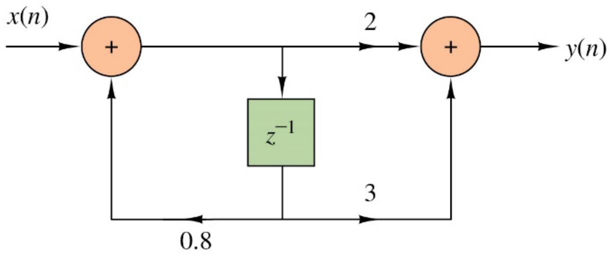
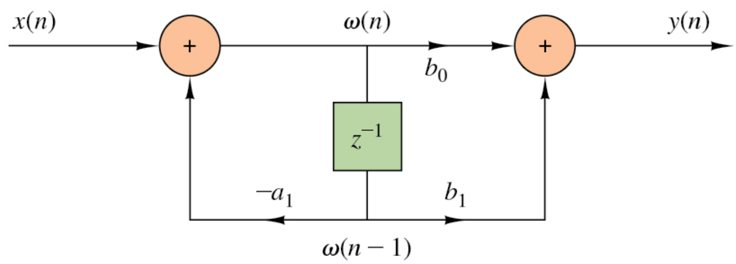
Impulse Response from Diagrams

Example 1:



From the diagram for direct form II:

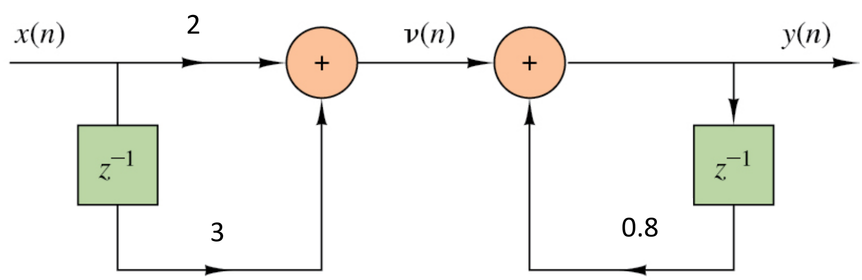


It can be seen that: $-a_1 = 0.8$, $b_0 = 2$, $b_1 = 3$

The difference equation is then:

$$y(n) = -a_1 y(n-1) + b_0 x(n) + b_1 x(n-1)$$
$$y(n) = 0.8y(n-1) + 2x(n) + 3x(n-1)$$

If you want to draw the direct form I diagram:



Impulse response determined using z-transform:

$$\mathcal{Z}\left\{y(n) - 0.8y(n-1) = 2x(n) + 3x(n-1)\right\}$$
$$Y(z) - 0.8z^{-1}Y(z) = 2X(z) + 3z^{-1}X(z)$$
$$\left(1 - 0.8z^{-1}\right)Y(z) = \left(2 + 3z^{-1}\right)X(z)$$
$$H(z) = \frac{Y(z)}{X(z)} = \frac{2 + 3z^{-1}}{1 - 0.8z^{-1}} = \frac{2z + 3}{z - 0.8}$$
$$H(z) = 2\frac{z}{z - 0.8} + 3z^{-1}\left(\frac{z}{z - 0.8}\right)$$

$$h(n) = \mathcal{Z}^{-1}\{H(z)\}$$

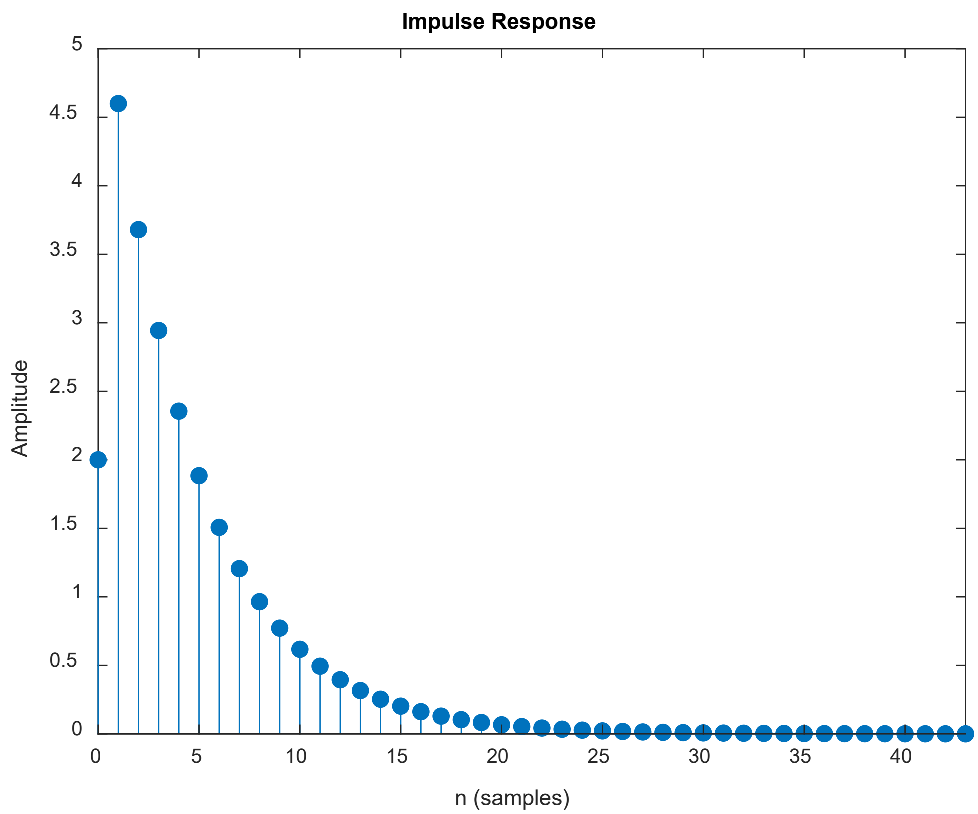
$$h(n) = 2\mathcal{Z}^{-1}\left\{\frac{z}{z-0.8}\right\} + 3\mathcal{Z}^{-1}\left\{z^{-1}\left(\frac{z}{z-0.8}\right)\right\}$$

$$\boxed{h(n) = 2(0.8)^{n}\,u(n) + 3(0.8)^{n-1}\,u(n-1)}$$

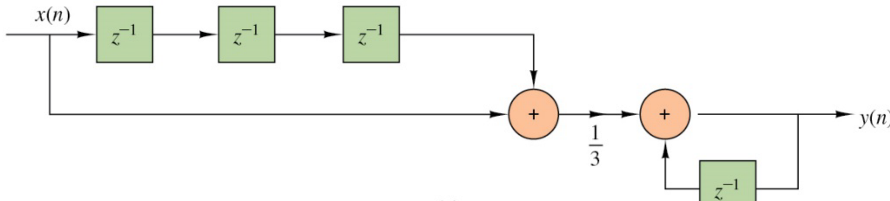
Writing out the first few terms: $y(0) = 2$, $y(1) = 4.6$, $y(2) = 3.68$, $y(3) = 2.944$

This looks correct comparing it to the Matlab plot using

```
impz([2, 3],[1, -0.8])
```



Example 2:



(a)

From the diagram:

$$y(n) = y(n-1) + \frac{1}{3}\left[x(n) + x(n-3)\right]$$

The difference equation is:

$$y(n) - y(n-1) = \frac{1}{3}\left[x(n) + x(n-3)\right]$$

Impulse response determined using z-transform:

$$\mathcal{Z}\left\{y(n) - y(n-1) = \frac{1}{3}\left[x(n) + x(n-3)\right]\right\}$$

$$\left(1 - z^{-1}\right)Y(z) = \frac{1}{3}\left(1 + z^{-3}\right)X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \left(\frac{1}{3}\right)\frac{1 + z^{-3}}{1 - z^{-1}}$$

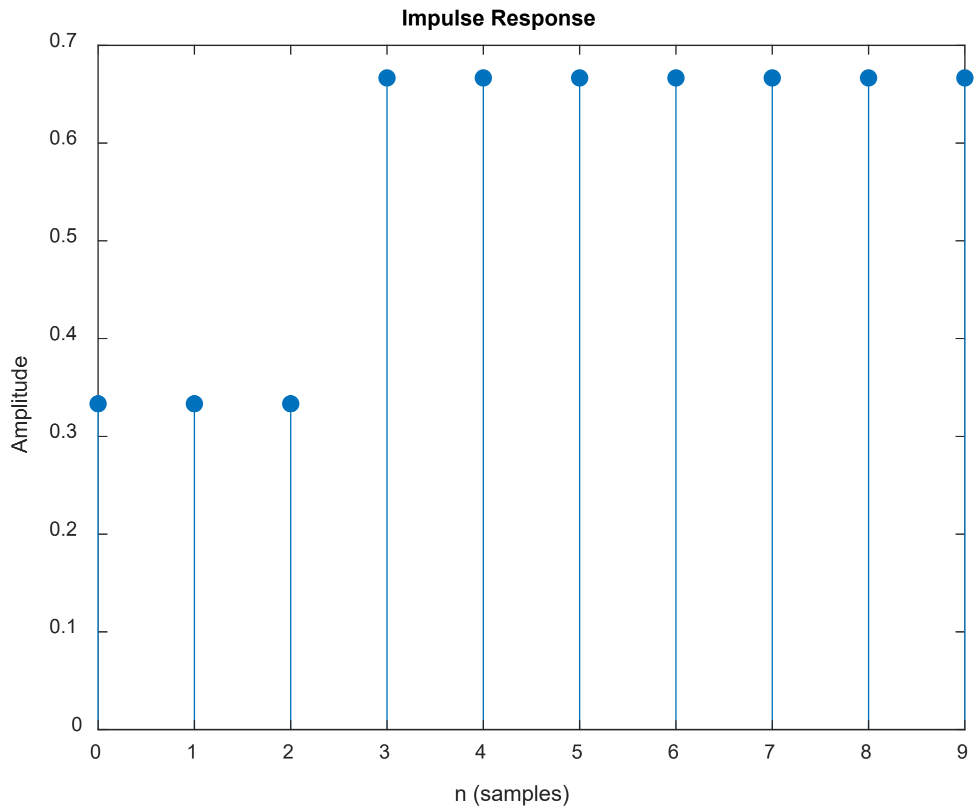
$$H(z) = \frac{1}{3}\frac{1}{1 - z^{-1}} + \frac{1}{3}z^{-3}\left(\frac{1}{1 - z^{-1}}\right)$$

$$h(n) = \mathcal{Z}^{-1}\left\{\frac{1}{3}\frac{1}{1 - z^{-1}} + \frac{1}{3}z^{-3}\left(\frac{1}{1 - z^{-1}}\right)\right\}$$

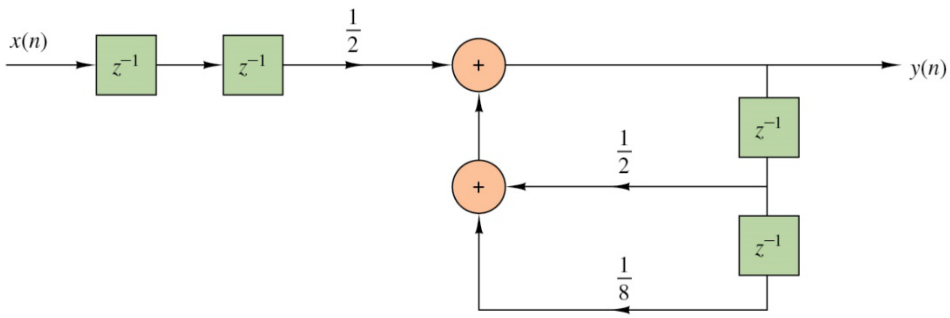
$$\boxed{h(n) = \frac{1}{3}\left[u(n) + u(n-3)\right]}$$

We can identify $b = [1/3, 0, 0, 1/3]$ and $a = [1, -1]$.

This looks correct comparing it to the Matlab plot using
`impz([1/3,0,0,1/3],[1, -1])`


Impulse Response
Amplitude
n (samples)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
7
8
9

Example 3:



$$y(n) = \frac{1}{2}y(n-1) + \frac{1}{8}y(n-2) + \frac{1}{2}x(n-2)$$

The difference equation is:

$$y(n) - \frac{1}{2}y(n-1) - \frac{1}{8}y(n-2) = 0x(n) + 0x(n-1) + \frac{1}{2}x(n-2)$$

(Note: you will need these zeros when you set up the *b* vector form impz.)

Impulse response determined using z-transform:

$$\mathcal{Z}\left\{y(n) - \frac{1}{2}y(n-1) - \frac{1}{8}y(n-2) = \frac{1}{2}x(n-2)\right\}$$

$$\left(1 - \frac{1}{2}z^{-1} - \frac{1}{8}z^{-2}\right)Y(z) = \frac{1}{2}z^{-2}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{2}\frac{z^{-2}}{1 - 1/2\,z^{-1} - 1/8\,z^{-2}}$$

$$H(z) = \frac{1}{2}\frac{1}{z^2 - 1/2\,z - 1/8}$$

Find the roots of the denominator:

$$z^2 - z^2 - 1/2\,z - 1/8 = 0$$

$$z_{p_{1,2}} = \frac{1/2 \pm \sqrt{(1/2)^2 - 4(1)(-1/8)}}{2} = \frac{1 \pm \sqrt{1+2}}{4} = \frac{1 \pm \sqrt{3}}{4}$$

$$z_{p_1} = \left(1+\sqrt{3}\right)/4 \quad ; \quad z_{p_2} = \left(1-\sqrt{3}\right)/4$$

$$\frac{1}{z^2 - z^2 - 1/2\,z - 1/8} = \frac{A}{z - z_{p_1}} + \frac{B}{z - z_{p_2}}$$

Solve for A and B from: $\dfrac{A}{z - z_{p_1}} + \dfrac{B}{z - z_{p_2}} = \dfrac{1}{\left(z - z_{p_1}\right)\left(z - z_{p_2}\right)}$

$$A\left(z - z_{p_2}\right) + B\left(z - z_{p_1}\right) = 1$$

Set $z = z_{p_1}$: $A = \dfrac{1}{z_{p_1} - z_{p_2}}$ Set $z = z_{p_2}$: $B = \dfrac{1}{z_{p_2} - z_{p1}} = -A$

$$A = \frac{4}{\left(1+\sqrt{3}\right) - \left(1-\sqrt{3}\right)} = \frac{2}{\sqrt{3}} \quad ; \quad B = -\frac{2}{\sqrt{3}}$$

$$\frac{1}{z^2 - z^2 - 1/2\, z - 1/8} = \frac{2}{\sqrt{3}}\left(\frac{1}{z - z_{p_1}} - \frac{1}{z - z_{p_2}}\right)$$

$$H(z) = \frac{1}{2}\frac{2}{\sqrt{3}}\left(\frac{1}{z - z_{p_1}} - \frac{1}{z - z_{p_2}}\right)$$

$$h(n) = \frac{1}{\sqrt{3}}\left(\mathcal{Z}^{-1}\left(\frac{1}{z - z_{p_1}}\right) - \mathcal{Z}^{-1}\left[\frac{1}{z - z_{p_2}}\right]\right)$$

$$\mathcal{Z}^{-1}\left(\frac{1}{z - z_{p_1}}\right) = z^{-1}\mathcal{Z}^{-1}\left(\frac{z}{z - z_{p_1}}\right) = \left(z_{p_1}\right)^{n-1} u(n-1) \quad ; \quad \text{Similarly } \mathcal{Z}^{-1}\left(\frac{1}{z - z_{p_2}}\right) = \left(z_{p_2}\right)^{n-1} u(n-1)$$

$$h(n) = \frac{1}{\sqrt{3}}\left(z_{p_1}^{n-1} - z_{p_2}^{n-1}\right)u(n-1)$$

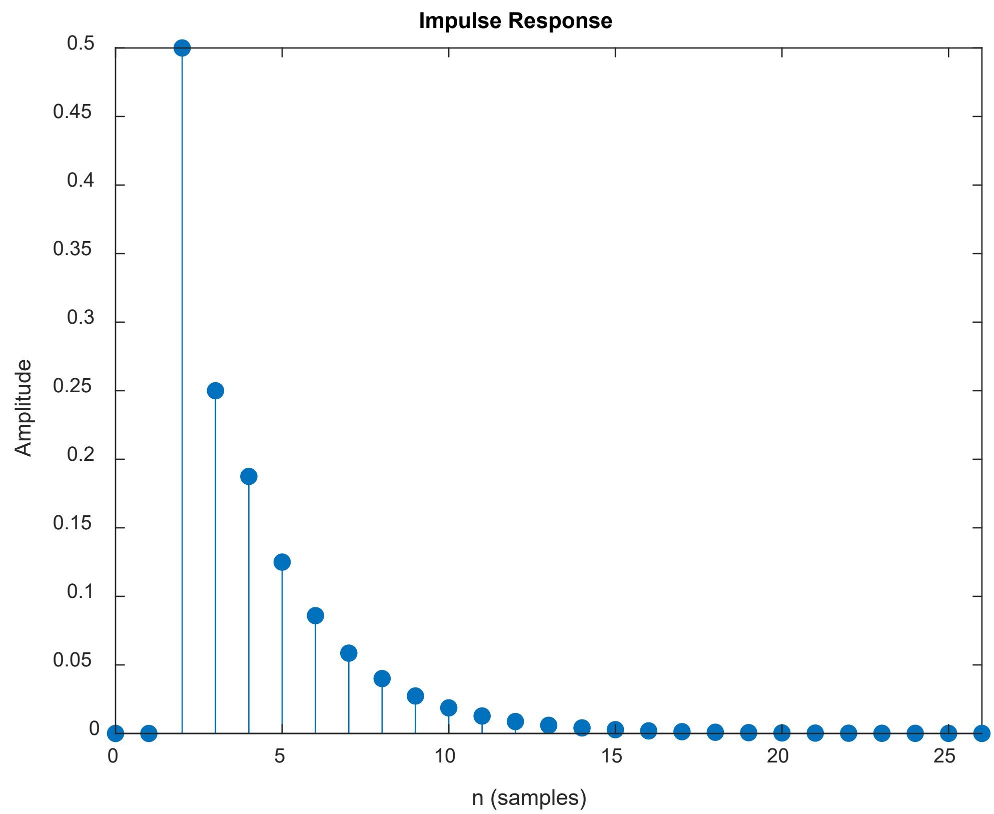
$$h(n) = \frac{1}{\sqrt{3}}\left(\left(\frac{1+\sqrt{3}}{4}\right)^{n-1} - \left(\frac{1-\sqrt{3}}{4}\right)^{n-1}\right)u(n-1)$$

$$h(n) = \frac{1}{\sqrt{3}}\frac{1}{4^{n-1}}\left[\left(1+\sqrt{3}\right)^{n-1} - \left(1-\sqrt{3}\right)^{n-1}\right]u(n-1)$$

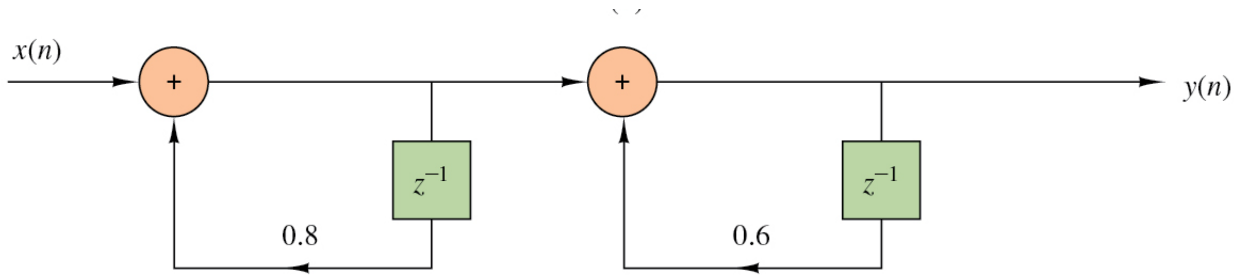
Since the input is delayed by 2, you expect the output to be delayed by 2 also. Notice that $h(1)$=0. Since $h(0) = 0$ and $h(1) = 0$, the output begins at $n = 2$, so the impulse response can be written as:

$$\boxed{h(n) = \frac{1}{\sqrt{3}}\frac{1}{4^{n-1}}\left[\left(1+\sqrt{3}\right)^{n-1} - \left(1-\sqrt{3}\right)^{n-1}\right]u(n-2)}$$

```
impz([0,0,1/2],[1,-1/2,-1/8])
```


Impulse Response
Amplitude
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
5
10
15
20
25
n (samples)

Example 4:



(c)

$$y(n) = \frac{3}{5}y(n-1) + x(n) + \frac{4}{5}x(n-1)$$

The difference equation is:

$$y(n) - \frac{3}{5}y(n-1) = x(n) + \frac{4}{5}x(n-1)$$

Impulse response determined using z-transform:

$$H(z) = \mathcal{Z}\left\{y(n) - \frac{3}{5}y(n-1) = x(n) + \frac{4}{5}x(n-1)\right\}$$

$$\left(1 - \frac{3}{5}z^{-1}\right)Y(z) = \left(1 + \frac{4}{5}z^{-1}\right)X(z)$$

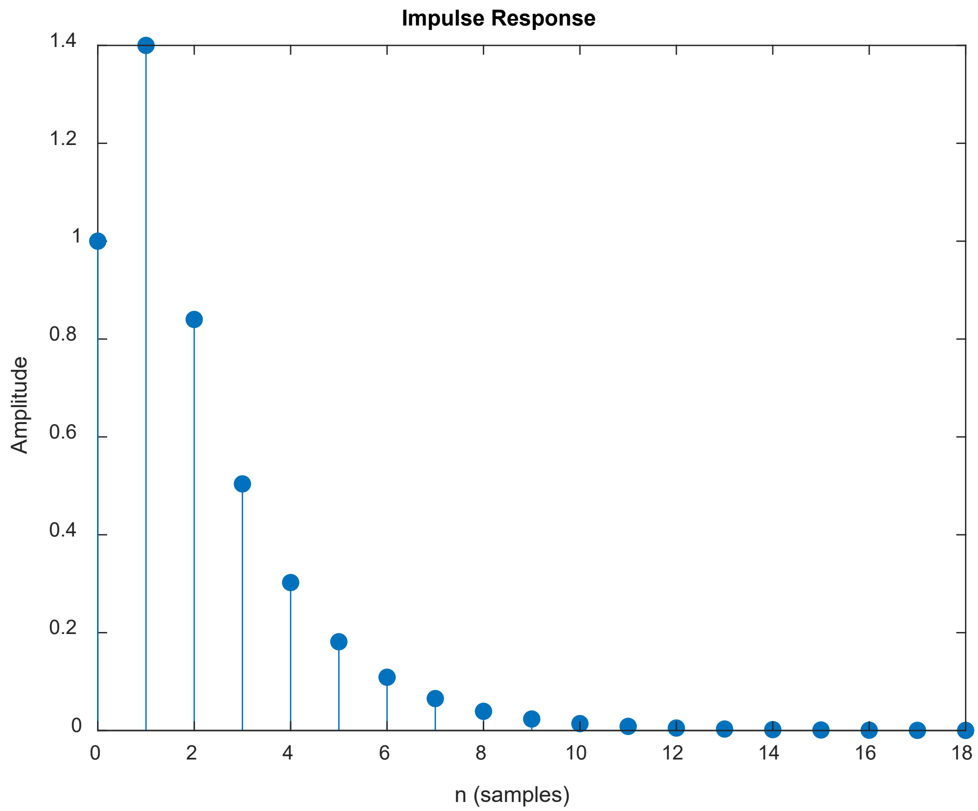
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 4/5z^{-1}}{1 - 3/5z^{-1}}$$

$$H(z) = \frac{z + 4/5}{z - 3/5} = \frac{z}{z - 3/5} + \frac{4/5}{z - 3/5}$$

$$h(n) = \mathcal{Z}^{-1}\left\{\frac{z}{z - 3/5}\right\} + z^{-1}\mathcal{Z}^{-1}\left\{\frac{4/5}{z - 3/5}\right\}$$

$$\boxed{h(n) = \left(\frac{3}{5}\right)^{n} u(n) + \frac{4}{5}\left(\frac{3}{5}\right)^{n-1} u(n-1)}$$

```
impz([1, 4/5],[1, 3/5])
```



Impulse Response
Amplitude
n (samples)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
16
18