Math 151 Lab 5

Use Python to solve each problem.

1. Find the values of r for which $y = e^{rx}$ is a solution to the following differential equations:
   a) $y'' + 5y' + 6y = 0$
   b) $y'' + 6y' + 13y = 0$
   c) Note the solutions in (b) are complex. Compute $y'' + 6y' + 13y$ when $y = e^{-3x}(\cos(2x) + \sin(2x))$. What can you conclude based on your answers to b) and c)?
2. Given the equation $(x^2 + y^2)^2 - 4(x^2 - y^2) + 3 = 0$ and assuming x and y are real numbers:
   a) Plot the graph of the equation using **plot_implicit** with $x \in [-3, 3]$ and $y \in [-3, 3]$**. (NOTE:** this type of curve is called the "ovals of Cassini". The plot should show you why!)
   b) Find $\dfrac{dy}{dx}$.
   c) Find the points (x and y coordinates) where the graph of the equation has a horizontal tangent line. (Make sure your answers agree with the graph in part (a)!)
   d) Find the equation of the tangent line to the curve at the point $(\sqrt{2}, \sqrt{\sqrt{17} - 4})$.
   e) Use the **extend** command to replot the equation with the tangent line found in part d).
3. Given $f(x) = x^n \ln(x)$:
   a) Use list comprehension to find $f^{(n+1)}(x)$ (the $(n+1)$th derivative) when $n = 2, 3, \cdots 8$.
   b) Using your answers to part a), write a print statement giving the formula for general n.
4. Given $y = x^{\sqrt{x}}$:
   a) Use logarithmic differentiation to find $\dfrac{dy}{dx}$. (NOTE: The logarithm step can be done by hand or by using **expand_log**).
   b) Find $\dfrac{dy}{dx}$ by differentiating directly.