Week 3 Homework

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1. Are the following formulas valid, satisfiable, or unsatisfiable?
   - P(a)→ExP(x) Valid, for any model M, a is a subset of all possible values Ex
   - ExP(x)→P(a) Satisfiable. Consider a model where the entire universe is {1, 2}. If a = 1, and P is also 1, then P(a) is True (and exists in this universe).
     However, if the universe was the same {1,2}, but a=2 and P is 1, then P(a) is False.
   - ExP(x) V Ex!P(x) → AxP(x) Valid, every instance of P(x) will be either True or False
2. Foe each of the following sequent, give a formal proof if entailment holds.
   - AxP(X) ⊢ ExP(x)
   - Ax(P(x)→Q(x)), AxP(x) ⊢ AxQ(x)
   - Ax(P(x)→Q(x)), ExP(x) ⊢ ExQ(x)
   - !ExP(x) ⊢ Ax!P(X)
   - !AxP(x) ⊢ Ex!P(x)

AxP(X) ⊢ ExP(x)

1. AxP(X)	premise
2. x_0	assumption
3. P(x_0)	A_e 1, 2
4. Ex(Px_0)	E_e, 2, 3
5. Ex(Px)	E_i, 2-3

Ax(P(x) → Q(x)), AxP(x) ⊢ AxQ(x)

1. Ax(P(x)→Q(x))	premise
2. AxP(x)	premise
3. X_0	assumption
4. P(x_0)→Q(X_0)	A_e, 1, 3
5. P(x_0)	$\rightarrow_i$, 4
6. Q(x_0)	$\rightarrow_i$, 4, 5
7. Ax_0Q(x_0)	A_e, 3-6
8. AxQ(x)	A_e, 3-7

Ax(P(x) → Q(x)), ExP(x) ⊢ ExQ(x)

1. Ax(P(x)→Q(x))	premise
2. AxP(x)	premise
3. x_0	assumption
4. $P(x_0) \rightarrow Q(x_0)$	A_e, 1, 3
5. $P(x_0)$	$\rightarrow_i$, 4
6. $Q(x_0)$	$\rightarrow_i$, 4, 5
7. $Ex_0Q(x_0)$	E_e, 3-6
8. ExQ(x)	E_e, 3-7

Note: almost the same proof as prior, except we just need to show there exists an X_0, not show for all X. Thus the slight variation

!ExP(x) ⊢ Ax!P(X)

1. !ExP(x)	premise
2. x_0	assumption
3. !Ax!P(x)	assumption
4. $!P(x_0)$	assumption
5. Ax!P(x)	A_i, 2, 4
6. $\perp$	$!_e$, 3, 5
7. $P(x_0)$	PBC, 4-6
8. AxP(x)	A_i, 3-7
9. $\perp$	!e, 1-8
10. Ax!P(x)	PBC, 2-9

!AxP(x) ⊢ Ex!P(X)

Note: same approach as prior, but using there exists elimination instead of for all elimination

1. !AxP(x)	premise
2. x_0	assumption
3. !Ex!P(x)	assumption
4. $!P(x_0)$	assumption
5. Ex!P(x)	E_i, 2, 4
6. $\perp$	$!_e$, 3, 5
7. $P(x_0)$	PBC, 4-6
8. AxP(x)	A_i, 3-7
9. $\perp$	!e, 1-8
10. Ex!P(x)	PBC, 2-9