Laboratory 3: Fourier Series

In this lab, you will study **Fourier series** and their relationship to linear systems. We started the semester by analyzing linear systems in the *time domain*; in this lab we will analyze the input-output behavior of an RC circuit in the *frequency domain*. Just as one can find the response of a linear system to any DC input signal by deriving its impulse response in the time domain, one can find the response to any periodic AC input signal by deriving the frequency response of the system to simple sinusoids.

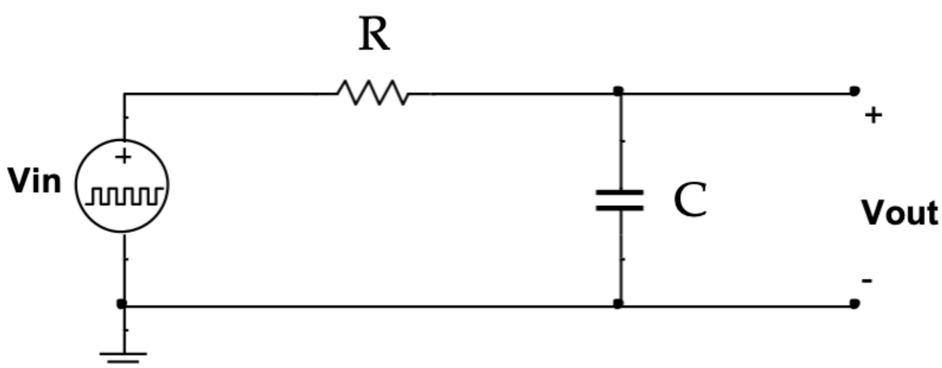

Figure 1: **RC circuit.**

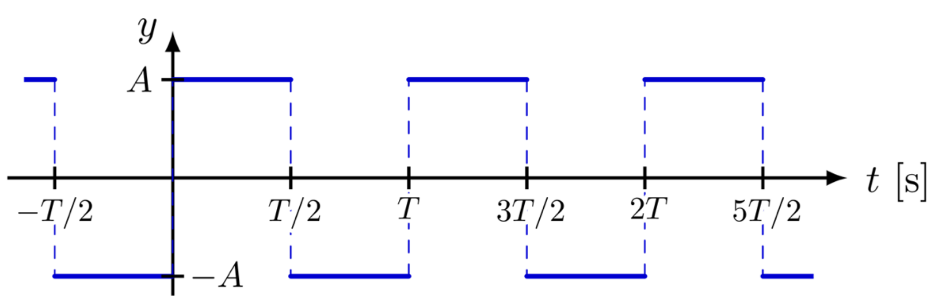

Figure 2: **Square wave.**

1 Lab instructions

1.1 Task 1: Output voltage across the capacitor

1. *Build the circuit.* Build an RC circuit using the resistance and capacitance boxes; ensure that the output voltage is measured across the capacitor. Select values of R and C so that the time constant $\tau = 10$ ms. Connect your circuit to the function generator and to the oscilloscope.

2. *Square wave response.* Set your input signal to a square wave with a frequency of 20 Hz and an amplitude of 2V with zero offset (i.e., your input signal should oscillate between -1V and 1V). Using the `Tekscope Utility`, collect data corresponding to the input and output channels of your circuit, ensuring that you capture two full periods.

3. *Sine wave response.* Set your input signal to a sine wave with the same amplitude as in the previous step; do not touch any other settings on the oscilloscope or function generator (i.e., do not readjust the scaling on the oscilloscope's screen). Collect data for the following frequencies:

$$[20, 60, 100, 140, 180, 220, 260, 300, 340, 380, 420, 460, 500] \tag{1.1}$$

1.2 Task 2: Output voltage across the resistor

1. *Build the circuit.* Swap the resistor and capacitor in your circuit so that the output voltage is measured across the resistor.

2. *Square wave response.* Repeat step 2 from the previous task.

3. *Sine wave response.* Repeat step 3 from the previous task, ensuring that you do not readjust any settings relative to step 2 apart from the input signal frequency.

1.3 Task 3: Theory

1. *Fourier series coefficients for the square wave.* Calculate the Fourier series for the square wave shown in Figure 2 in terms of the period T. Assume that the amplitude $A = 1$.

2. *RC response to sinusoid.* Use the (complex) impedance method to calculate the response of the circuits you built in Tasks 1 and 2 to the input signal $V_{\text{in}}(t) = \sin(\omega t)$. Do not plug in constants; leave your final answer in terms of R, C, and ω.

1.4 Task 4: Coding (generated data)

1. *Fourier approximation of the square wave.* Use the theoretical calculations you did in step 1 of Task 3 to calculate and plot an approximation of the square wave by summing sinusoids weighted by the Fourier coefficients you derived.

2. *Fourier approximation of the RC response.* Use the theoretical calculations you did in step 2 of Task 3 and the code you wrote in the previous step of Task 4 to calculate and plot an approximation of the responses of the RC circuits you built in Tasks 1 and 2 to a square wave input.

1.5 Task 5: Coding (real data)

1. *Fourier approximation of the square wave.* Use the data you collected in lab to show that your input signal (i.e., the square wave) can be reconstructed by summing the sine wave input signals you collected in step 3 of Tasks 1 and 2.
2. *Fourier approximation of the RC response.* Use the data you collected in lab to show that your the responses to the square wave input for both Task 1 and Task 2 can be reconstructed by summing the responses of your RC circuits to the sinusoid inputs.

2 Your report

For this lab, each group will submit a single formal report. As a group, you are expected to share the work equally; every member of the lab group must read and approve of the content, formatting, and writing for each section of the lab report. The formatting requirements for this report are as follows.

- Section 1: **Title (5 points)**
  - ☐ Title of the lab.
  - ☐ Date of submission.
  - ☐ Names of the members of your lab group.
  - ☐ Emails of the members of your lab group.
- Section 2: **Abstract (5 points)**
  - ☐ A short paragraph describing the main question(s) considered in the lab, the experiments you conducted, and the main results. The abstract should be no more than 5-6 sentences.
- Section 3: **Introduction (5 points)**
  - ☐ Motivation: Explain why Fourier are important and what they are used for in the context of linear systems, ensuring that you cite any sources that you use.
  - ☐ Purpose: Explain the purpose of this lab and the main question(s) under consideration.
  - ☐ Experiments: Explain (at a high level) the experiments you performed and why you believe they address the main question(s) considered in this lab.
  - ☐ Results: Explain (at a high level) your main results and whether they agree with your theoretical expectation.
- Section 4: **Theory (20 points)**
  - ☐ Derive the Fourier coefficients for the square wave shown in Figure 2.
  - ☐ Derive the responses of the RC circuits you built for Tasks 1 and 2 to the input signal $V_{\text{in}}(t) = \sin(\omega t)$.
- Section 5: **Methods (5 points)**
  - ☐ Describe the steps you followed to produce your results. You may refer to the lab handout (which you should properly cite) and report any deviations from the specified procedure.
- Section 6: **Experimental results (20 points)**

- ☐ Tasks 1 and 2
  - ☐ Plot all of the data you collected for both RC circuit configurations.
- ☐ Tasks 4 and 5
  - ☐ Step 1: Plot a figure with two columns and several rows of subplots. Each row will correspond to a different frequency. On the left, show the sinusoids (of different frequencies) that you have added to your Fourier series approximation. On the right, show the current approximation of the square wave. For an example, see the Sawtooth example plot in the lecture slides.
  - ☐ Step 2: Create a similar plot to the one described above for the RC responses found in Tasks 1 and 2.

- **Section 7: Discussion (5 points)**
  - ☐ Discuss the improvement in your approximation as you add more terms to your Fourier series approximations.
  - ☐ Bonus points: Can you quantify the error between your approximation and the ground truth signal? If so, plot the error as a function as the number of terms in your Fourier series.

- Section 8: **Conclusion (5 points)**
  - ☐ Summarize your report—the motivation, experiments, and main results—in one or two short paragraphs.

- Section 9: **Acknowledgments (5 points)**
  - ☐ If you discussed this lab with anyone other than the members of your lab group, acknowledge them here. Include a brief description of what was discussed with each person.

- Section 10: **References (5 points)**
  - ☐ Properly cite any sources (including this document) that you used when for this lab.

- Section 11: **Appendices (5 points)**
  - ☐ Appendix A: Include the statement of each lab member's contribution(s) to this lab.
  - ☐ Appendix B: Include the (well-commented) code you wrote for Tasks 4 and 5.

In addition to including the aforementioned sections, a portion of your grade for this lab will depend on the writing quality and formatting of this report.

- **Formatting (5 points)**
  - ☐ *Figures:* Each figure is labeled and is referenced in the main text. All figures include a caption directly below the figure which describes the main idea in 1-2 short sentences.
  - ☐ *Plots:* All plots include a title and, if appropriate, a legend. The vertical and horizontal axes are labeled and units are included (if appropriate).
  - ☐ *Sections:* The paper is broken up into clearly defined and numbered sections (e.g., for the introduction, experiments, conclusion, etc.).

- **Grammar, spelling, and conciseness (5 points)**
  - ☐ *Grammar/Spelling:* The report is generally free of grammatical and spelling mistakes.

- ☐ *Conciseness:* The report is concise; the writing is clear and matter of fact.

- **Audience & reproducibility (5 points)**
  - ☐ *Audience:* Readers with technical knowledge of the subject, but who otherwise have no knowledge of the lab, should be able to understand your report.
  - ☐ *Reproducibility:* The report is written such that a knowledgeable reader could reproduce your results after reading your report and the references therein.