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CIS623 – Winter 2021
Tuesday @ 6:00pm EST

CIS 623 Assignment 2 Due Week 3 Live

For each of the following set of formulas, show if there is an entailment relation from formulas in a to the formulas in b. Give a formal proof if the entailment relation holds from the formulas in a to b.

1.
   a. $(p \wedge q) \vee (p \wedge r)$
   b. $p \wedge (q \vee r)$
2.
   a. Nothing
   b. $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
3.
   a. $p \rightarrow q$
   b. $\neg q \rightarrow \neg p$
4.
   a. $\neg p \vee q$
   b. $p \rightarrow q$

1.

a. $(p \wedge q) \vee (p \wedge r)$

b. $p \wedge (q \vee r)$

Truth Table:

p	**q**	**r**	**p/\q**	**p/\r**	**(p/\q) \/ (p/\r)**	**q\/r**	**p /\ (q\/r)**
T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	F	F	F
F	T	T	F	F	F	T	F
F	T	F	F	F	F	T	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

Proof:

1. (p/\q) \/ (p/\r)	premise
2. p/\ (q\/r)	premise
3. (p/\q)	assumption
4. p	/\e, 3
5. q	/\e, 3
6. q\/r	\/i, 5
7. p/\(q\/r)	/\i, 4, 6
8. (p/\r)	assumption
9. r	/\e, 8
10. (q\/r)	\/i, 9
11. p	/\e, 8
12. p/\(q\/r)	\/i, 10, 11
13. p/\(q\/r)	\/e, 3-7, 8-12

2.

a. Nothing

b. $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

Truth Table:

p	**q**	**r**	**p → q**	**q → r**	**(p → q)/\q → r)**	**p → r**	**((p → q)/\(q → r)) → (p → r)**
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Proof:

1. ((p → q)/\(q → r)) → (p → r)	premise
2. p → q	assumption
3. p	→ e, 2
4. q	→ e, 2
5. q → r	assumption
6. r	→ e, 5
7. (p → q)/\(q → r)	/\i, 2-4, 5-6
8. p → r	→ i, 3, 6
9. ((p → q)/\(q → r)) → (p → r)	→ e, 2-8, 5-7

3.

a. $p \rightarrow q$

b. $\neg q \rightarrow \neg p$

Truth Table:

p	**q**	**$p \rightarrow q$**	**$\neg q$**	**$\neg p$**	**$\neg p \rightarrow \neg q$**
T	T	T	F	F	T
T	F	F	T	F	F
F	T	F	F	T	T
F	F	F	T	T	T

Proof:

1. $(p \rightarrow q)$	premise
2. $\neg q$	assumption
3. $\neg p$	MT, 1, 2
4. $\neg q \rightarrow \neg p$	$\rightarrow$i, 2-3

4.

a. ¬p V q

b. p→q

Truth Table:

p	**q**	**¬p∨q**	**p → q**
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Proof unnecessary as p and ¬p cannot both be true. Therefore, ¬p∨q cannot entail for p → q