Assignment 8 Due Week 11 Live Session

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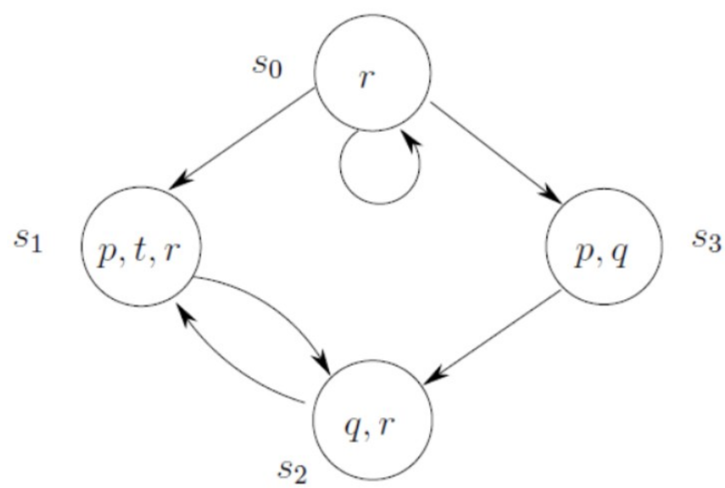
M1:

1. Does the model M1 and s1 satisfy the following formulas?
   a. AG AF p False – S1 → S1 → S1… will never get to p
   b. AG EF p True – can always find a way to p

M2:



2. Does the model M2 and s0 satisfy the following formulas?
   a. ! EG r False – can always get to r
   b. AF q False – S0 → S0, S0 → S1 → S2 → S1...
   c. AG AF q False – same as above, cannot always find path to q
3. Prove or construct counterexamples for the following CTL formulas
   a. EG (p & q) → (EG p & EG q)
   b. EG (p | q) → (EG p | EG q)

   See page 2
4. Give a model and a world in which only one of the following two formulas is true while the other is false.

   $\Diamond(p \wedge q)$ and $\Diamond p \wedge \Diamond q$ See page 3

5. Find natural deduction proofs for the following sequent over the basic modal logic K.

   $\Diamond(p \to q) \vdash \Box p \to \Diamond q$ See page 3

3. Prove or construct counterexamples for the following CTL formulas
    a. EG (p & q) → (EG p & EG q) Valid (Proof)
    b. EG (p | q) → (EG p | EG q) Invalid (Counterexample)

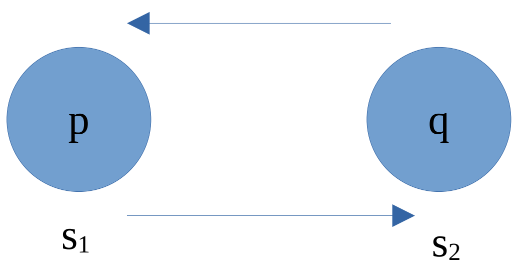
a.

M is a model of CTL, s is a given state in M
M, s |= EG(p & q)
if EG(p & q) is satisfied, it must also be the case that (EGp & EGq) is satisfied
Therefore, for all the states in M, s_i, M, s_i |= (p&q) must be satsified
Therefore, for all paths starting with s, all states s_i must satisfy M, s_i |= p and all paths must also satisfy M, s_i |= q
This means that M,s |= EGp and M, s|= EGq
We can now say that M, s |= (EGp & EGq)

b.

To show a counterexample, M is a model of CTL and s is a given state in M
M satisfies M, s |= EG(p or q), but not M, s |= (EGp or EGq)
Consider M to have two states, s_1, and s_2 and edges (s_1, s_2), (s_2, s_1)
If p holds only in s_1, and q holds only in s_2, then EG(p or q) holds but not (EGp or EGq)

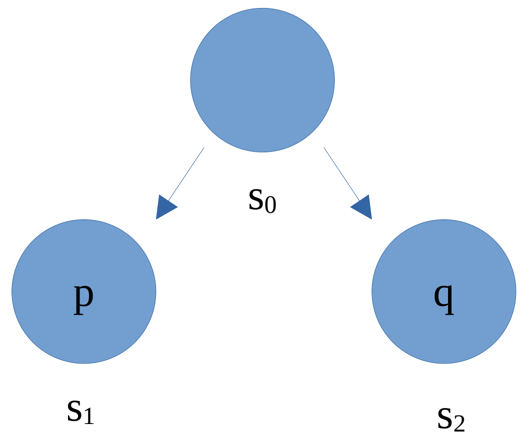
Example:



4. Give a model and a world in which only one of the following two formulas is true while the other is false.

$$\Diamond(p \wedge q) \text{ and } \Diamond p \wedge \Diamond q$$

The solution is similar to problem 3. Here we will show an example where ♦(p & q) is false and ♦p & ♦q is true.
Consider the world W with s_0, s_1, and s_2, R (s_0, s_1) and (s_0, s_2), and L(s_1) = p, L(s_2) = q.
The following is a representation of W:



Thus s0 !||- ♦(p & q), but s0 ||- ♦p & ♦q

5. Find natural deduction proofs for the following sequent over the basic modal logic K.

$$\Diamond(p \rightarrow q) \vdash \Box p \rightarrow \Diamond q$$

1. □p → ♦q	premise
2. ¬ □¬(p → q)	premise
3. □p	assumption
4. □¬q	assumption
5. p	□e 2
6. p → q	assumption
7. q	5, → e 6
8. ¬q	□e 4
9. ⊥	¬e 8, 7
10. ¬(p → q)	→ i 5-9
11. □¬(p → q)	□i 4-10
12. ⊥	¬e 11, 2
13. ¬ □¬q	¬i 3-12
14. □p → ¬ □¬q	→ i 2-13
15. □p → ♦q	