Math 151 Lab 3

Use Python to solve each problem.

1. Let $g(x) = 5 + x^2 \cos\left(\frac{\pi}{x^2}\right)$
   a) Find $\lim_{x \to 0} g(x)$
   b) This limit can be proven using the Squeeze Theorem. Find functions f and h which satisfy the Squeeze Theorem and graph all three functions on one set of axes in the domain $x \in [-0.5, 0.5]$.

2. To estimate the minimum velocity needed for a round flat stone to skip when it hits the water, Lyderic Bocquet ("The Physics of Stone Skipping", American Journal of Physics) obtained the following equation:

$$V = \frac{\sqrt{\frac{16Mg}{\pi C \rho_W d^2}}}{\sqrt{1 - \frac{8M \tan^2(\beta)}{\pi d^3 C \rho_W \sin(\theta)}}}$$

   where $M = 0.1$ kg is the mass of the stone, d is the stone diameter, $\rho_W = 1000\, kg/m^3$ is the water density, $C = 1$ is a constant, $\theta = 10°$ is the tilt angle of the stone, $\beta = 10°$ is the angle of incidence, and $g = 9.81\, m/s^2$ is the acceleration due to gravity.

   a) Using the given values, plot $V(d)$ on the given domain $[0.05, 0.1]$ and range (use **ylim**) $[0.7, 0.9]$.
   b) Note that V is decreasing. Use your graph to estimate values a and b such that $a < b$, $b - a \leq 0.01$, and $V(a) > 0.8$, $V(b) < 0.8$. In your print statement, state the values of a and b as well as the Theorem you are applying to find a solution to $V(d) = 0.8$.
   (**NOTE**: In Jupyter, clicking on the graph or using "Zoom to Rectangle" will allow you to more easily find a and b. Later, ENGR 102 students will learn an easier way to find these values with conditional statements and loops).
   c) Numerically solve the equation $V = 0.8$ in Python to confirm your answers in part b).

(Lab continues on the next page...)

3. Given $f(x) = \dfrac{\sqrt{2x^2+1}}{3x-5}$:

   a) Use the **for** command (list comprehension) to evaluate the function at $x = 10,\ 50,\ 100$, then at $x = -10,\ -50,\ -100$ to numerically approximate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
   (**NOTE**: Data types are important here! Remember there is a difference between 10 and 10.0. Consider which type you want to use here)

   b) Compute the limits in part a). Give exact and approximate answers.

   c) Plot f and the horizontal asympotote(s) on the domain $[-10,\ 10]$ and range $[-5,\ 5]$ to graphically verify your answers.

4. Given $f(x) = \begin{cases} 2x - x^2 & \text{if } 0 \le x \le 2 \\ 2 - x & \text{if } 2 < x \le 3 \\ \dfrac{x^2 - 7x + 12}{x - 3} & \text{if } 3 < x < 4 \\ \pi & \text{if } x \ge 4 \end{cases}$

   a) Graph the function in the domain $[0,\ 6]$ to visually determine whether f is continuous at the "break points" or not.

   b) Find the left and right hand limits of f at all "break points" to algebraically confirm your answers to part a).