Claim. *For any integer $n \geq 1$,* $2^1 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 2$.

Proof. By induction. Base case: Let n = 1. Then $2^1 = 2^{1+1} - 2$. The right hand side evalutes to 4 - 2, which is equal to the left hand side 2. From Inductive Hypothesis: assume claim holds for some n. We will show it holds for $n + 1$. $2^1 + ... + 2^n + 2^{n+1} = 2^{n+1} - 2 + 2^{n+1}$. Combining like terms, we get $2^{n+2} - 2 = 2^{(n+1)+1} - 2$, which is what we were looking for.

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