1. $z = x + iy$

Show that $zz^* = |z|^2$

$|z| = \sqrt{x^2 + y^2}$ $\qquad$ $|z|^2 = x^2 + y^2$

$zz^* = (x+iy)(x-iy) = x^2 - xiy + xiy + y^2$

$= x^2 + y^2$

$\therefore \quad zz^* = |z|^2$

2 a) $(2-i) - (1-i2) = 1 + i$

$|1+i| = \sqrt{2}$ $\qquad$ $\theta = 45° = \pi/4$

$\cancel{\sqrt{2} \angle 45°}$ $\qquad$ $1 + i, \quad \sqrt{2} \angle \pi/4 = \sqrt{2}\, e^{\pi/4\, i}$

b) $(\sqrt{2} - i) - i(1 - i\sqrt{2}) = \sqrt{2} - i - i - \sqrt{2} = -2i$

$|-2i| = \sqrt{2^2} = 2$

$0 - 2i \quad , \quad 2 \angle -\pi/2 = 2e^{-\pi/2\, i}$

c) $(2-i3)(-2+i)$

$-4+2i+6i+3\cancel{i} = -1+8i$

$\sqrt{65}\ \angle\tan^{-1}(8/-1)$

$\sqrt{65}\ \angle 1.6952 = \sqrt{65}\ e^{1.6952i}$

d) $(3+i)(3-i)(1/5 + i\,1/10)$

$\sqrt{10}\ e^{+0.321i} \cdot \sqrt{10}\ e^{-0.321i} \cdot \sqrt{0.05}\ e^{0.463i}$

$\sqrt{100 \times 0.05}\ e^{0.463i} \cdot e^{0} = \sqrt{5}\ e^{0.463i}$

$\sqrt{5}\left(\cos(0.463) + i\sin(0.463)\right)$

Actual: from MATLAB

$2+i$

$2+i$, $\sqrt{5}\ e^{0.463i}$ $\sqrt{5}\ \angle 0.463$

e) $\dfrac{1+i2}{3-i4} + \dfrac{2-i}{5i}$

$\dfrac{(1+i2)(3+i4)}{(3-i4)(3+i4)} + \dfrac{(2-i)(5i)}{5i\cdot(5i)}$

$\dfrac{3+4i+6i-8}{25} - \dfrac{5+10i}{25} = \dfrac{-10}{25} = -0.4$

$-0.4 = \underline{0.4\, e^{\pi i}} = \underline{0.4 \angle \pi}$

f) $\dfrac{5e^{0}}{(1-i)(2-i)(3-i)} = \dfrac{5e^{0}}{\sqrt{2}\, e^{-\pi/4 i}\ \sqrt{5}\, e^{-0.463 i}\ \sqrt{10}\, e^{-0.321}}$

$\dfrac{5\, e^{0-(-\pi/4 - 0.463 - 0.321)i}}{\sqrt{10 \times 10}}$

$\frac{5}{10} e^{-(-\pi/4 - 0.463 - 0.321)i} = \frac{5}{10} e^{1.5693 i}$

$5/10 \cos 1.5693 + i \frac{5}{10} \sin 1.5693$

Rounding Errors $0.0007 + 0.5i \approx 0.5i = 0.5 \angle \pi/2$

Actual from MATLAB $\underline{0.5i}$

g) $(1-i)^4 = \left(\sqrt{2}\, e^{-\pi/4 i}\right)^4 = (\sqrt{2})^4\, e^{-\pi i}$

$4e^{-\pi i}$

$= 4\left(\cos -\pi + i \sin(-\pi)\right)$

$= -4 + 0$

$= -4 = 4 \angle -\pi, \quad 4e^{-\pi i}$

3. $2e^{i\pi/4}$

$2(\cos \pi/4 + i \sin \pi/4)$

$1.414 + 1.414i$

4. $8(\cos \pi/3 + i \sin \pi/6)$

$8(0.5 + 0.5i) = 4 + 4i$

$\angle \tan^{-1}(4/4) = \angle \pi/4$

$4e^{\pi/4 i}$, phase $= \pi/4$

5 $\left(\cos \pi/3 + i \sin(\pi/3)\right)^2$

$\left(e^{\pi/3 i}\right)^{2} = e^{2/3 \pi i}$

$e^{2/3 \pi i} = \cos 2\pi/3 + i \sin 2\pi/3$

6. $\sqrt[3]{i} = \left(e^{\pi/2 i}\right)^{1/3} = \left(e^{i(\pi/2 + 2k\pi)}\right)^{1/3}$

$\boxed{k=0, \quad e^{i\pi/6}}$, $\boxed{k=1, \quad e^{i5\pi/6}}$, $\boxed{k=2, \quad e^{i\frac{3\pi}{2}}}$

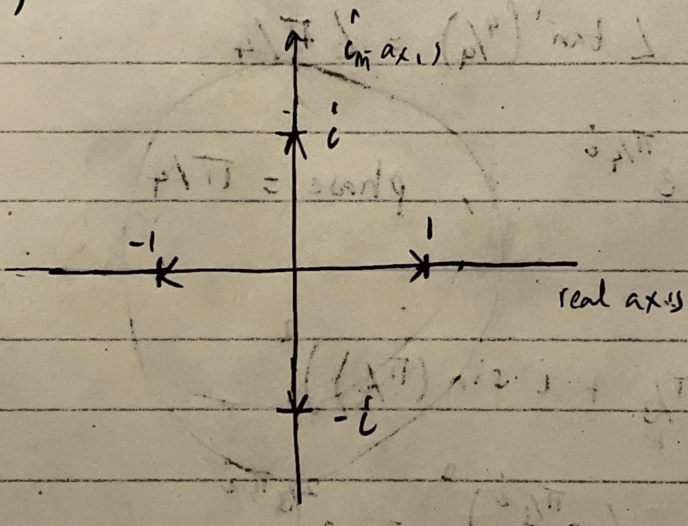
7. $\sqrt[3]{1+i} = \left(e^{i\pi/4}\right)^{1/3} = \left(e^{i(\pi/4 + 2k\pi)}\right)^{1/3}$

$k=0 \quad e^{i\pi/12} \qquad k=1 \quad e^{i3\pi/4} \qquad k=2 \quad e^{i5\pi/4}$

8. $(1)^{1/4} = \left(e^{i0}\right)^{1/4} = \left(e^{i(0+2k\pi)}\right)^{1/4}$

$k=0, \quad 1 \qquad k=1 \quad e^{i\pi/2} = i$

$k=2, \quad e^{i\pi} = -1 \qquad k=3 \quad e^{i\frac{3}{2}\pi} = -i$



9. $(x+1+i3)(x-1+i2)^* = (x+1+i3)(x-1-i2)$

$x^2 \cancel{-x} - 2ix \cancel{+x} - 1 + i2 + 3ix + 3i + 6$

$x^2 + ix - 5i + 5 = \underline{x^2 + ix - 5i + 5}$

10. $e^{ix}(\cos x - i \sin x)$

$e^{ix} e^{-ix} = e^{0} = 1$