

Example

$$\mathscr{L}[e^{-at}1(t)] = \frac{1}{s+a}$$





Example

$$\mathscr{L}[t1(t)] = \frac{1}{s^2}$$