

Exercise 3.4.2/4

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Claim: a directed graph has a cycle if and only if its DFS reveals a back edge.

Proof. First, show that if DFS tree has a back edge, then the graph G has a cycle. A back edge is an edge from a descendent to one of its ancestors. So there is a path from a to d in the DFS tree, but now we have an edge from d to a (back edge). So, there is a cycle.

Suppose G has a cycle. Define u to be the first node in the cycle that the DFS visits. Define node v to be the node right before u in the cycle. Because there is a path from u to v in G , v will be a descendent of u in the tree. (node u was the first node in the cycle). That is going to make the edge from v to u a back edge. Thus, the tree has a back edge. ■