HW 3

2.9 a) $\mathcal{J}$ an LTI, relaxed, and BIBO stable system with input $x(n)$ and output $y(n)$.

Show that:

a) if $x(n)$ is periodic with period N [ie, $x(n) = x(n+N)$ for all $n \geq 0$], the output $y(n)$ tends to ~~be~~ a periodic signal with the same period.

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$y(n+N) = \sum_{k=0}^{n+N} h(k)\, x(n+N-k)$$; system is causal, sum up to $k = n+N$, zero output for $k<0$, ignore values $k<0$

$$y(n) = \sum_{k=0}^{n} h(k)\, x(n-k)$$

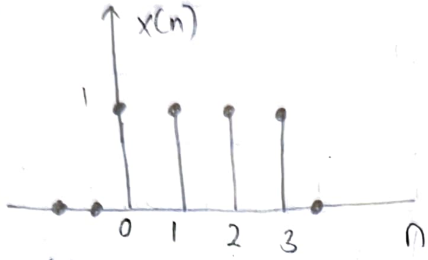
since $x(n)$ is periodic with period N

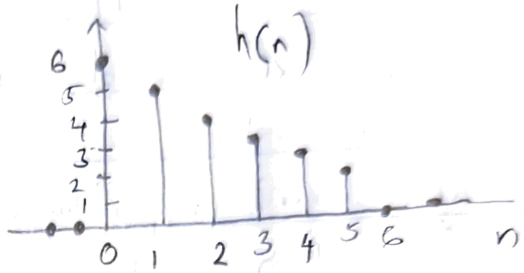
$$x(n+N-k) = x((n-k)+N) = x(n-k)$$

Since the system is BIBO stable, the tail of the convolution where $n-k<0$ or near 0 vanishes as $n \to \infty$ making

$$y(n+N) - y(n) = 0, \Rightarrow y(n+N) = y(n)$$

2.17 a) Convolutions $x(n) * h(n)$ and $h(n) * x(n)$

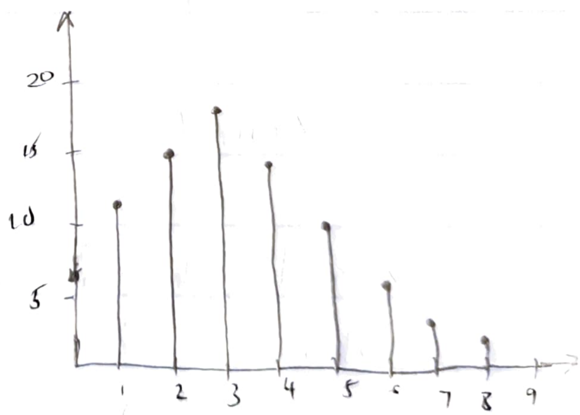




$$y(n) = \sum_{k=}^{n} h(k)\, x(n-k)$$

$h(0) = 6$, $h(3) = 3$
$h(1) = 5$, $h(4) = 2$
$h(2) = 4$, $h(5) = 1$
$h(6) = 0$

$y(0) = h(0) = 6$
$y(1) = h(1) + h(0) = 11$
$y(2) = h(2) + h(1) + h(0) = 15$
$y(3) = h(3) + h(2) + h(1) + h(0) = 18$
$y(4) = h(4) + h(3) + h(2) + h(1) = 14$
$y(5) = h(5) + h(4) + h(3) + h(2) = 10$
$y(6) = h(6) + h(5) + h(4) + h(3) = 6$
$y(7) = h(6) + h(5) + h(4) = 3$
$y(8) = h(6) + h(5) = 1$
$y(9) = h(6) = 0$
$y(n) = 0$, $n < 0$, $n > 9$



Since convolution is commutative, $y(n) = x(n) * h(n)$

$$x(n) * h(n) = h(n) * x(n)$$

2.28 a) let $x(n)$, $N_1 \le n \le N_2$ and $h(n)$, $M_1 \le n \le M_2$ be two finite-duration signals
Determine the range $L_1 \le n \le L_2$ of their convolution in terms of N_1, N_2, M_1, M_2

$x(n)$ is non zero for $N_1 \le n \le N_2$, $h(n)$ for $M_1 \le n \le M_2$
$h(n-k)$ should be non zero for $M_1 \le n-k \le M_2$
and $x(k)$ for $N_1 \le k \le N_2$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$y(n)$ is nonzero on $N_1+M_1 \le n \le M_2+N_2$
$L_1 = N_1 + M_1$, $L_2 = N_2 + M_2$

c) $x(n) = \begin{cases} 1, & -2 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$

illustrate the validity of your results by computing the convolution of the signals

$h(n) = \begin{cases} 2, & -1 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$

$$y(n) = \sum_{k=-3}^{6} x(k)\, h(n-k)$$

$y(-3) = 2$	$y(2) = 8$
$y(-2) = 4$	$y(3) = 8$
$y(-1) = 6$	$y(4) = 6$
$y(0) = 8$	$y(5) = 4$
$y(1) = 8$	$y(6) = 2$

2.35 Determine the response of the system with impulse response

$$h(n) = a^n u(n)$$

to the signal

$$x(n) = u(n) - u(n-10)$$

$$y_{step}(n) = \sum_{k=0}^{n} a^k u(n) = (1 + a + a^2 + \dots + a^n) u(n) = \frac{1-a^{n+1}}{1-a} u(n)$$

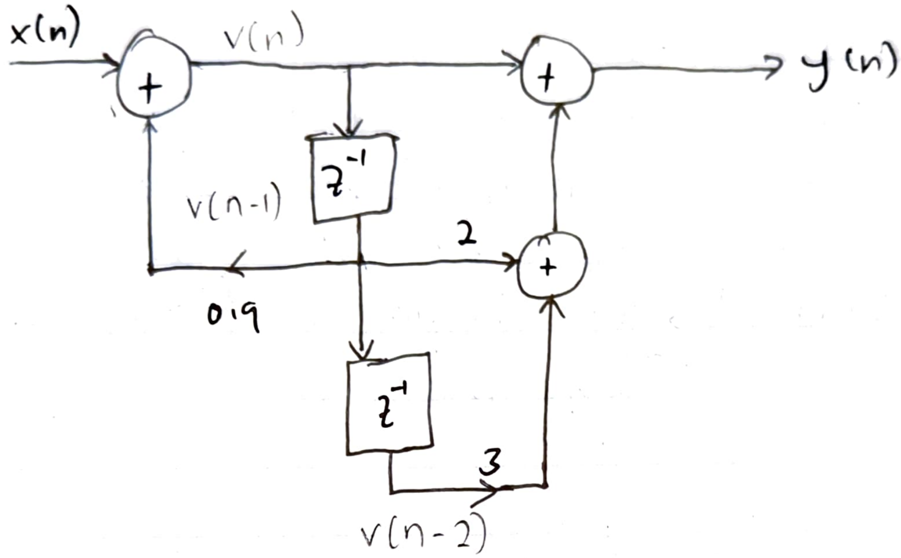
$$y_{step}(n-10) = \frac{1 - a^{(n-10)+1}}{1-a} u(n-10)$$

$$x(n) = u(n) - u(n-10)$$

$$y(n) = h * x = h * u(n) - h * u(-10)(n)$$

$$y(n) = \frac{1-a^{n+1}}{1-a} u(n) - \frac{1-a^{(n-10)+1}}{1-a} u(n-10)$$

2.46



a) Compute first 6 values of the impulse response of the system

$$y(n) = -a_1\, y(n-1) - a_2\, y(n-2) + b_0 x(n) + b_1 x(n-1) + b_2 x(n-2)$$

$-a_1 = 0.9$ $\quad b_0 = 1$ $\quad b_2 = 3$

$-a_2 = 0$ $\quad b_1 = 2$

$$y(n) = -0.9\, y(n-1) + x(n) + 2x(n-1) + 3x(n-2)$$

$x(n) = \delta(n)$

$h(n) = -0.9h(n-1) + \delta(n) + 2\delta(n-1) + 3\delta(n-2)$

$h(0) = 1$ $\quad h(1) = -0.9 + 2 = 1.1$ $\quad h(2) = -0.9(1.1) + 3 = 2.01$

$h(3) = -0.9(2.01) + 3 = 1.191$ $\quad h(4) = -0.9(1.191) = -1.0719$

$h(5) = -0.9(-1.0719) = 0.96471$

C 2.14

d) The nature of response of part (c) ~~is~~ ~~similar~~ for the first 20 samples is similar to the tail end of the plot in (b).
The plot in ~~b~~ (c) remains unchanged after the first 20 samples. The plot in (b) however is still changing even after 100 samples as expected since it is an IIR filter, whereas (c) is an FIR system.

Matlab Live Script with Figures Attached.

C 2.8

a) The system is stable, it appears to decay to 0 as n becomes larger