Lecture Outline

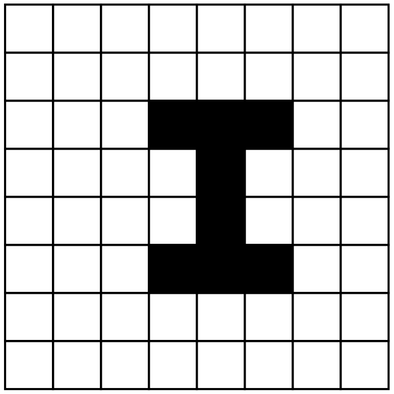
- Median filter
- Rank order filter
- Bilevel Morphological filters
  - Dilation and erosion
  - Opening and closing
- Grayscale Morphological filters

Morphological Processing

- Morphological operations are originally developed for bilevel images for shape and structural manipulations.
- Basic functions are *dilation* and *erosion*.
- Concatenation of dilation and erosion in different orders result in more high level operations, including *closing* and *opening*.
- Morphological operations can be used for smoothing or edge detection or extraction of other features.
- Belongs to the category of spatial domain filter.

Morphological Filters for Bilevel Images

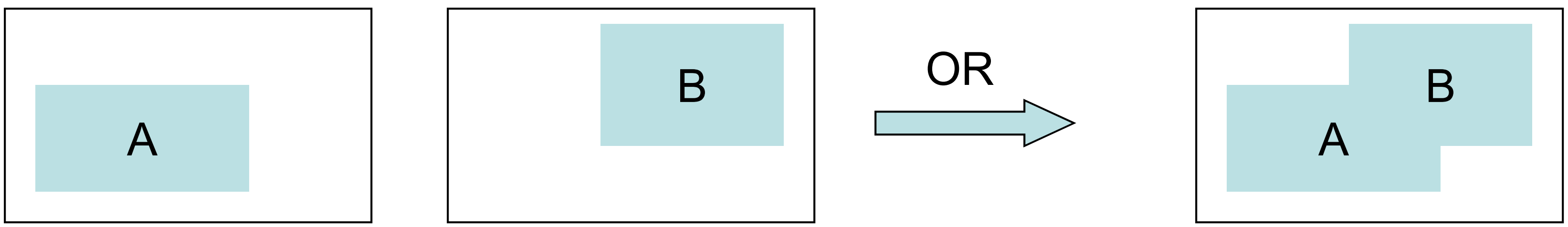
- A binary image can be considered as a set by considering “black” pixels (with image value “1”) as elements in the set and “white” pixels (with value “0”) as outside the set.



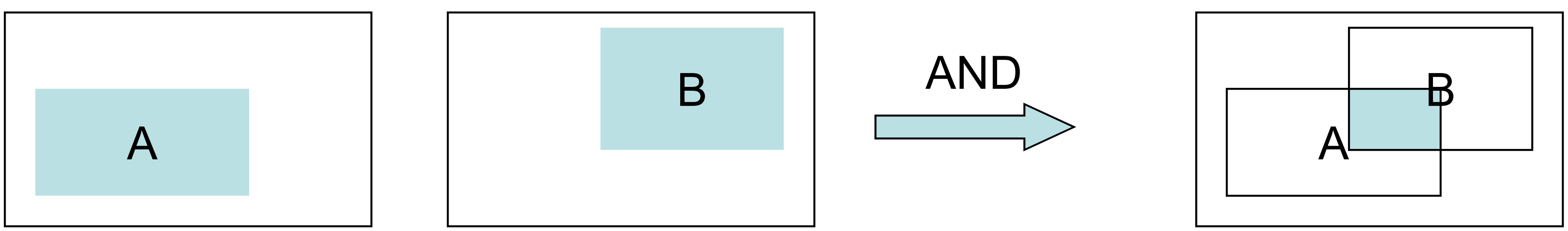
- Morphological filters are essentially set operations

Basic Set Operations

- Let x, y, z, … represent locations of 2D pixels, e.g. $x = (x_1, x_2)$, S denote the complete set of all pixels in an image, let A, B, … represent subsets of S.
- Union (OR) $A \cup B = \{x : x \in A \; or \; x \in B\}$



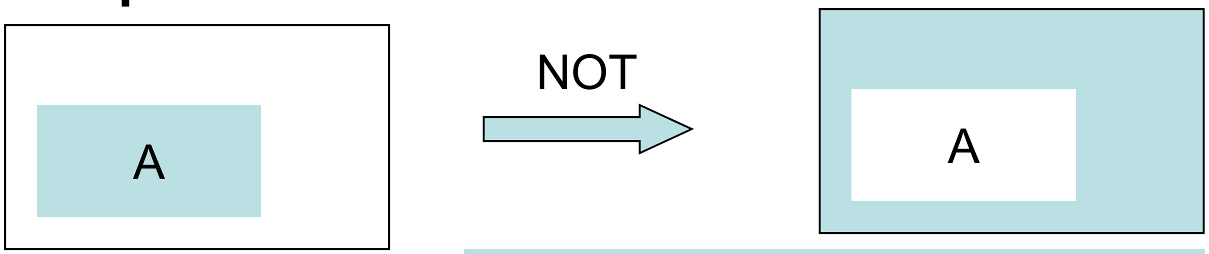
- Intersection (AND) $A \cap B = \{x : x \in A \; and \; x \in B\}$



Basic Set Operations

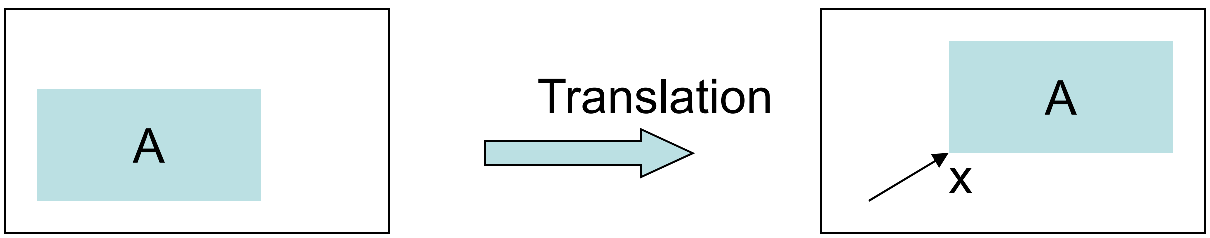
- Complement

$$\bar{A} = \{x : x \in S \ and \ x \notin A\}$$



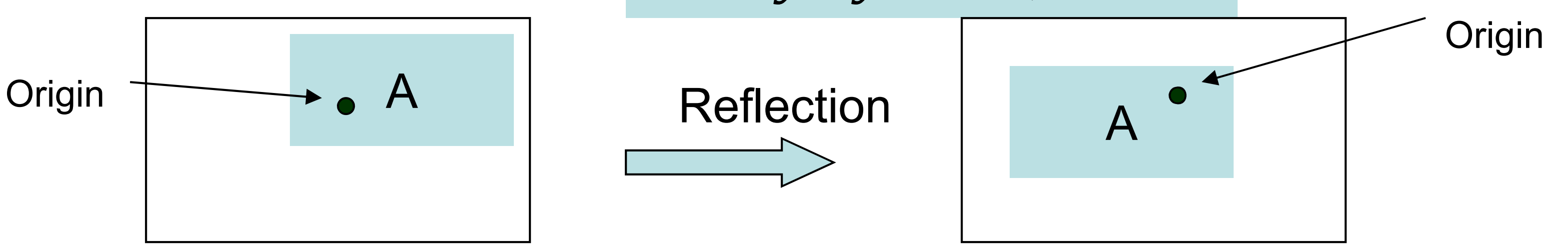
- Translation

$$(A)_x = \{z : z = y + x,\ y \in A\}$$



- Reflection

$$\hat{A} = \{y : y = -x,\ x \in A\}$$



Dilation

- Dilation of set F with a structuring element H is represented by $F \oplus H$

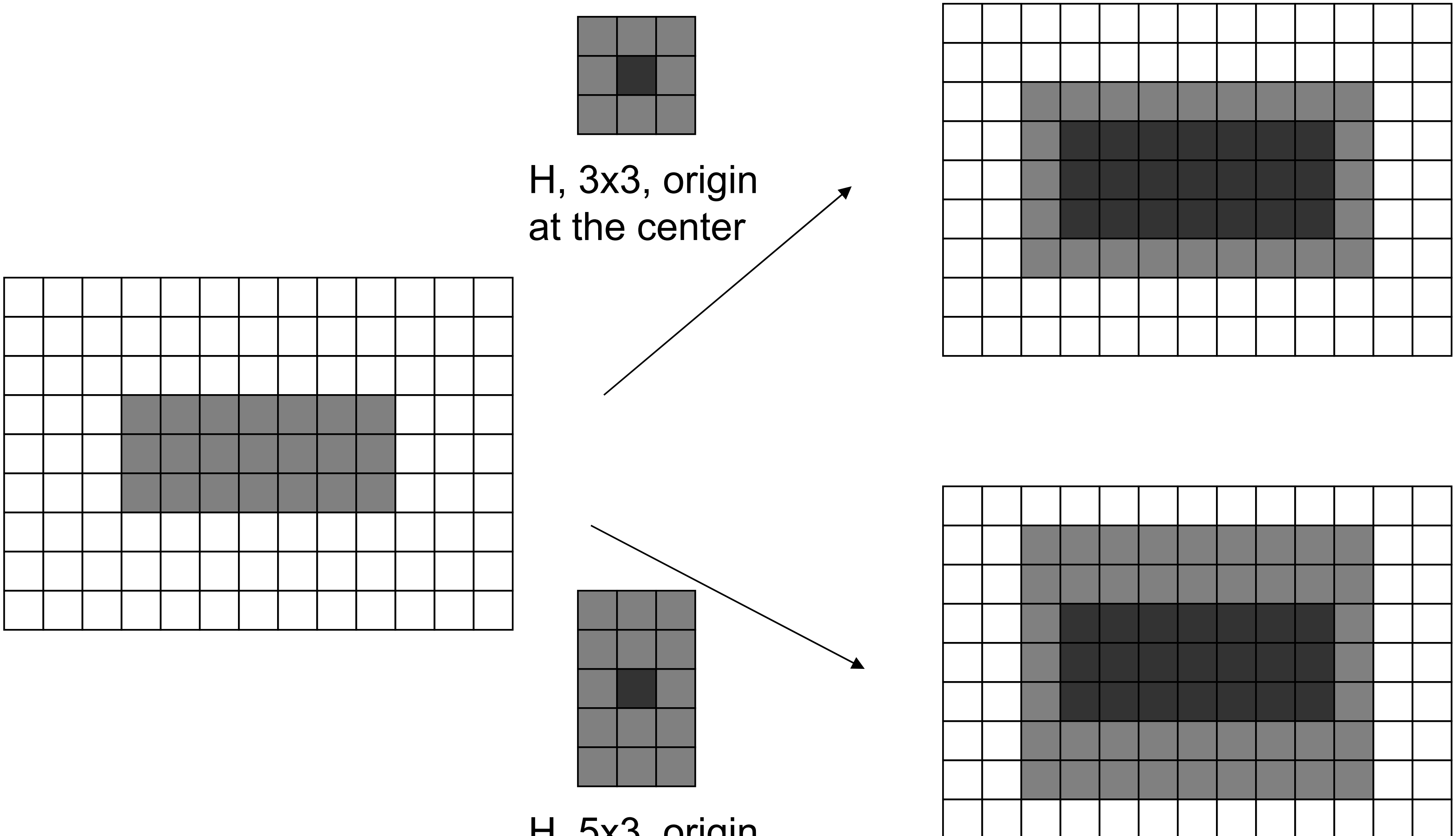
$$F \oplus H = \{x : (\hat{H})_x \cap F \neq \Phi\}$$

  where Φ represent the empty set.

- $G = F \oplus H$ is composed of all the points that when Ĥ shifts its origin to these points, at least one point of Ĥ is included in F.
- If the origin of H takes value "1",

$$F \subset F \oplus H$$

Example of Dilation (1)



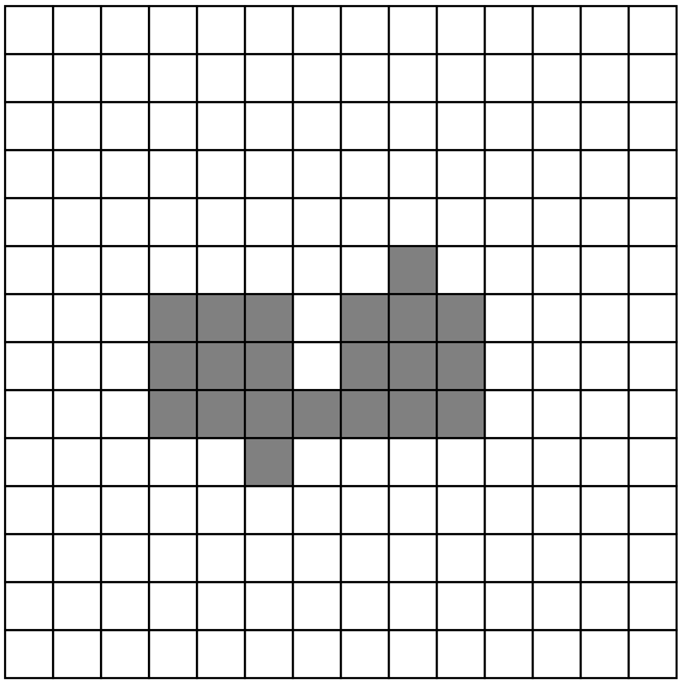
H, 3x3, origin at the center

H, 5x3, origin at the center

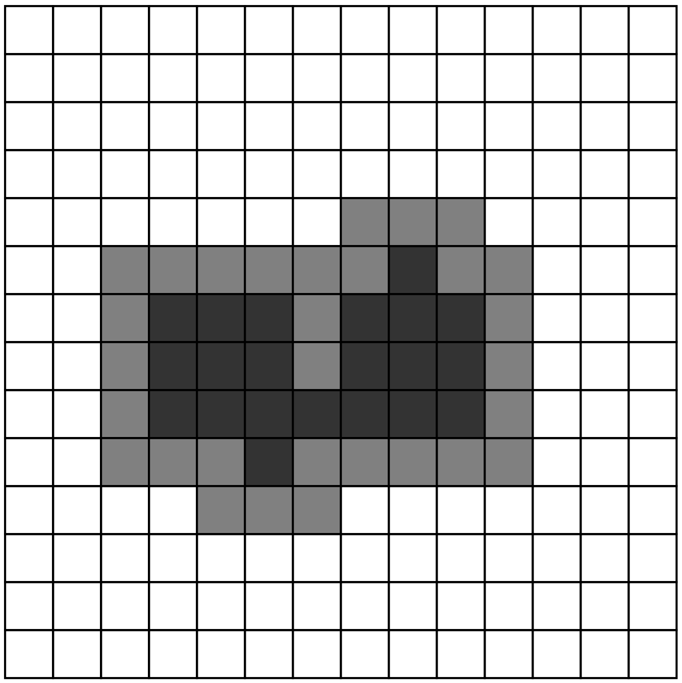
Dilation enlarges a set.

Example of Dilation (2)

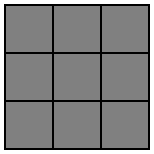
Note that the narrow ridge is closed



F



G



H, 3x3, origin at the center

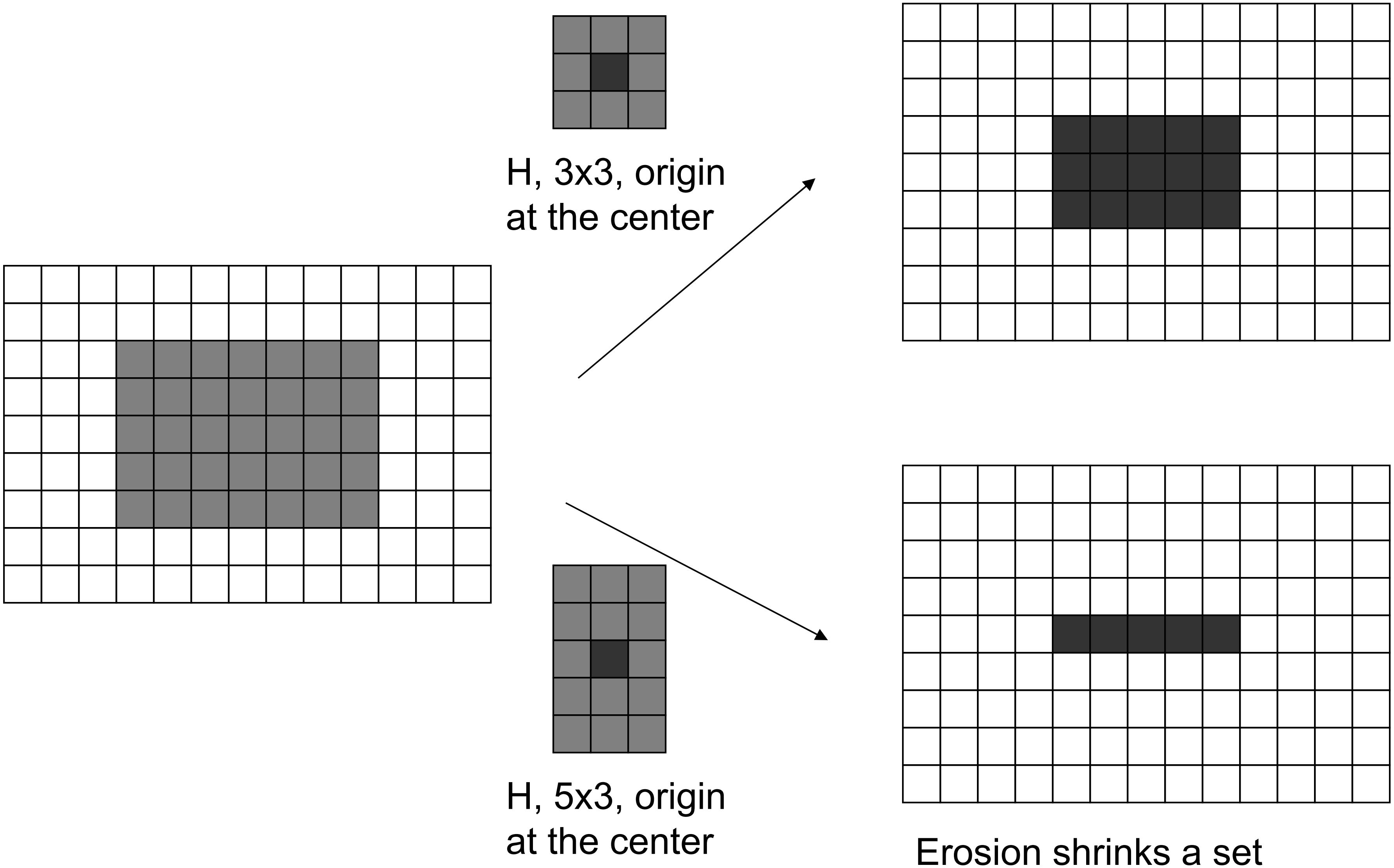
Erosion

- Erosion of set F with a structuring element H is represented by $F \Theta H$, and is defined as,

$$F \Theta H = \{x : (H)_x \subset F\}$$

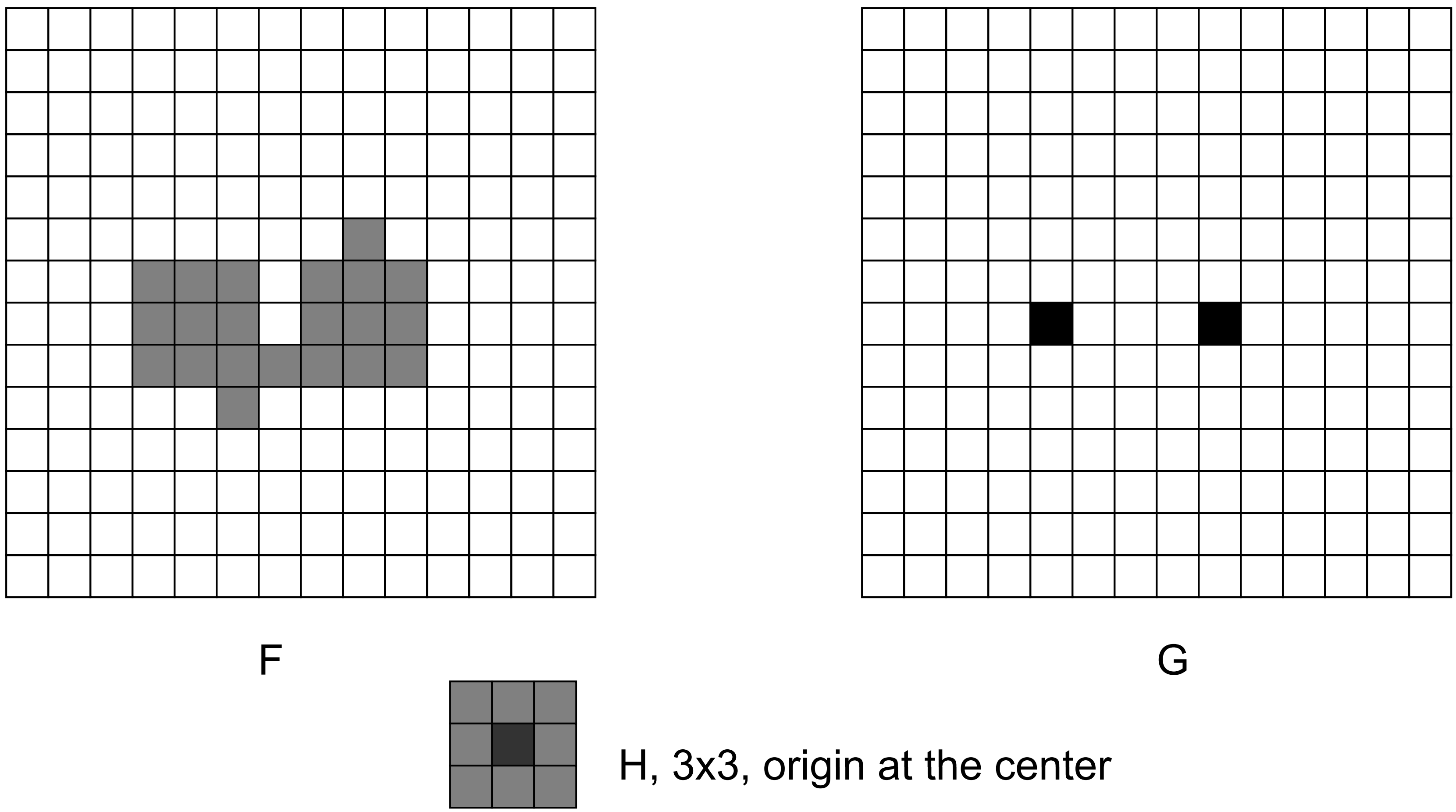
- $G = F \Theta H$ is composed of points that when H is translated to these points, every point of H is contained in F.
- If the origin of H takes value of “1”, $F \Theta H \subset F$

Example of Erosion (1)



Erosion shrinks a set

Example of Erosion (2)



Structuring element

- The shape, size, and orientation of the structuring element depend on application.
- A symmetrical one will enlarge or shrink the original set in all directions.
- A vertical one, will only expand or shrink the original set in the vertical direction.

Closing and Opening

- Closing

$$F \bullet H = (F \oplus H) \Theta H$$

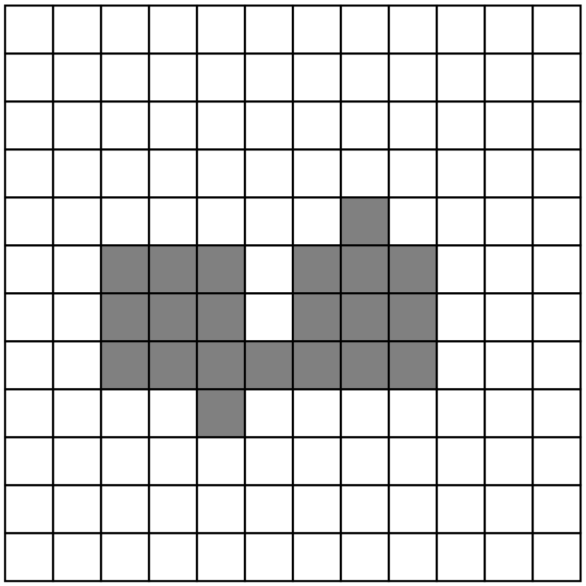
  – Smooth the contour of an image
  – Fill small gaps and holes

- Opening

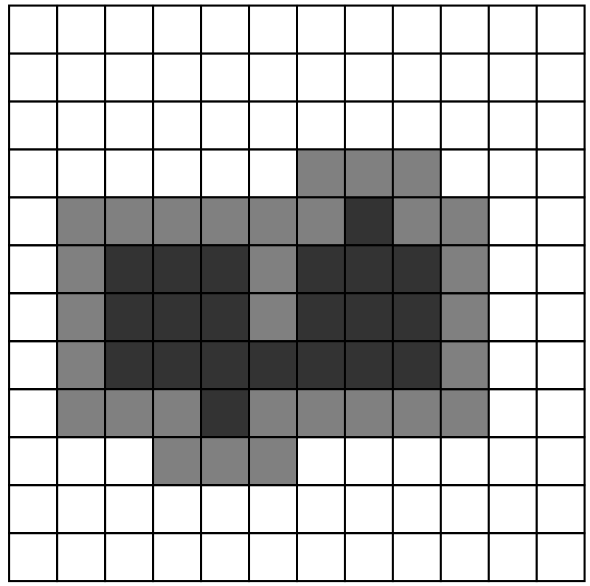
$$F \circ H = (F \Theta H) \oplus H$$

  – Smooth the contour of an image
  – Eliminate false touching, thin ridges and branches

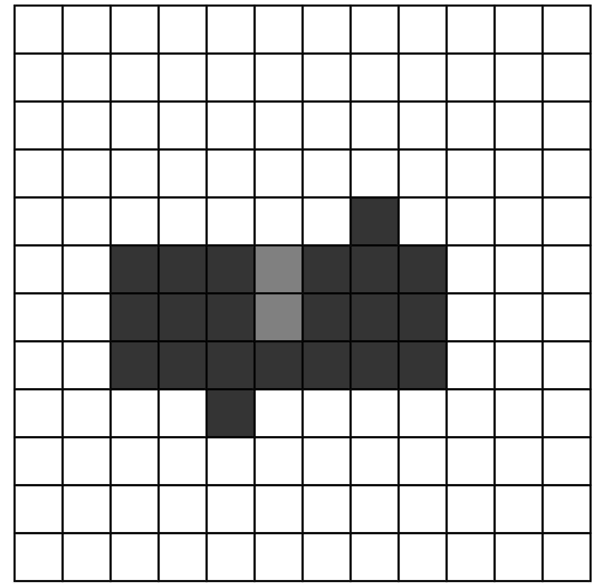
Example of Closing



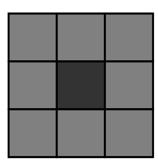
F



$F \oplus H$

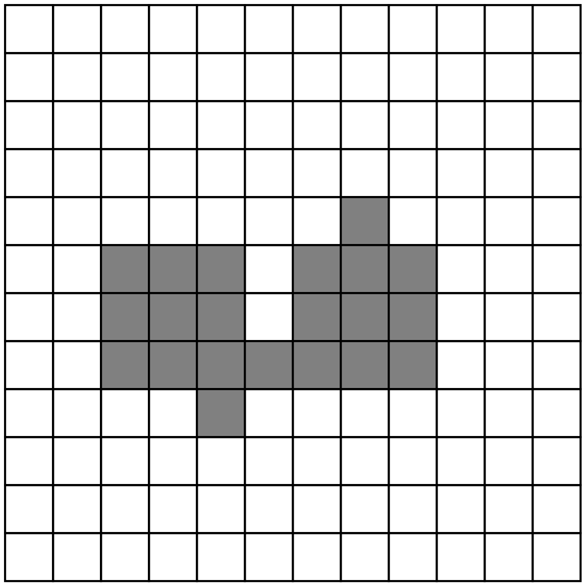


$(F \oplus H)\Theta H$

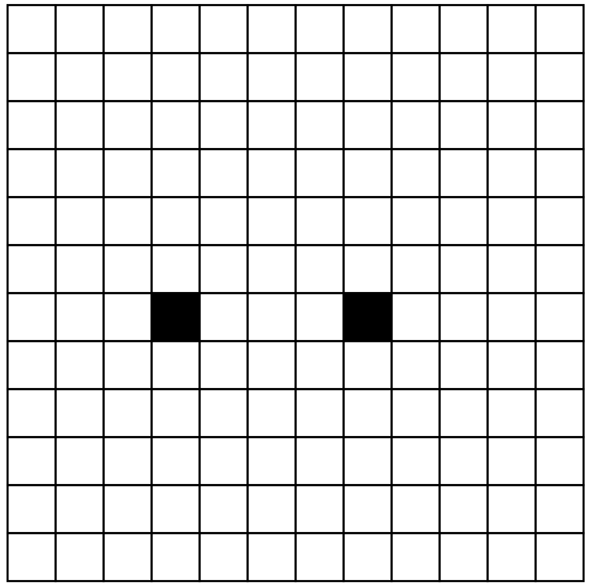


H, 3x3, origin at the center

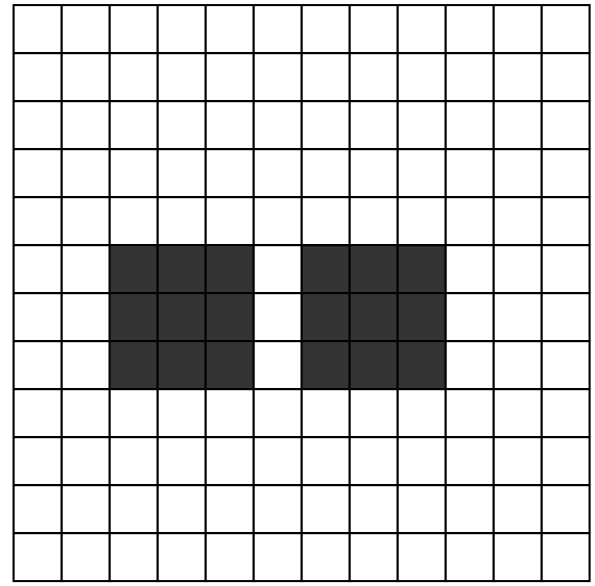
Example of Opening



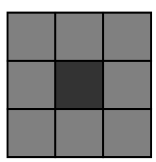
F



$F \ominus H$



$(F \ominus H) \oplus H$



H, 3x3, origin at the center

Example of Opening and Closing

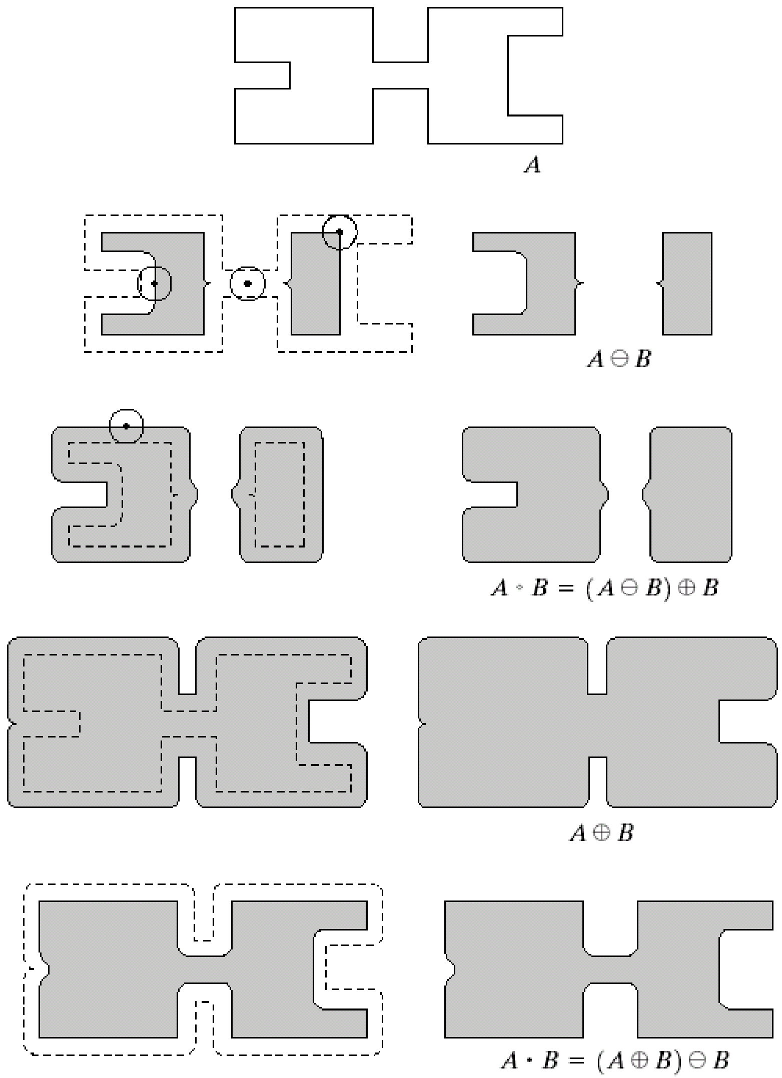


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

Morphological Filters for Grayscale Images

- The structure element h is a 2D grayscale image with a finite domain (D_h), similar to a filter
- The morphological operations can be defined for both continuous and discrete images.

Dilation for Grayscale Image

- Dilation

$$(f \oplus h)(x,y) = \max\{f(x-s, y-t) + h(s,t); (s,t) \in D_h, (x-s, y-t) \in D_f\}$$

  - Similar to linear convolution, with the max operation replacing the sums of convolution and the addition replacing the products of convolution.
  - The dilation chooses the maximum value of f+h in a neighborhood of f defined by the domain of h.
  - If all values of h are positive, then the output image tends to be brighter than the input, dark details (e.g. dark dots/lines in a white background) are either reduced or eliminated.

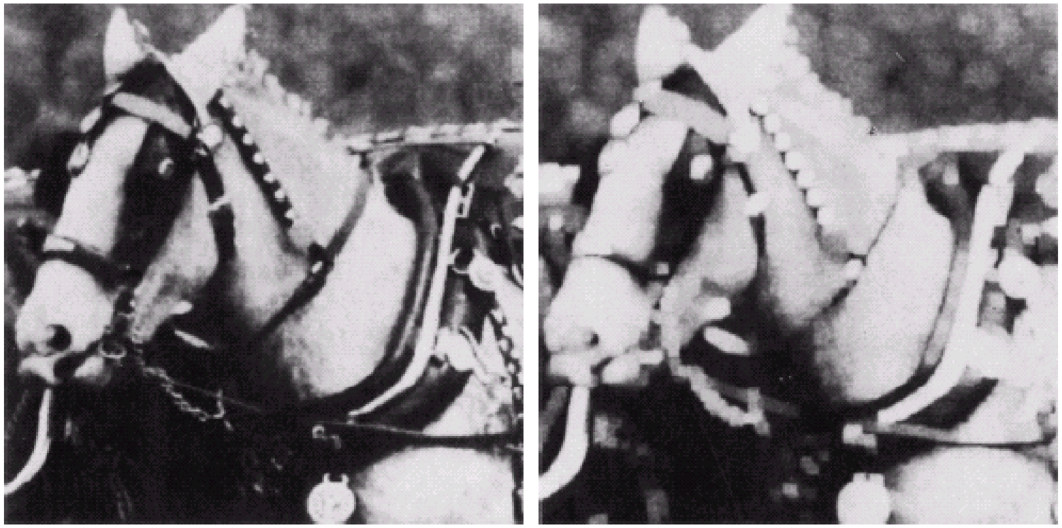
Erosion for Grayscale Image

- Erosion

$$(f \Theta h)(x,y) = \min\{ f(x+s, y+t) - h(s,t); (s,t) \in D_h, (x+s, y+t) \in D_f \}$$

  - Similar to linear correlation, with the min operation replacing the sums of correlation and the subtraction replacing the products of correlation.
  - The erosion chooses the minimum value of f-h in a neighborhood of f defined by the domain of h.
  - If all values of h are positive, then the output image tends to be darker than the input, brighter details (e.g. white dots/lines in a dark background) are either reduced or eliminated.

Example of Grayscale Dilation and Erosion



a b
c

FIGURE 9.29 (a) Original image. (b) Result of dilation. (c) Result of erosion. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Opening for Grayscale Image

- Opening

$$f \circ h = (f \ominus h) \oplus h$$

- Geometric interpretation
  - Think f as a surface where the height of each point is determined by its gray level.
  - Think the structure element has a gray scale distribution as a half sphere.
  - Opening is the surface formed by the highest points reached by the sphere as it rolls over the entire surface of f from underneath.

Closing for Grayscale Image

- Closing

$$f \bullet h = (f \oplus h) \Theta h$$

- Geometric interpretation
  - Think f as a surface where the height of each point is determined by its gray level.
  - Think the structure element has a gray scale distribution as a half sphere.
  - Closing is the surface formed by the lowest points reached by the sphere as it slides over the surface of f from above.

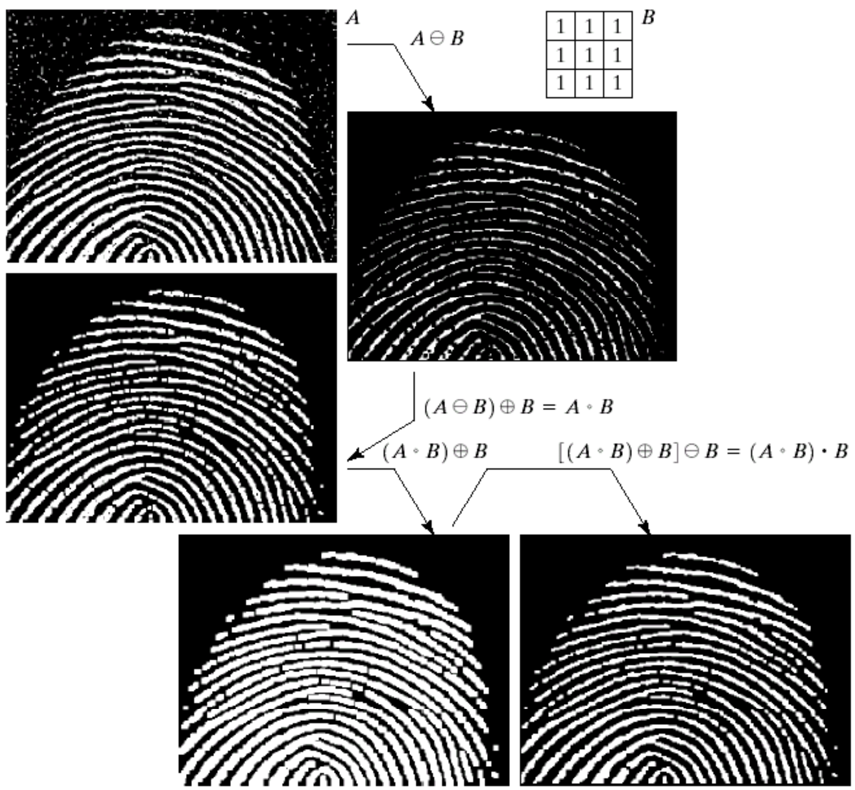
Example of Grayscale Opening and Closing

a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Morphological Operation for Image Enhancement

- Morphological smoothing
  - Opening followed by closing, $(f \circ h) \bullet h$
  - Attenuated both bright and dark details

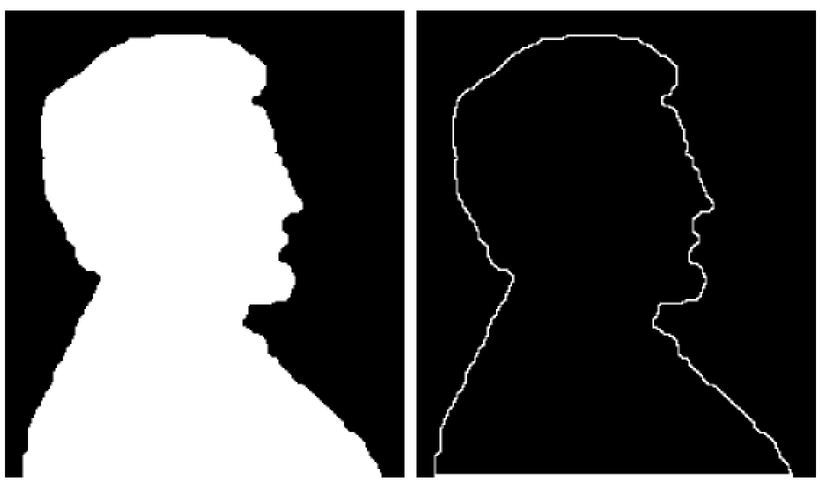


a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(c) Eroded image.
(d) Opening of A.
(d) Dilation of the opening.
(e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Morphological Operation for Image Enhancement

- Morphological gradient $(f \oplus h) - (f \Theta h)$
  - The difference between the dilated and eroded images,
- Valley detection $f \bullet h - f$
  - Detect dark text/lines from a gray background
- Boundary detection $f - f \Theta h$



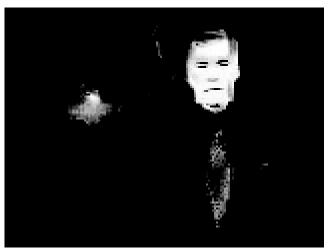
a b

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

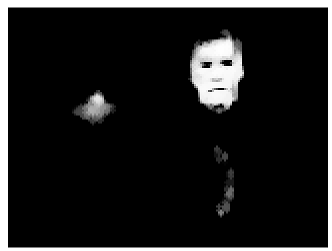
Application in Face Detection

- Use color information to detect candidate face region
- Verify the existence of face in candidate region

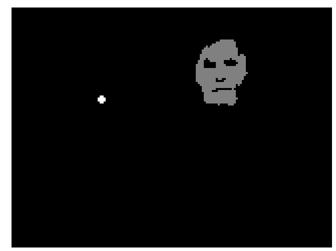
Input image



Skin-tone color likelihood



Opening processed image



Blob coloring

Face detection result