Laboratory 4: The coupled Pendulum

In this lab, you will analyze and perform experiments with the coupled pendulum system shown in Figure 1. The coupled pendulum is a complex linear physical system, and at this point in the semester, you have all of the tools you need to derive the equations of motion, solve these equations to predict the motion of the system, and compare your theoretical results to empirical data.

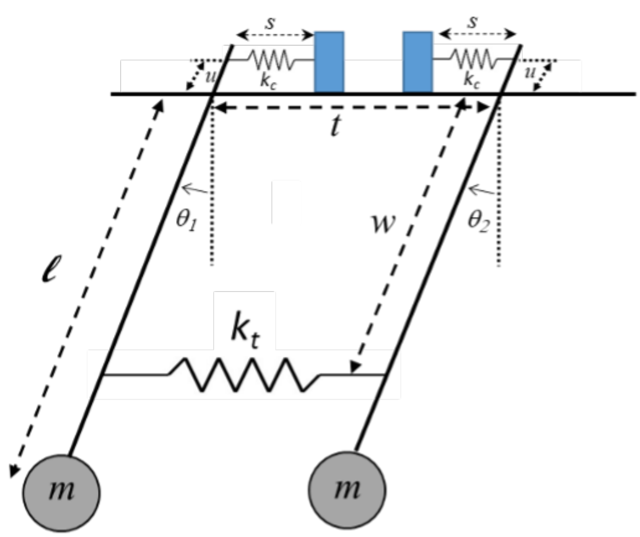

Figure 1: **Coupled pendulum.**

Symbol	Value	Description
k_t	2.5 N/m	Spring constant for tension spring connecting pendulum rods
k_c	53.0 N/m	Spring constant for coupling spring connected to dynamic force sensor
m	0.201 kg	Mass of each weight (plus plastic screw)
t	0.248 m	Distance between pivot points on two pendulums
ℓ	0.360 m	Distance from pivot to center of weight
w	0.270 m	Distance from pivot to attachment point for tension spring
u	0.030 m	Distance from pivot to spring connected to force sensor
s	0.060 m	Distance from rod to force sensor (for undeflected spring)
–	14 mV/mN	Scale factor to convert volts to force for sensor

Table 1: **Constants for the components in Figure 1.**

1 Lab instructions

A word of caution. A word of caution about the force sensors: The sensors can be damaged if the hooks are loaded with more than 5 N in the axial direction or 1 N in transverse direction. Be especially careful with the maximum loading force if you remove or reattach the springs from the hooks.

Data collection. The pendulums are instrumented so that you can measure angles as voltages on an oscilloscope. You don't need to know the actual relationship between voltage and angle because of linearity (i.e., 1 V could correspond to 2 radians, or 0.1 radians – it won't matter for your results). Collect around 30 seconds worth of data for three different initial conditions:

1. **Same angles.** $\theta_1(0) = \theta_2(0) = \theta_0$.
2. **Opposite angles.** $\theta_1(0) = \theta_0$, $\theta_2(0) = -\theta_0$.
3. **One angled, one not.** $\theta_1(0) = \theta_0$, $\theta_2(0) = 0$.

Here θ_0 is some small angle that you can choose. Ensure that you measure θ_0 with the protractors; this measurement can be inexact. Make the angles small enough so that the spring doesn't go slack.

Theory. Derive a set of *linear* equation(s) describing the motion of the system. Note that the system is inherently nonlinear, so you will have to decide how to approximate the equations of motion to ensure that they are linear. Next, either (a) create a Simulink model for the system and run the simulation for all three sets of initial conditions, or (b) derive an analytical solution for the equations you derived. Extra credit is available if you would like to attempt both (a) and (b). Assume that the pendulum arms are massless and that the system is lossless, i.e., there is no friction.

Some pointers. Here are a few pointers that you may need when completing the theory:

- Use the small angle approximation, i.e., $\sin(\theta) \approx \theta$ and $\cos(\theta) \approx 1$ when θ is small.
- Recall the trigonometric identity $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.
- There is a formalism to solve systems of differential equations by writing the system in matrix form and using eigenvalues and eigenvectors to find the normal modes. This page–which was written by Swarthmore's Professor (Emeritus) Erik Cheever–outlines all of the basic principles.

Project. The next time you come to lab, we will discuss your project. Having a project selected and a project group chosen when you come to lab constitutes 5% of the project grade.

2 Your report

For this lab, each group will submit a single formal report. As a group, you are expected to share the work equally; every member of the lab group must read and approve of the content, formatting, and writing for each section of the lab report. The formatting requirements for this report are as follows.

- Section 1: **Title (5 points)**
  - ☐ Title of the lab.
  - ☐ Date of submission.
  - ☐ Names of the members of your lab group.
  - ☐ Emails of the members of your lab group.
- Section 2: **Abstract (5 points)**
  - ☐ A short paragraph describing the main question(s) considered in the lab, the experiments you conducted, and the main results. The abstract should be no more than 5-6 sentences.
- Section 3: **Introduction (5 points)**
  - ☐ Motivation: Explain why the coupled pendulum is difficult to analyze.
  - ☐ Purpose: Explain the purpose of this lab and the main question(s) under consideration.
  - ☐ Experiments: Explain (at a high level) the experiments you performed and why you believe they address the main question(s) considered in this lab.
  - ☐ Results: Explain (at a high level) your main results and whether they agree with your theoretical expectation.
- Section 4: **Theory (20 points)**
  - ☐ Derive the equations of motion for the coupled pendulum system.
  - ☐ If you choose to derive an analytical solution to these equations, type up your results and carefully explain each step. On the other hand, if you choose to create a Simulink model, show a screenshot of your model and describe how it works.
- Section 5: **Methods (5 points)**
  - ☐ Describe the steps you followed to produce your results. You may refer to the lab handout (which you should properly cite) and report any deviations from the specified procedure.
- Section 6: **Experimental results (20 points)**
  - ☐ Plot the data that you collected in lab for all three sets of initial conditions.
  - ☐ Plot the results from your theoretical derivation/Simulink model for all three sets of initial conditions.
- **Section 7: Discussion (5 points)**
  - ☐ Discuss the agreement between (1) your theoretical calculations or simulation and (2) your experimental data.
  - ☐ Discuss any sources of error in your empirical results.
- Section 8: **Conclusion (5 points)**

- ☐ Summarize your report—the motivation, experiments, and main results—in one or two short paragraphs.

- Section 9: **Acknowledgments (5 points)**
  - ☐ If you discussed this lab with anyone other than the members of your lab group, acknowledge them here. Include a brief description of what was discussed with each person.
- Section 10: **References (5 points)**
  - ☐ Properly cite any sources (including this document) that you used when for this lab.
- Section 11: **Appendices (5 points)**
  - ☐ Appendix A: Include the statement of each lab member's contribution(s) to this lab.
  - ☐ Appendix B: Include the (well-commented) code you wrote for Tasks 4 and 5.

In addition to including the aforementioned sections, a portion of your grade for this lab will depend on the writing quality and formatting of this report.

- **Formatting (5 points)**
  - ☐ *Figures:* Each figure is labeled and is referenced in the main text. All figures include a caption directly below the figure which describes the main idea in 1-2 short sentences.
  - ☐ *Plots:* All plots include a title and, if appropriate, a legend. The vertical and horizontal axes are labeled and units are included (if appropriate).
  - ☐ *Sections:* The paper is broken up into clearly defined and numbered sections (e.g., for the introduction, experiments, conclusion, etc.).
- **Grammar, spelling, and conciseness (5 points)**
  - ☐ *Grammar/Spelling:* The report is generally free of grammatical and spelling mistakes.
  - ☐ *Conciseness:* The report is concise; the writing is clear and matter of fact.
- **Audience & reproducibility (5 points)**
  - ☐ *Audience:* Readers with technical knowledge of the subject, but who otherwise have no knowledge of the lab, should be able to understand your report.
  - ☐ *Reproducibility:* The report is written such that a knowledgeable reader could reproduce your results after reading your report and the references therein.