Math 151 Lab 6

Use Python to solve each problem.

1. Given the parametric equations $x = \cos(t)$ and $y = \sin(t) + 3$:
   a) Find the Cartesian equation of the line tangent to the curve when $t = \frac{\pi}{4}$ (exact and approximate)
   b) Find the points on the curve where the tangent line is vertical
   c) Sketch the graph using the domain $t \in [0, 2\pi]$ and ALL tangent lines from part a) and b).
2.
   a) The curve parametrized by $x = t^2$, $y = t^3 - 3t$ crosses itself at the point (3, 0). Find the equations of both tangent lines at that point.
   b) Plot the parametrized curve (using a domain of $t \in [-2, 2]$) and both tangent lines (using a domain of $x \in [0, 4]$) on the same graph.
3. A bacteria culture grows exponentially. After 1 hour, the bacteria count was 200 and after 1.5 hours, the bacteria count was 360.
   a) Solve a system of equations to find k and y_0 (Approximate answers OK. NOTE: it will help to assume k and y_0 are real in the symbols command).
   b) Use this to determine when the bacteria count reaches 2,000 (exact and approximate).
   c) Suppose 200 was the "initial" amount and 360 the amount after 0.5 hours. Find k and the amount of bacteria 1 hour BEFORE the "initial" time. In a print statement, explain what you notice when comparing these answers to part a).
   (Problem 4 on the next page...)

4. The frequency of vibrations of a vibrating violin string is given by

$$f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$$

where L is the length of the string, T is its tension, and ρ is its linear density.

a) Find the rate of change in f with respect to L, T, and ρ (in each case, assume the other parameters are constant).

b) The pitch of a note (how high or low it sounds) is determined by the frequency (the higher the frequency, the higher the pitch). Based on your answers to part a), explain what happens to the pitch if:

   i. the length is decreased (by placing a finger on the string)

   ii. the tension is increased (by adjusting the tuning peg of a string)

   iii. the linear density is increased (by choosing a different string)

c) A violin string is typically tuned by setting the A string to 440 Hz. Solve the equation for ρ, and plot ρ if the frequency is 440 Hz, the length is 0.3 m, and the domain of the tension is $T \in [0, 100]$.

d) The linear density of the A string on a violin is .00078 kg/m. Determine the tension needed to tune the string using the parameters in part c).