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# Assignment 5

Show that the following Hoare triples are valid.

1.

**[ $y \geq 0$ ]** **Precondition**

$0 = x.y$ Set equal to zero (implied)

a = 0;

$0 = ax$ Set equal to zero (implied)

z = 0;

$z = ax$ Invariant

while (a != y) {

$(z = ax)$ && $(a! = y)$ Added guard

$z + x = (a + 1)x$ Increment by 1 (implied)

z = z + x;

$z = (a + 1)x$ Assignment for x

a = a + 1;

$z = ax$ Assignment for x to reach invariant

}

$z = ax$ && $a = y$ Invariant and not guard (exit loop)

**[ $z = x.y$ ]** **Postcondition**

2.

$[y = y_0 \;\&\; y \geq 0\,]$ **Precondition**

$[0 = x.(y_0-y)]$ Set equal to zero (implied)

$z = 0;$

$[z = x.(y_0-y)]$ Invariant

while (y != 0) {

$[z = x.(y_0-y)$ && $y \,!= 0]$ Added guard

$[z + x = x.(y_0-y+1)]$ Increment by 1 (implied)

$z = z + x;$

$[z = x.(y_0-y+1)]$ Assignment of y for all x

$y = y - 1;$

$[z = x.(y_0-y)]$ Assignment of y for all x to reach invariant

}

$[z = x.(y_0-y)$ && $!(y \,!= 0)]$ Invariant and not guard (exit loop)

$[z = x.\,y_0\,]$ **Postcondition**

3.

$[\,\top\,]$ First "if"

$[(x>y) \rightarrow ((w>x \rightarrow w = \max(x,y,w))$ && $(!(w>x) \rightarrow x = \max(x,y,w)))$ &&

$(!(x>y) \rightarrow ((w>y) \rightarrow w = \max(x,y,z))$ && $(!(w>y) \rightarrow y = \max(x,y,w))]$

"else"

if (x > y)

$[(w > x \rightarrow w = \max(x,y,w))$ && $(!(w>x) \rightarrow x = \max(x,y,w))]$

$z = x;$

$[(w > z \rightarrow w = \max(x,y,w))$ && $(!(w>z) \rightarrow z = \max(x,y,w))]$

else Assignment of z for x

$[(w > y \rightarrow w = \max(x,y,w))$ && $(!(w>x) \rightarrow y = \max(x,y,w))]$

$z = y;$

$[(w > z \rightarrow w = \max(x,y,w))$ && $(!(w>z) \rightarrow z = \max(x,y,w))]$

Assignment of z for y

if (w > z)

$[w = \max(x,y,w)]$ After replacing both x and y with z, check against w

$z = w;$

$[z = \max(x,y,w)]$ Assignment of z for w

$[\,z = \max(x,y,w)\,]$