

Exercise 1.5.4

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To make $L \neq 0$ but less than infinity (and thus $f(x) = O(g(x))$), $g(x)$ must grow about as fast as $f(x)$. Looking at the dominant term of $f(x)$, x^4 , let $g(x) = x^4$. Using the Limit Theorem, and applying L'Hopital's Rule: $L = \lim_{x \rightarrow \infty} f/g$

$$\begin{aligned} &= \frac{5x^4 + 3x^2 + 10,000}{x^4} \\ &= \frac{20x^3 + 6x}{4x^3} \\ &= \frac{60x^2 + 6}{12x^2} \\ &= \frac{120x + 24}{24x} \\ &= \frac{120 + 24/x}{24} \\ &= 5 \end{aligned}$$

Because $L = 5$, $f(x) = o(g(x))$ and $f(x) = O(g(x))$