Math 151 Lab 8

Use Python to solve each problem.

1. Let $f(x) = 2\sin^{-1}(x)$ and $g(x) = \cos^{-1}(1 - 2x^2)$ (with domain $x \geq 0$)
   a) Find and simplify the derivative of $f(x) - g(x)$.
   b) In a print statement, explain what your answer to part a) tells you about $f(x) - g(x)$.
   c) Evaluate $f(0) - g(0)$ and give further detail about the explanation in part b).

2. Given $g(x) = -5x^4 - 20x^3 + 18$:
   a) Find the critical values of g.
   b) By observing a graph of g' OR by testing numerically, find the intervals on which g is increasing and the intervals on which g is decreasing. Give your answers using interval notation (use "oo" for infinity and "U" for union where applicable).
   c) Find the x-values where $g''(x) = 0$ (basically, the critical values of g')
   d) By observing a graph of g'' OR by testing numerically, find the intervals on which g is concave up and the intervals on which g is concave down. Give your answers using interval notation (use "oo" for infinity and "U" for union where applicable).
   e) Plot the function g in an appropriate domain and range to graphically confirm your answers to parts (b) and (d). All local extrema and inflection points should be clearly visible in your graph.

3. Given $f(x) = \dfrac{x^2(x+1)^3}{(x-2)^2(x-4)^4}$:
   a) Plot f on the domain $x \in [-10, 10]$. In a print command, indicate how many local extrema and how many inflection points there appear to be.
   b) Find $f'(x)$ and the critical values of f (approximate real values only).
   c) Determine the intervals where f is increasing and decreasing (you may try to do this graphically or numerically as discussed in the Overview and described in #2 above).
   d) Find $f''(x)$ and the intervals of concavity (approximate real x-values only. As with part c), you may determine the intervals graphically or numerically.)
   e) How many local extrema and inflection points actually exist?

   (#4 on the next page...)

4. Given $y = (1 - 6x)^{1/x}$:

   a) By hand, write $\ln(y)$ as a fraction $\dfrac{f(x)}{g(x)}$. Define f and g in Python.

   b) Find the limits of f and g as $x \to 0$.

   c) If the answers in part (b) allow for it, use L'Hospital's Rule to compute $\lim\limits_{x\to 0} \ln(y)$ and state the resulting $\lim\limits_{x\to 0} y$.

   d) Evaluate $\lim\limits_{x\to 0} y$ directly in Python to verify your answer in part (c).