Lecture 8

Last time: Reviewed Stability

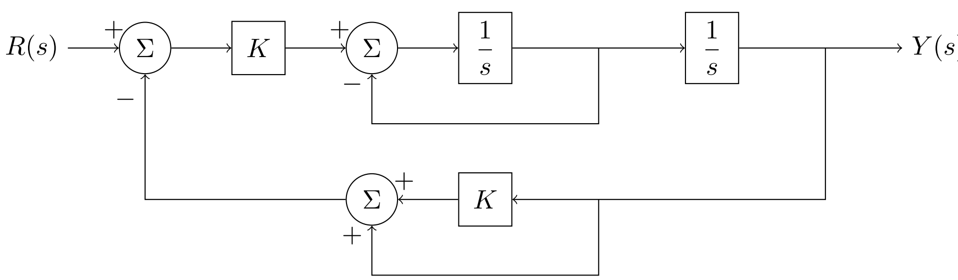
- Stability requirements
- Routh-Hurwitz Criterion

Last lecture, we mentioned that we have not yet addressed the issue of accuracy of the final value of a system's output, in other words how close a system's final value is to the desired value. This lecture we will learn about block diagram algebra, which is a tool we will use to assess steady-state error in the next lecture.

Block Diagram Reduction

Block Diagrams are used quite a bit in the study of control systems — mainly in their representation. Sometimes, we are interested in reducing a relatively complex block diagram system to a simpler form.

Example



How can we reduce this to

$$R(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s)$$

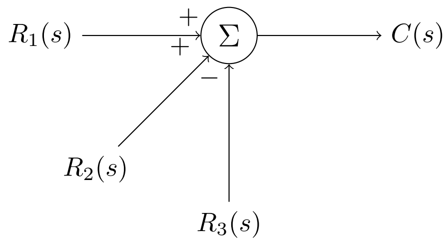
How do we find $G(s)$?

Basic Elements

First, let's consider two basic elements that occur when several subsystems are connected together:

- The summing junction
- The pick-off point

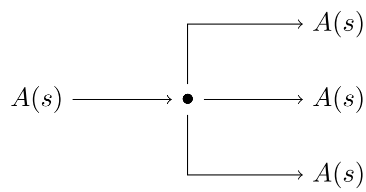
The summing junction



$C(s)$ is the algebraic sum of all the signals going into the junction.

$$C(s) = R_1(s) + R_2(s) - R_3(s)$$

The pick-off point

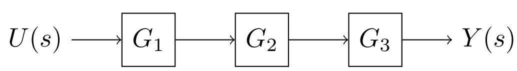


The pickoff point distributes the input signal undiminished to all its output points.

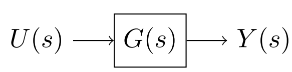
Common Interconnection Topologies

Now, lets look at some common topologies, and solve them using our knowledge of the basic elements.

1. The Cascade Form

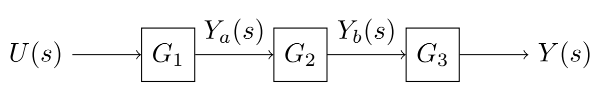


We have seen that, in the Laplace domain, the output of a block equals the input times the transfer function.



$$Y(s) = G(s)U(s)$$

If we have multiple blocks **in cascade**,



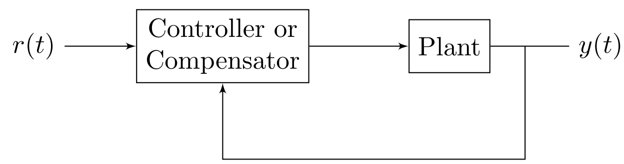
So, we have

$$\begin{aligned} Y(s) &= G_3(s)Y_b(s) \\ &= G_3(s)G_2(s)Y_a(s) \\ &= G_3(s)G_2(s)G_1(s)U(s) \end{aligned}$$

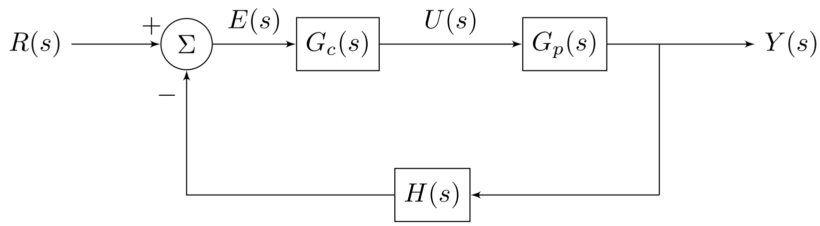
$$G(s) = G_1G_2G_3$$

2. The Feedback Form

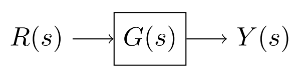
We have seen that a typical feedback control system has the schematic form



In block diagram form (in the Laplace domain), we have:



and we want:



where $G(s)$ is such that $Y(s) = G(s)U(s)$. From the block diagram:

$$
\begin{aligned}
Y(s) &= G_p(s)U(s) \\
U(s) &= G_c(s)E(s) \\
E(s) &= R(s) - H(s)Y(s)
\end{aligned}
$$

So,

$$Y(s) = G_pU(s) = G_pG_cE(s) = G_pG_c(R(s) - HY(s))$$

or,

$$Y(s) = G_pG_cR(s) - G_pG_cHY(s)$$

then,

$$(1 + G_cG_pH)Y(s) = G_cG_pR(s)$$

$$\boxed{\frac{Y}{R}(s) = \frac{G_cG_p}{1 + G_cG_pH} = G(s)}$$

We define $L(s)$ as

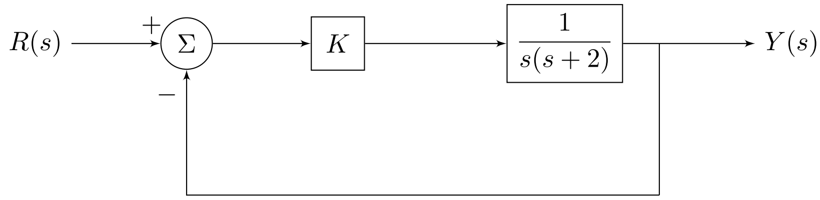
$$L(s) = G_cG_pH$$

called the return ratio, loop gain, loop transmission, or open-loop transfer function. In general:

$$\frac{output}{input} = \frac{direct\ path}{1 + return\ ratio}$$

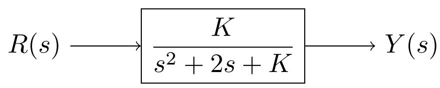
This is true for simple closed-loop systems with negative feedback.

Example

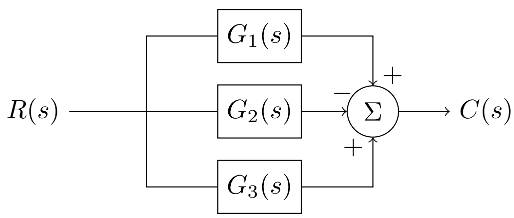


$$\frac{Y}{R}(s) = \frac{K\dfrac{1}{s(s+2)}}{1+K\dfrac{1}{s(s+2)}} = \frac{K}{s(s+2)+K}$$

$$\frac{Y}{R}(s) = \frac{K}{s^2+2s+K}$$



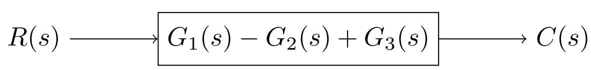
3. The Parallel Form



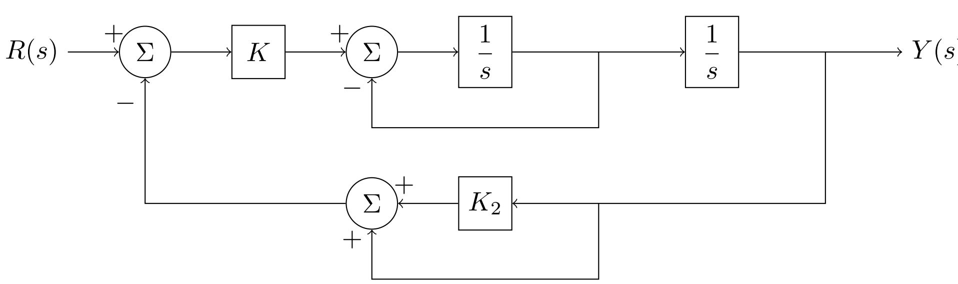
Using our knowledge of pickoff points and summing junctions,

$$C(s) = G_1(s)R(s) - G_2(s)R(s) + G_3(s)R(s)$$

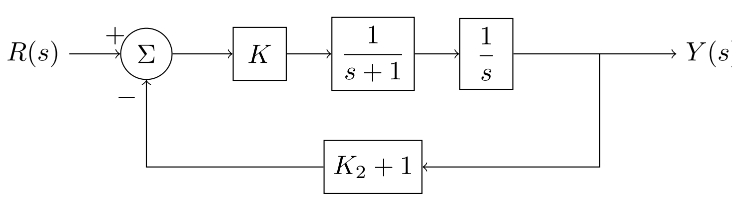
$$C(s) = \left(G_1(s) - G_2(s) + G_3(s)\right) R(s)$$



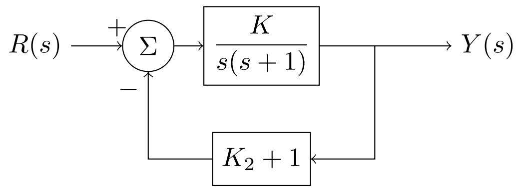
Example



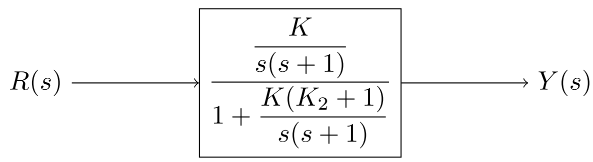
Reduce the inner feedback loop and bottom parallel form.



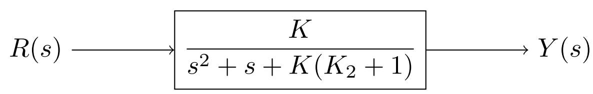
Reduce the cascading form.



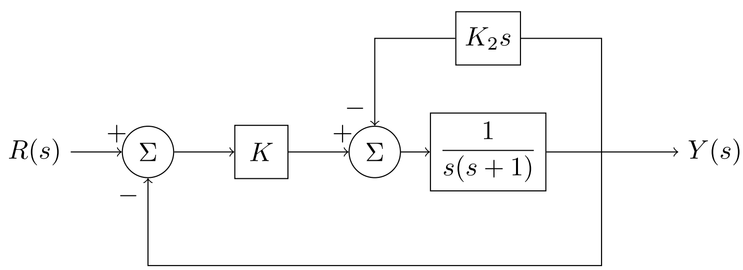
Condense the feedback loop.



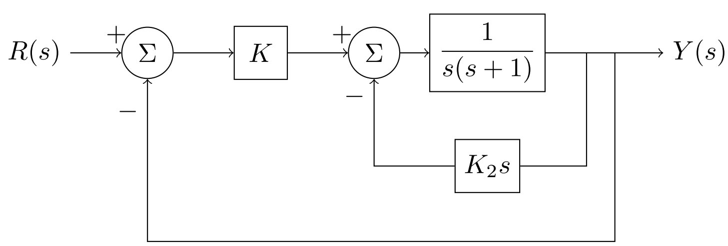
Simplify the transfer function.



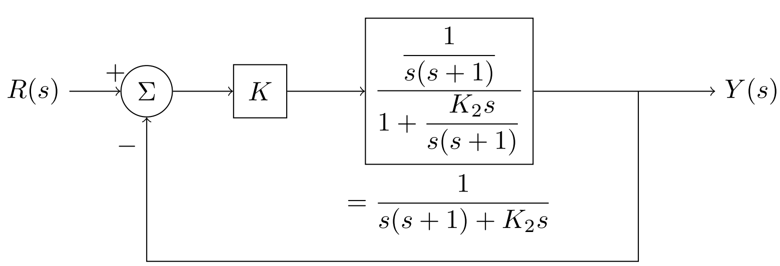
Example



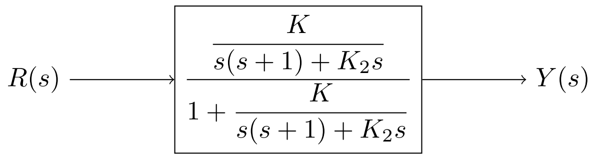
Rearrange (split the pickoff point apart).



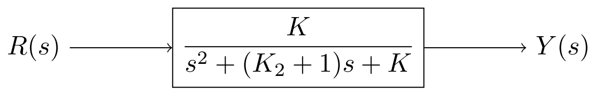
Condense the feedback loop.



Combine the two cascading blocks, and then reduce the feedback loop.



Simplify the transfer function

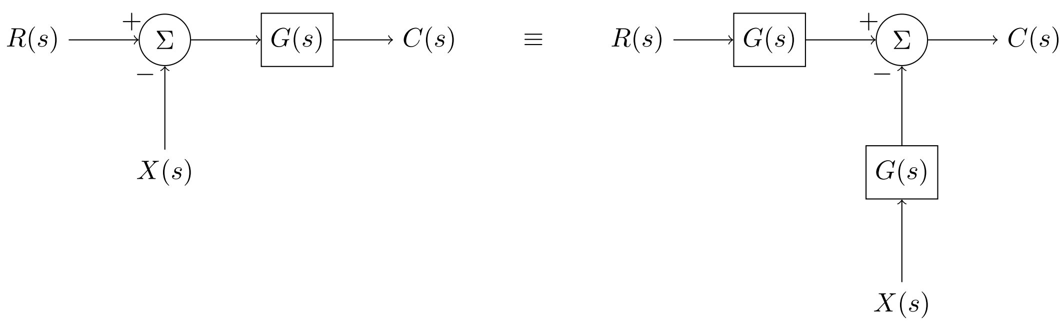


Block Diagram Manipulation

Let's go over a few ways to move blocks around in a block diagram.

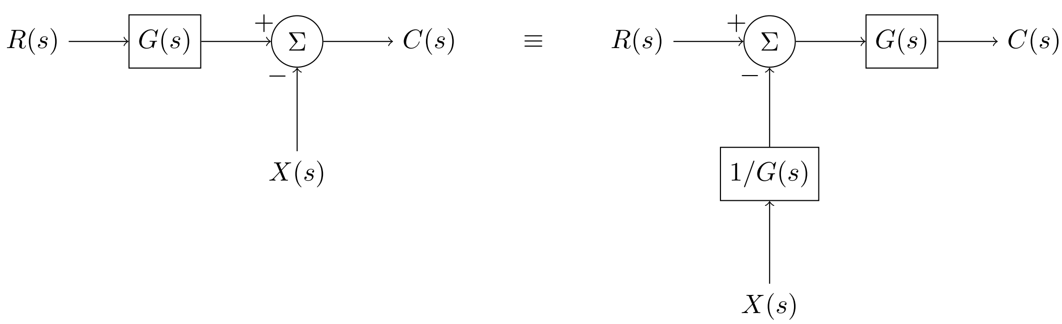
Moving a block across a summing junction

Moving backwards:



$$C(s) = (R(s) - X(s))G(s) \qquad \Leftrightarrow \qquad C(s) = R(s)G(s) - X(s)G(s)$$

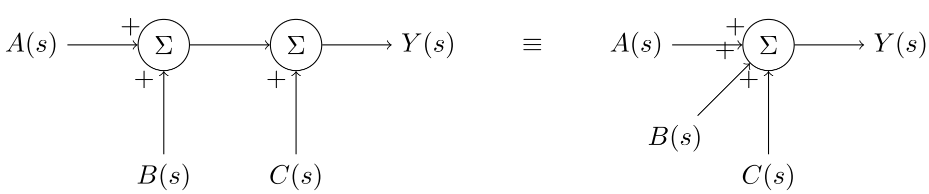
Moving forwards:



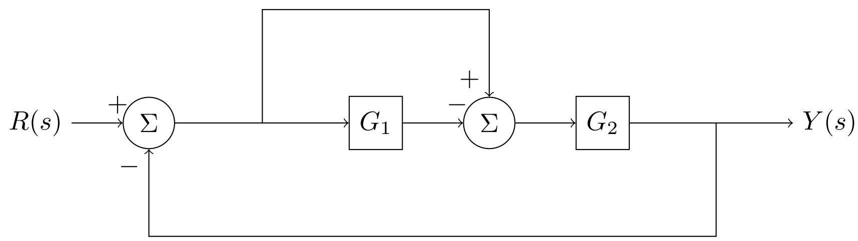
$$C(s) = R(s)G(s) - X(s) \qquad \Leftrightarrow \qquad C(s) = \left(R(s) - \frac{X(s)}{G(s)}\right)G(s)$$

Combining summing junctions

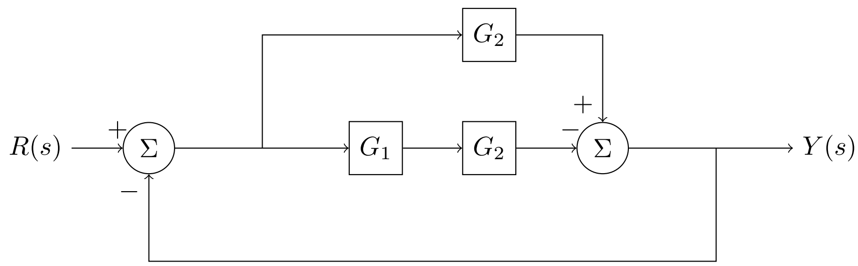
Summing junctions can be combined together as follows:



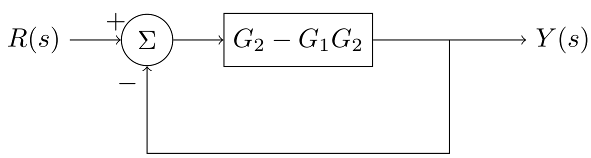
Example



Move G_2 behind the summing junction.



Combine the parallel form:



Reduce the feedback loop.