12/1

Exam 3

Question 1

Student A. $P_2^2(x) = -3x^2 + 2 \cdot + 1$

$(0,1)$
$(1,0)$
$(\frac{2}{3}, \frac{1}{2})$

$P_2(x) = y_0 L(x_0) + y_1(x_1) + y_2(x_2)$.

$= 1 \cdot \frac{(x-1)(x-\frac{2}{3})}{(0-1)(0-\frac{2}{3})} + 0 + \frac{1}{2} \frac{(x-0)(x-1)}{(\frac{2}{3}-0)(\frac{2}{3}-1)}$

$= \frac{(x-1)(x-\frac{2}{3})}{(-1)(-\frac{2}{3})} + \frac{1}{2} \cdot \frac{(x)(x-1)}{(\frac{2}{3})(-\frac{1}{3})}$

$\frac{3}{3} \frac{2}{3}$

$= \frac{(x^2 - \frac{5}{3}x + \frac{2}{3})}{\frac{2}{3}} + \frac{1}{2} \cdot \frac{x^2 - x}{-\frac{2}{9}}$

$= \frac{3}{2}(x^2 - \frac{5}{3}x + \frac{2}{3}) + (-\frac{9}{4}) \cdot (x^2 - x)$

$\frac{\frac{1}{2}}{-\frac{2}{9}}$

$= \frac{3}{2}x^2 - \frac{5}{2}x + 1 - \frac{9}{4}x^2 + \frac{9}{4}x$

$= \frac{6}{4}x^2 - \frac{10}{4}x - \frac{9}{4}x^2 + \frac{9}{4}x + 1$

$= \frac{-3}{4}x^2 - \frac{1}{4}x + 1$

$0: 1$

$1: -\frac{3}{4} - \frac{1}{4} + 1 = -1 + 1 = 0$

$\frac{2}{3}: -\frac{3}{4}(\frac{4}{9}) - \frac{1}{4} \cdot \frac{2}{3} + 1 = -\frac{1}{3} - \frac{1}{6} + 1 = -\frac{2}{6} - \frac{1}{6} + \frac{6}{6} = \frac{3}{6} = \frac{1}{2}$

$(0,1)$
$(1,0)$
$(\frac{2}{3}, \frac{1}{2})$

Student, B.

$$P_2^u(x) = -0.75(x^2) - 0.25(x) + 1$$

$$P_2(x) = f(x_0) + (x - x_1) f[x_0, x_1] + (x - x_0)(x - x_1) f[x_0, x_1, x_2]$$

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{0 - 1}{1 - 0} = -1$$

$$\frac{2}{3} - 1 \quad \frac{3}{3} \quad \frac{2}{3}$$

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \frac{(-\frac{3}{2}) - (-1)}{\frac{2}{3} - 0} = \frac{-\frac{1}{2}}{\frac{2}{3}} = \frac{-3}{4}$$

$$f[x_1, x_2] = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{\frac{1}{2} - 0}{\frac{2}{3} - 1} = \frac{\frac{1}{2}}{-\frac{1}{3}} = \frac{-3}{2}$$

$$\frac{-3}{2} - (-1) \qquad \frac{-3}{2} - (-\frac{2}{2}) \qquad \frac{-3 + 2}{2} \qquad \frac{-1}{2}$$

$$P_2(x) = f(x_0) + (x - x_0) f[x_0, x_1] + (x - x_0)(x - x_1) f[x_0, x_1, x_2]$$
$$= 1 + (x - 0) \cdot (-1) + (x - 0)(x - 1) \cdot (-\frac{3}{4})$$
$$= 1 - x + 0 - \frac{3}{4} x^2 + \frac{3}{4} x$$
$$= -\frac{3}{4} x^2 - \frac{1}{4} x + 1$$

$$-1 + \frac{3}{4} \qquad -\frac{4}{4} + \frac{3}{4}$$

$$P_2(0) = 1$$
$$P_2(1) = -\frac{3}{4} - \frac{1}{4} + 1 = 0$$
$$P_2(\tfrac{2}{3}) = -\frac{3}{4}(\tfrac{2}{3})^2 - \frac{1}{4} \cdot \frac{2}{3} + 1 = \frac{1}{2}$$

Student B

Kw Anem
12/1

Exam 2.

Question 2
Part 1

$D_h^+ f(x)$. $D_h^- f(x)$.

$x = 0.6 \quad h = 0.3$

$$D_h^+ f(x) = \frac{f(x+h) + f(x)}{h}$$

$$D_h^+ f(0.6) = \frac{f(0.6+0.3) + f(0.6)}{0.3}$$

$$= \frac{f(0.9) + f(0.6)}{0.3}$$

$$= \frac{15 + 7}{0.3} = \frac{22}{0.3}$$

$$D_h^- f(x) = \frac{f(x) - f(x-h)}{h}$$

$$D_h^- f(0.6) = \frac{f(0.6) - f(0.6-0.3)}{0.3}$$

$$= \frac{7 - 1.5}{0.3}$$

$$= \frac{5.5}{0.3}$$

Question 2

Part 2

$$D_h^{(2)} f(x) = \frac{D_h^+ f(x) - D_h^- f(x)}{h}, \quad h = 0.3, \quad x = 0.6$$

$h = 0.3$ $$D_h^{(2)} f(0.6) = \frac{D_h^+ f(0.6) - D_h^- f(0.6)}{0.3}$$

$$= \frac{\frac{22}{0.3} - \frac{5.5}{0.3}}{0.3}$$

$\frac{16.5}{0.3} \div \frac{0.3}{1}$

$$= \frac{\frac{16.5}{0.3}}{0.3} = \frac{\frac{16.5}{0.3}}{\frac{0.3}{1}} = \frac{16.5}{0.09}$$

False

Exam 3

Ken Amé

12/1

Question 3

$x^2 - 2x + 1 = 0$

$f(x) = x^2 - 2x + 1$

$$x_{n+1} = \frac{x_{n-1}^2 \cdot x_n + x_{n-1} \cdot x_n^2 + 1}{x_n^2 + (x_{n-1})^2 + x_{n-1} \cdot x_n - 2}$$

$$x_{n+1} = x_n - \frac{f(x_n)\cdot(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

$$= x_n - \frac{(x_n^2 - 2x_n + 1)(x_n - x_{n-1})}{(x_n^2 - 2x_n + 1) - ((x_{n-1})^2 - 2(x_{n-1}) + 1)}$$

$$= x_n - \frac{(x_n^2 - 2x_n + 1)(x_n - x_{n-1})}{(x_n^2 - 2x_n + 1) - x_{n-1}^2 + 2x_{n-1} - 1}$$

$$= x_n - \frac{(x_n^2 - 2x_n + 1)(x_n - x_{n-1})}{x_n^2 - 2x_n - x_{n-1}^2 + 2x_{n-1}}$$

$$= x_n - \frac{(x_n^2 - 2x_n + 1)(x_n - x_{n-1})}{(x_n - x_{n-1})(x_n + x_{n-1}) - 2(x_n - x_{n-1})}$$

$$= x_n - \frac{(x_n^2 - 2x_n + 1)}{(x_n + x_{n-1}) - 2}$$

$$= \frac{x_n(x_n + x_{n-1} - 2) - (x_n^2 - 2x_n + 1)}{(x_n + x_{n-1} - 2)}$$

$$= \frac{\cancel{x_n^2} + x_n x_{n-1} - \cancel{2x_n} - \cancel{x_n^2} + \cancel{2x_n} - 1}{(x_n + x_{n-1} - 2)}$$

$$= \frac{x_n x_{n-1} - 1}{x_n + x_{n-1} - 2}$$

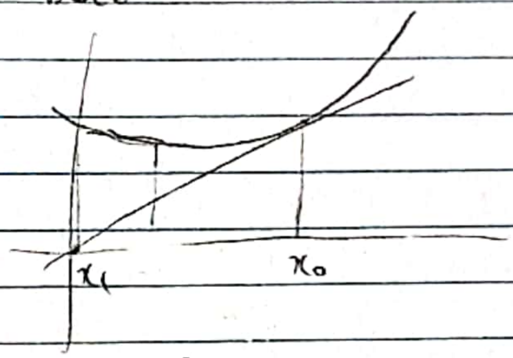
Question 4

Describe the Newton's method

$x_{n+1} = x - \frac{f(x_0)}{f'(x_0)}$

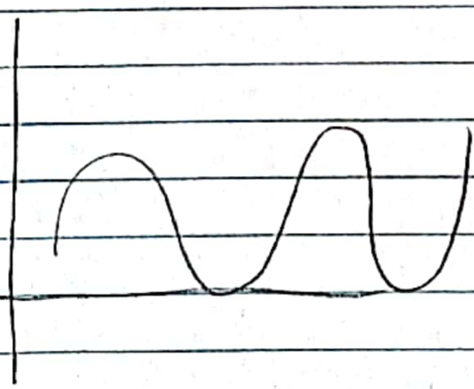
Newton's method used to find approximation to the root of a function.

Use the previous point to approximate as shown below.



$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Failing situation



It is fail situation because there is no initial point and It repeats the same movement for infinity.

Newton; $x_n - \frac{f(x_n)}{f'(x_n)}$

Part i

$f(x) = -e^{-x} - \cos(x)$ $x_0 = 0$ $x_1 = 2$

$f'(x) = e^{-x} + \sin(x)$ $x_0 = 0$, $f(x_0) = -1 - 1 = -2$ $f'(x_0) = 1$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{-2}{1} = 2$$

True