~~A#~~

1) $$\begin{bmatrix} \dot{x} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix}$$

a) $$\begin{bmatrix} -\lambda & 1 \\ -k/m & -c/m - \lambda \end{bmatrix}$$

$$-\lambda\left(-c/m - \lambda\right) + k/m = 0$$

$$\lambda \frac{c}{m} + \lambda^2 + k/m = 0$$

$$\lambda^2 + \lambda \frac{c}{m} + k/m = 0$$

$$\lambda = \frac{-c/m \pm \sqrt{c^2/m^2 - 4k/m}}{2}$$

$$\lambda_1 = -c/2m + \frac{\sqrt{c^2/m^2 - 4k/m}}{2}$$

$$\lambda_2 = -\frac{c}{2m} - \frac{\sqrt{c^2/m^2 - 4k/m}}{2}$$

b)

$$\tau = -c/m \qquad \Delta = k/m$$

c) Case 1: $k = 2$, $m = 5$, $c = 7.5$

$$\lambda_1 = -\frac{7.5}{10} + \frac{\sqrt{7.5^2/5^2 - 8/5}}{2} = -0.425$$

$\Delta = 2/5 = 0.4$

$\tau = \cancel{7} - 7.5/5 = -1.5$

$$\lambda_2 = -0.75 - 0.325 = -1.075$$; stable node

Case 2: $k = 3$, $m = 2$, $c = 1$

$$\lambda_1 = -\tfrac{1}{4} + \frac{\sqrt{\tfrac{1}{4} - \tfrac{12}{2}}}{2} = -\tfrac{1}{4} + \frac{\sqrt{-\tfrac{9}{4}}}{2} = -\tfrac{1}{4} + \frac{\sqrt{-1}\ \tfrac{3}{2}}{2}$$

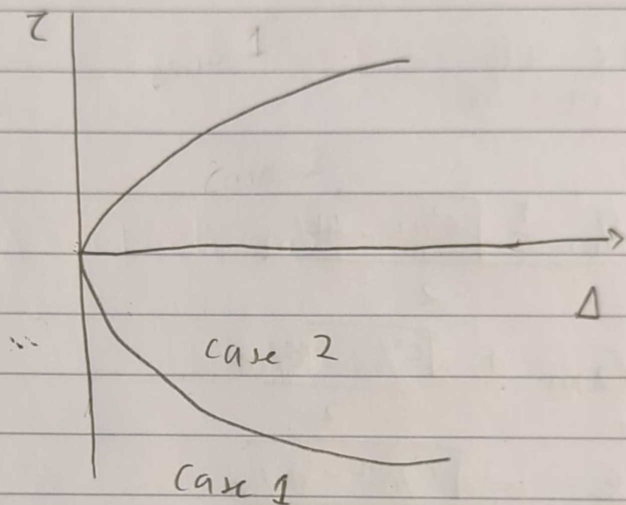
$\Delta = 1.5$

$\tau = -0.5$

$$= -\tfrac{1}{4} + \tfrac{3}{4} i$$

$\lambda_2 = -\tfrac{1}{4} - \tfrac{3}{4} i$; stable spiral

d)



e)

$\dot{x} = x$

$\dot{v} = -\frac{k}{m} x - \frac{c}{m} v$

case 1: $\rightarrow$ $\dot{x} = x$

$\dot{v} = -\frac{2}{5} x - 1.5 v$

cases

case 2: $\rightarrow$ $\dot{x} = x$

$\dot{v} = -\frac{3}{2} x - \frac{1}{2} v$

Computer generated plots included in order. 1st one is case 1, second case 2

f) Case 1 has real eigenvalues, and is stable. Therefore, mass-spring-dashpot system when ~~rotated~~ released from a non-zero starting point comes to a stop exponentially. It will therefore be smooth. This is commonly known as the overdamped case.

Case 2 on the other hand has complex eigenvalues, and is stable. Therefore the system comes to a stop but with oscillations due to the imaginary part of the eigenvalues. You'll therefore see some oscillation unlike in case 1. This is commonly known as the underdamped case.

g) The regions allowed are the stable spirals and stable nodes. This is because first, the quantities mass, spring constant and damping coefficient can never be negative in such a system. ~~That is~~ ~~The~~

The system also has to be stable since no external energy is being added to the system. Therefore it must always eventually settle to an equilibrium value.

This ~~ensures~~ guarantees that we have the two regions ignoring the edge case.

2. $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$\tau = a + d \qquad \Delta = ad - cb$

A: $\tau = a+d > 0$; $\Delta = ad - cb < 0$: Saddle Point

$\begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} \qquad \tau = 2 , \Delta = -1$

B: $\tau = a+d > 0$; $\Delta = ad - cb = 0$

$\cancel{\begin{bmatrix} 2 & [illegible] \\ 4 & 2 \end{bmatrix}} \quad \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \qquad \tau = 4 \qquad \Delta = 0$

c: $\tau = a+d > 0$; $\Delta = ad - cb > 0$: Unstable node

$\cancel{\tau^2 < 4\Delta} \quad \tau^2 > 4\Delta$

$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \qquad \tau = 5 \qquad \Delta = 6$

D: $\tau = a+d > 0$; $\Delta = ad - cb > 0$: Unstable ~~spiral~~ node

$\tau^2 = 4\Delta \qquad \tau^2 = 4 , 4\Delta = 4$

$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad \tau = 2 \qquad \Delta = 1$

E: $\tau = a + d > 0$; $\Delta = ad - cb > 0$: Unstable Spiral

$\tau^2 < 4\Delta$

$$\begin{bmatrix} -1 & -4 \\ 4 & -1 \end{bmatrix}$$

$\tau = -2$ $\Delta = 17$

F: $\tau = a + d = 0$ $\Delta = ad - cb > 0$: Center

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$\tau = 0$ $\Delta = 1$

G: $\tau = a + d < 0$ $\Delta = ad - cb > 0$: Stable Spiral

$\tau^2 < 4\Delta$

$$\begin{bmatrix} -1 & -4 \\ 4 & -1 \end{bmatrix}$$

$\tau^2 = 4$; $\tau = -2$

$4\Delta = 68$ $\Delta = 17$

H: $\tau = a + d < 0$ $\Delta = ad - cb > 0$: Stable ~~Spiral~~ Node

$\tau^2 = 4\Delta$

$$\begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$$

$\tau^2 = 4$, $4\Delta = 4$ $\tau = -2$

$\Delta = 1$

I: $\tau = a + d < 0$ $\Delta = ad - cb > 0$: Stable Node

$$\begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$$

$\tau^2 > 4\Delta$

$\tau = -5$ $\tau = 6$

J: $\tau = a + d < 0$ $\qquad \Delta = ad - cb = 0$

$$\begin{bmatrix} -2 & +1 \\ +4 & -2 \end{bmatrix} \qquad \tau = -4 \qquad \Delta = 0$$

K: $\tau = a + d < 0$ $\qquad \Delta = ad - cb < 0$ $\qquad$ Saddle

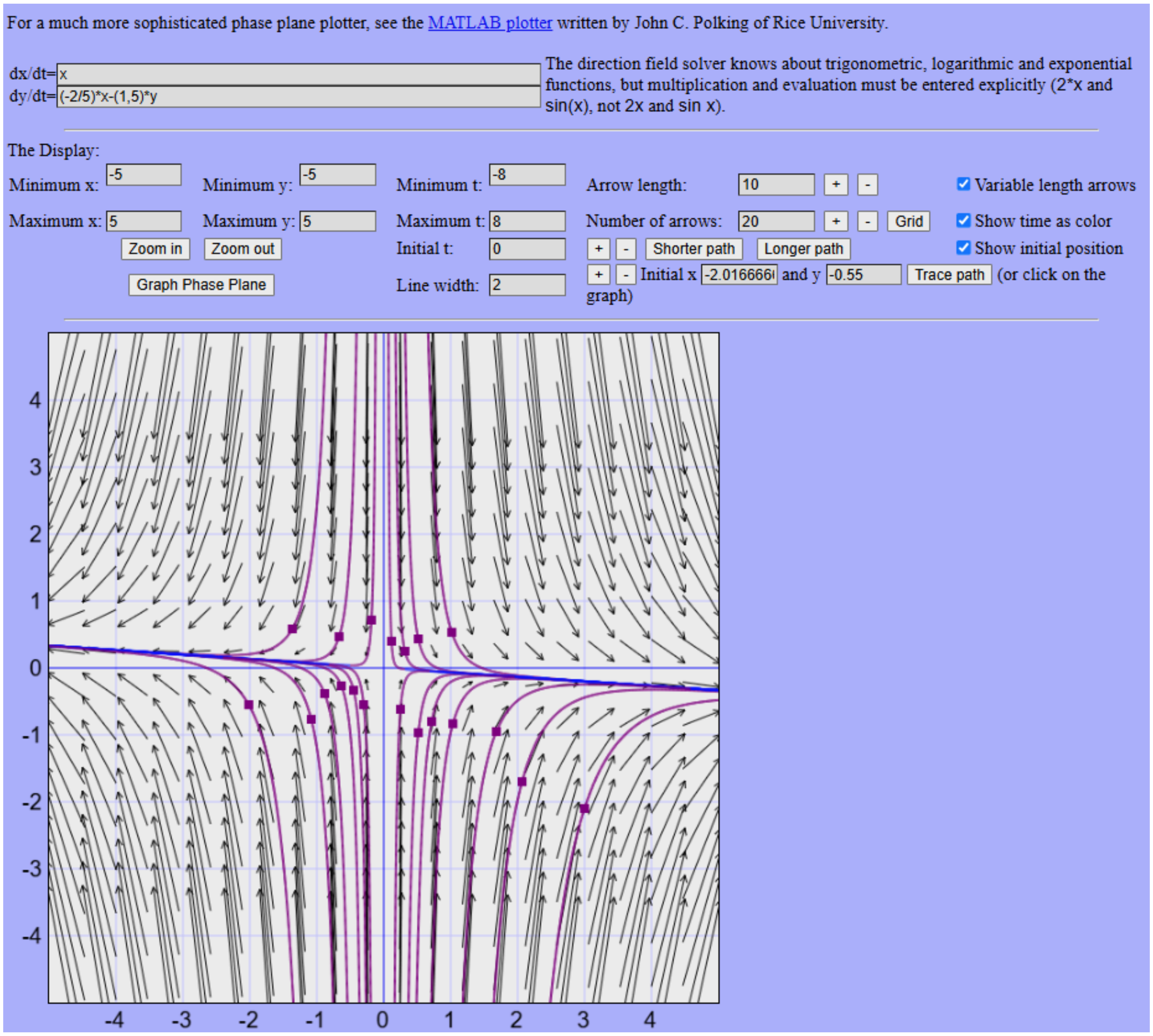
$$\begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \qquad \tau = -1 \qquad \Delta = -1$$

L: $\tau = a + d = 0$ $\qquad \Delta = ad - cb < 0$ $\qquad$ Saddle

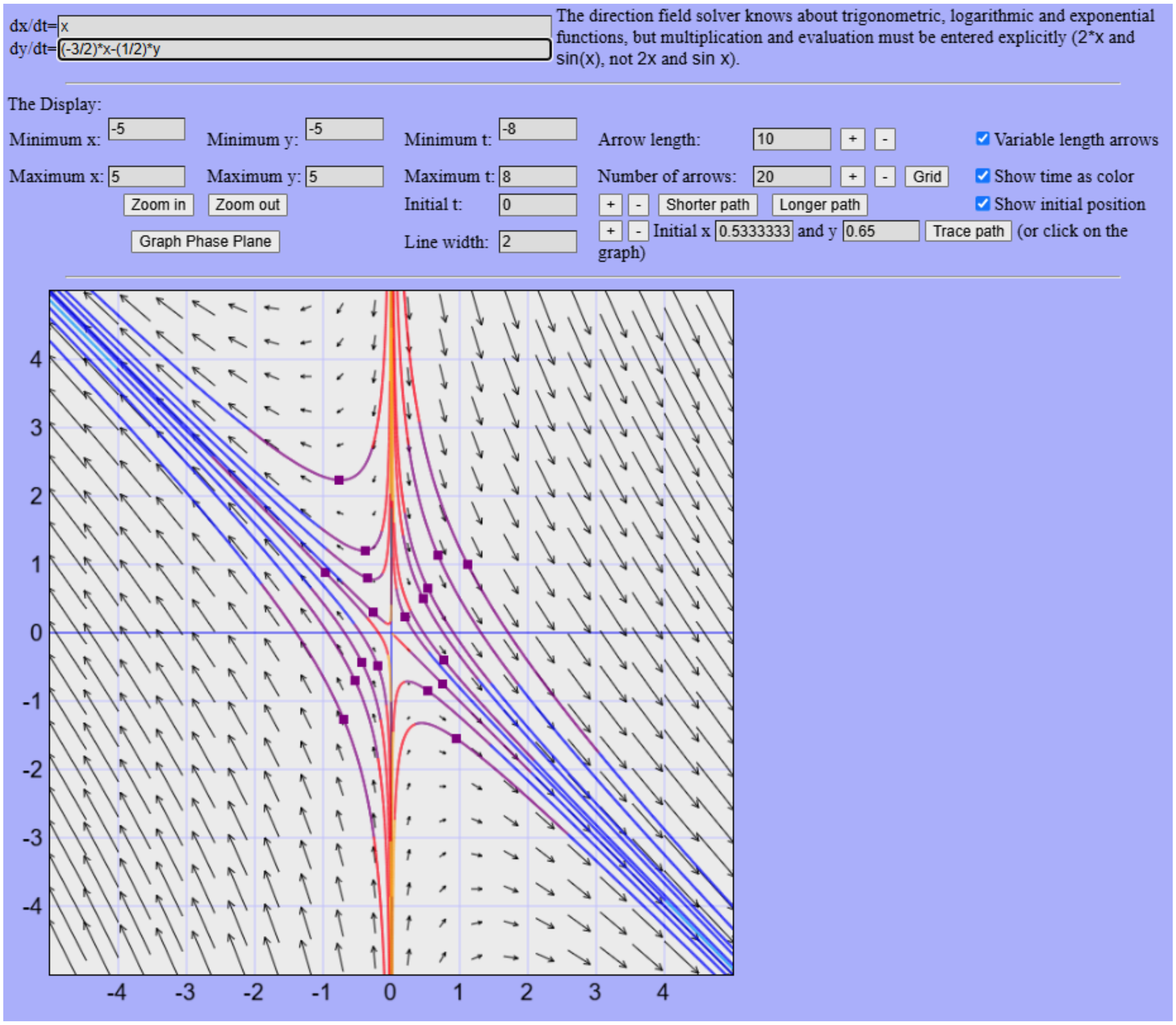
$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \tau = 0 \qquad \Delta = -1$$

See Next Page for All figures

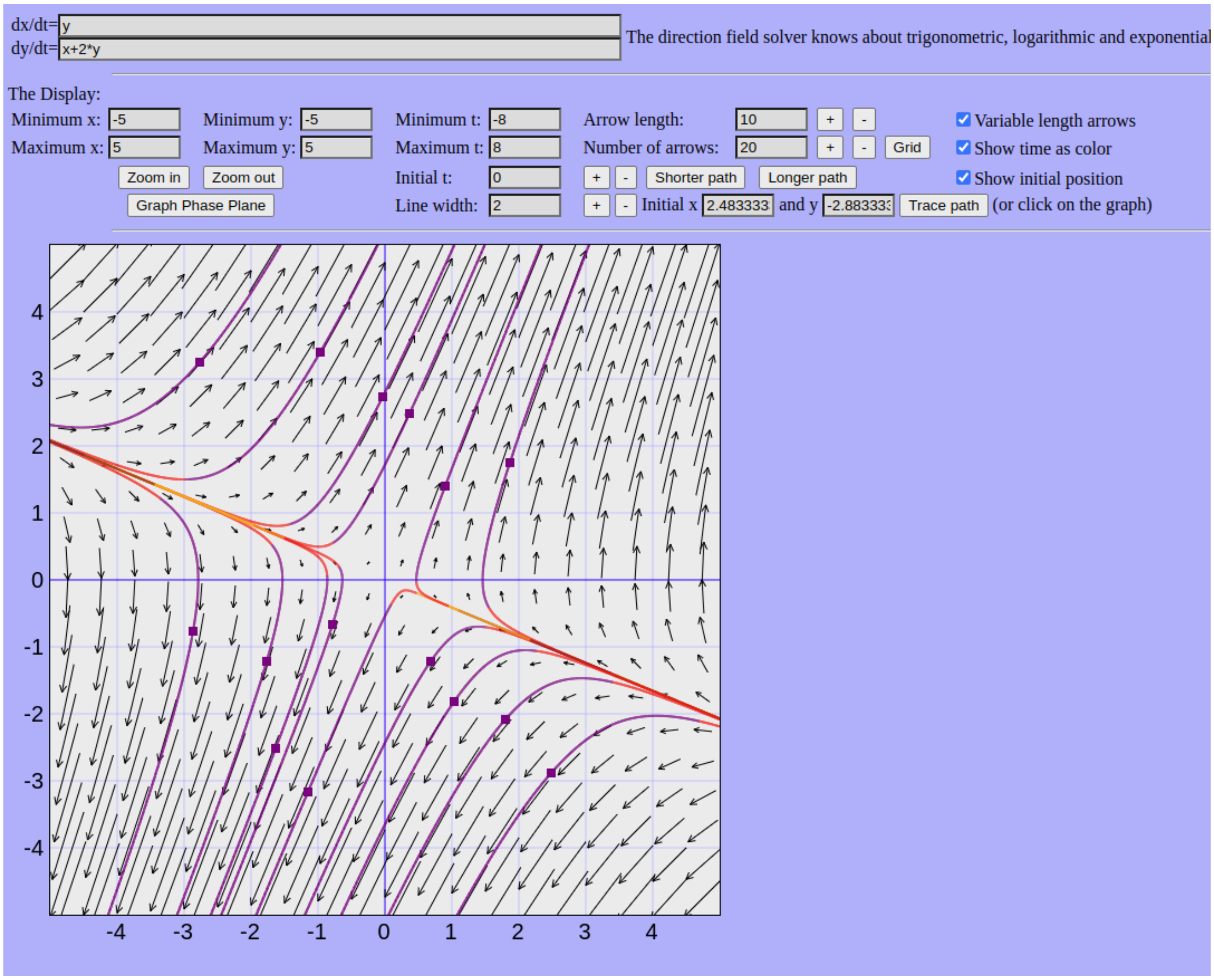
Ques 1e Case 1



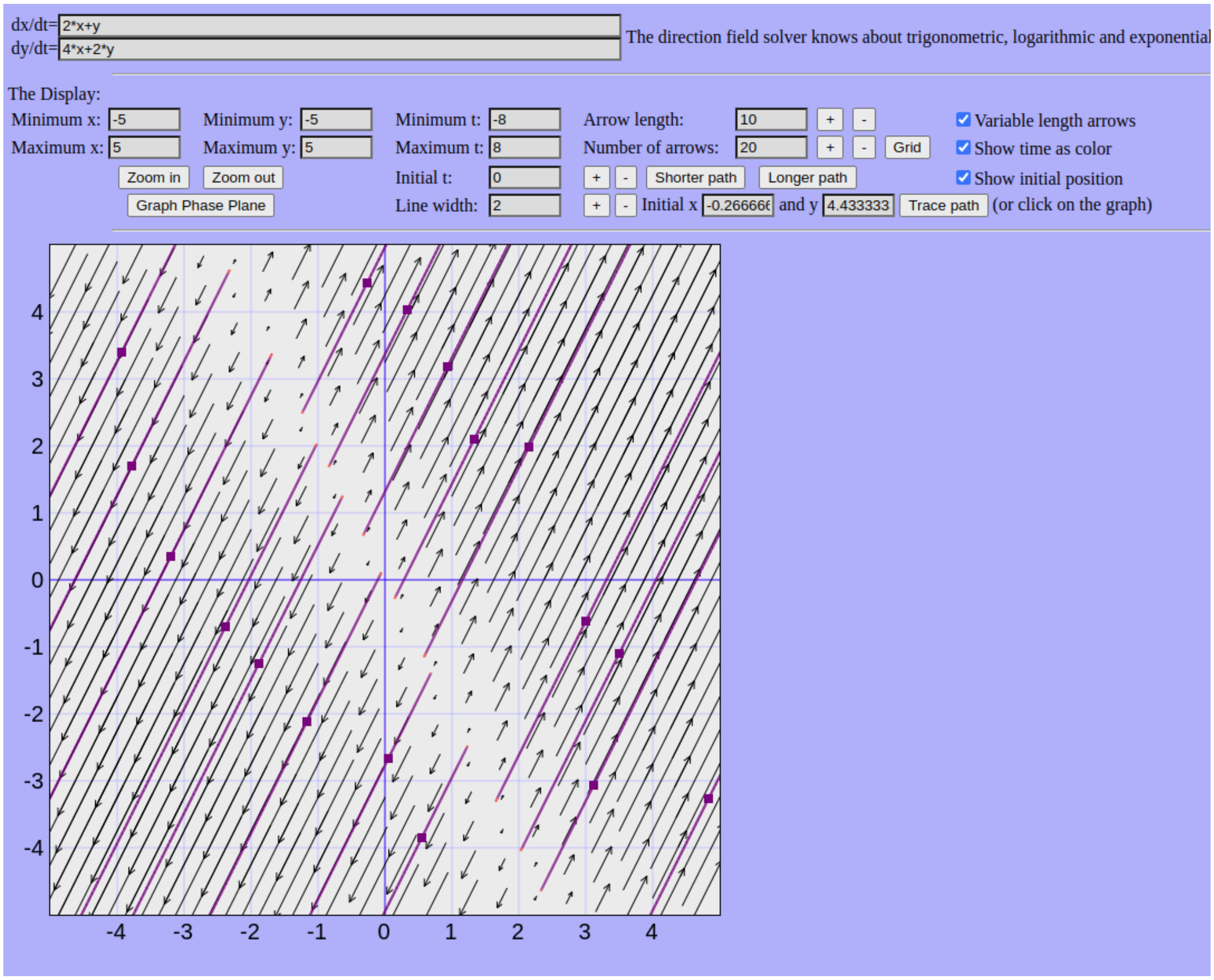
Ques 1e Case 2



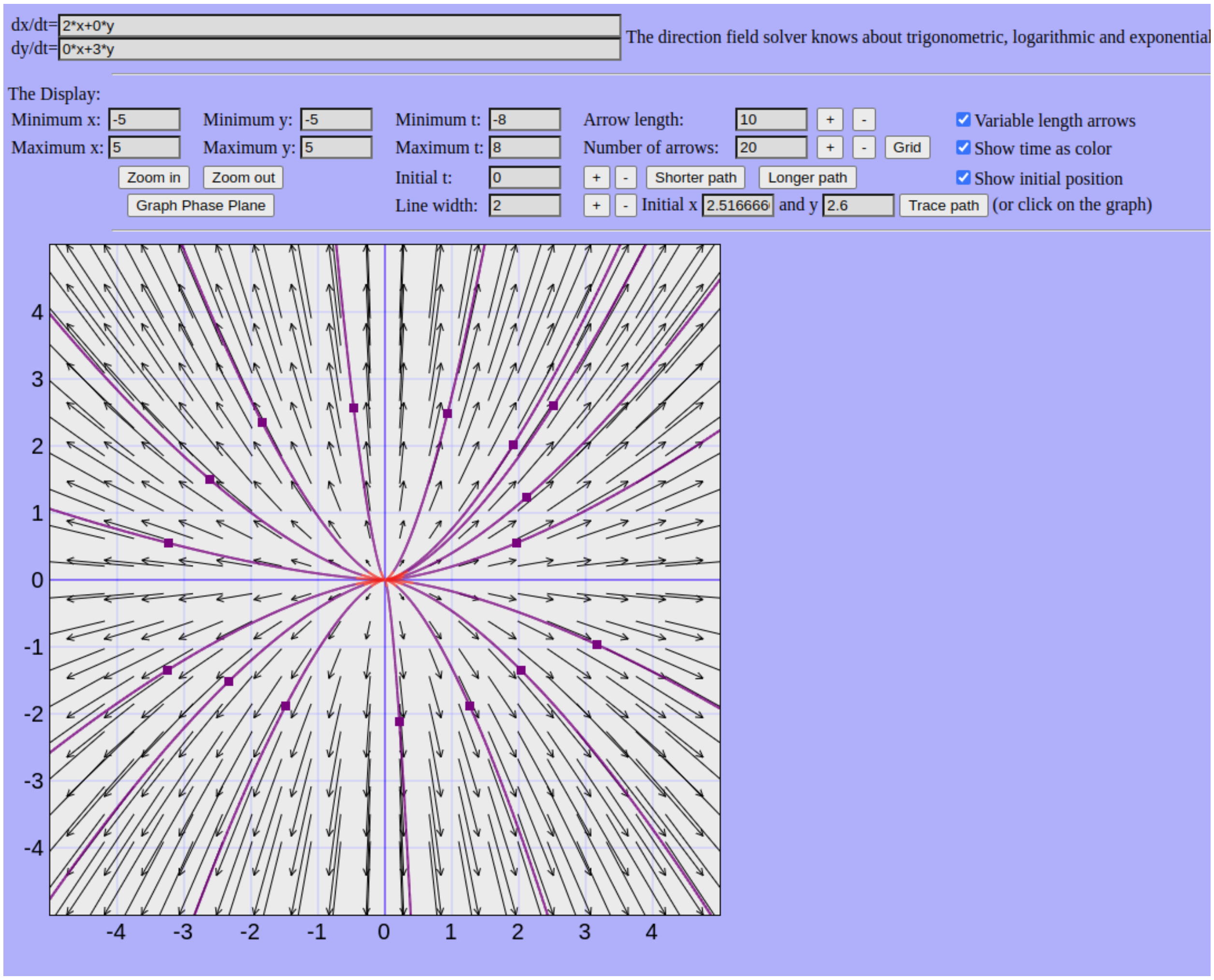
Point A


dx/dt= y
dy/dt= x+2*y
The direction field solver knows about trigonometric, logarithmic and exponential
The Display:
Minimum x: -5
Maximum x: 5
Minimum y: -5
Maximum y: 5
Minimum t: -8
Maximum t: 8
Initial t: 0
Line width: 2
Arrow length: 10
Number of arrows: 20
Grid
Variable length arrows
Show time as color
Show initial position
Zoom in
Zoom out
Graph Phase Plane
Shorter path
Longer path
Initial x 2.483333
and y -2.883333
Trace path
(or click on the graph)
4
3
2
1
0
-1
-2
-3
-4
-4
-3
-2
-1
0
1
2
3
4

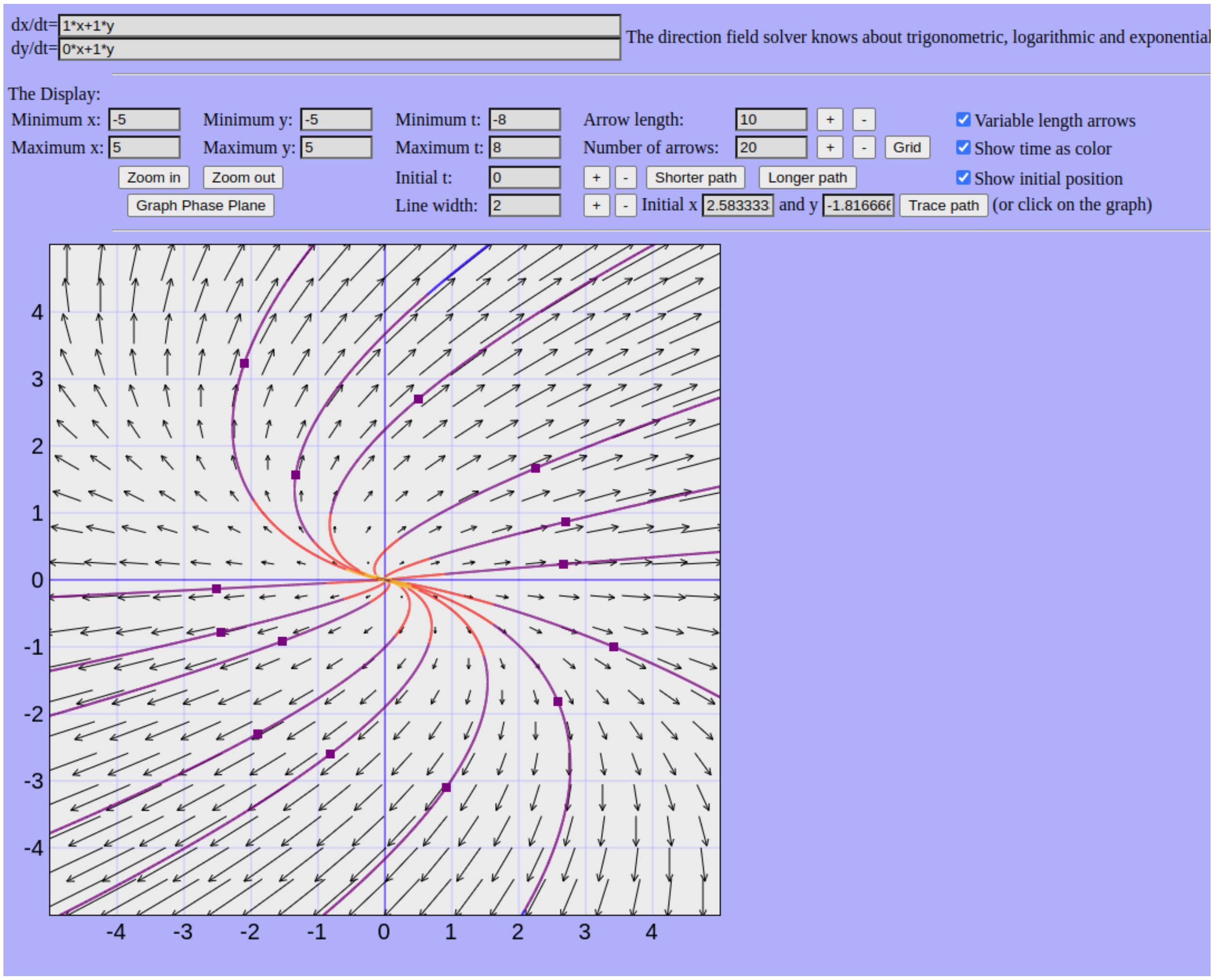
Point B


dx/dt= 2*x+y
dy/dt= 4*x+2*y
The direction field solver knows about trigonometric, logarithmic and exponential
The Display:
Minimum x: -5
Maximum x: 5
Minimum y: -5
Maximum y: 5
Zoom in
Zoom out
Graph Phase Plane
Minimum t: -8
Maximum t: 8
Initial t: 0
Line width: 2
Arrow length: 10
Number of arrows: 20
Grid
Shorter path
Longer path
Initial x -0.266666 and y 4.433333
Trace path
(or click on the graph)
Variable length arrows
Show time as color
Show initial position
4
3
2
1
0
-1
-2
-3
-4
-4
-3
-2
-1
0
1
2
3
4

Point C

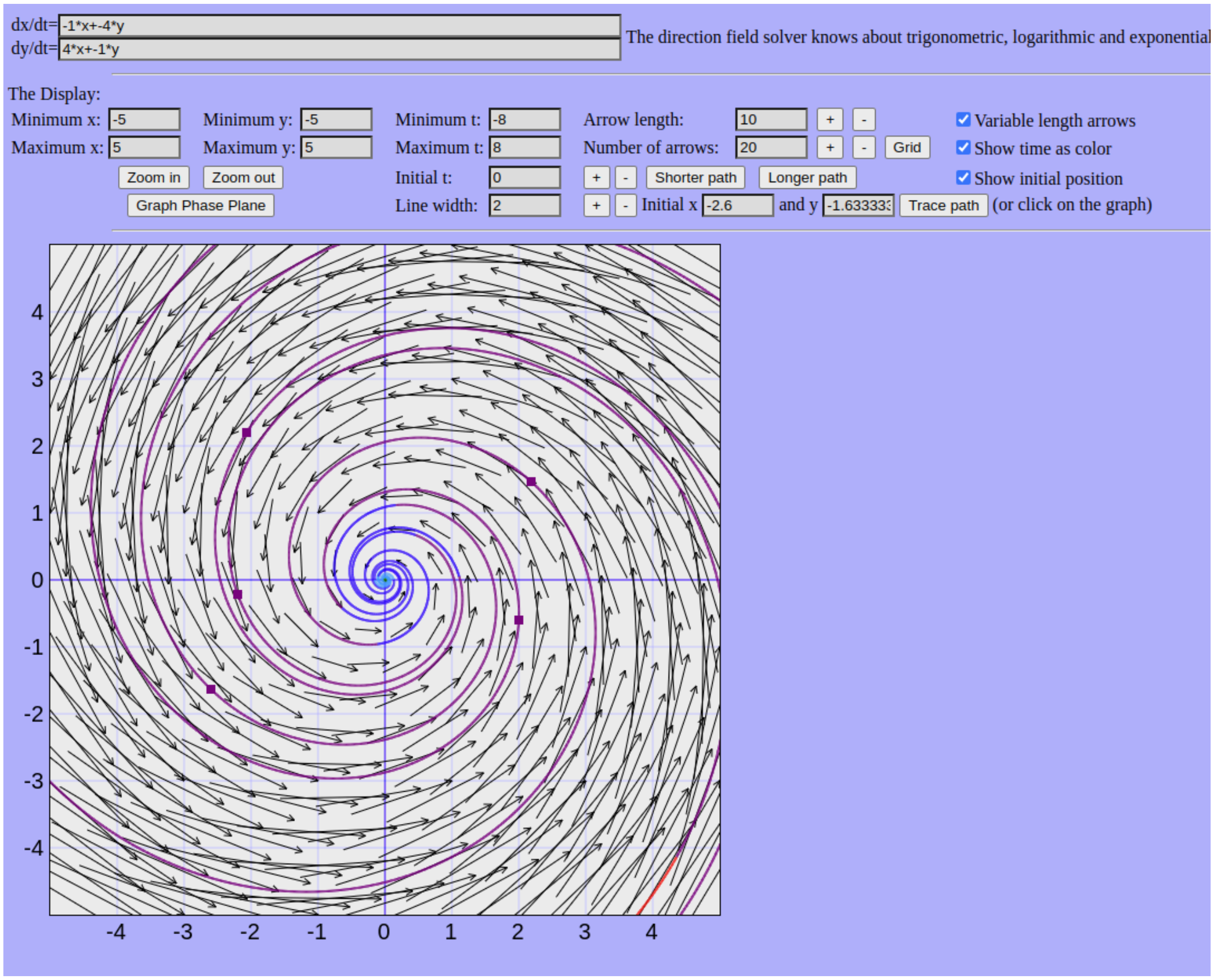

dx/dt= 2*x+0*y
dy/dt= 0*x+3*y
The direction field solver knows about trigonometric, logarithmic and exponential
The Display:
Minimum x: -5
Maximum x: 5
Minimum y: -5
Maximum y: 5
Minimum t: -8
Maximum t: 8
Initial t: 0
Line width: 2
Arrow length: 10
Number of arrows: 20
Grid
Zoom in
Zoom out
Graph Phase Plane
Shorter path
Longer path
Initial x 2.516666 and y 2.6
Trace path
(or click on the graph)
Variable length arrows
Show time as color
Show initial position
4
3
2
1
0
-1
-2
-3
-4
-4
-3
-2
-1
0
1
2
3
4

Point D

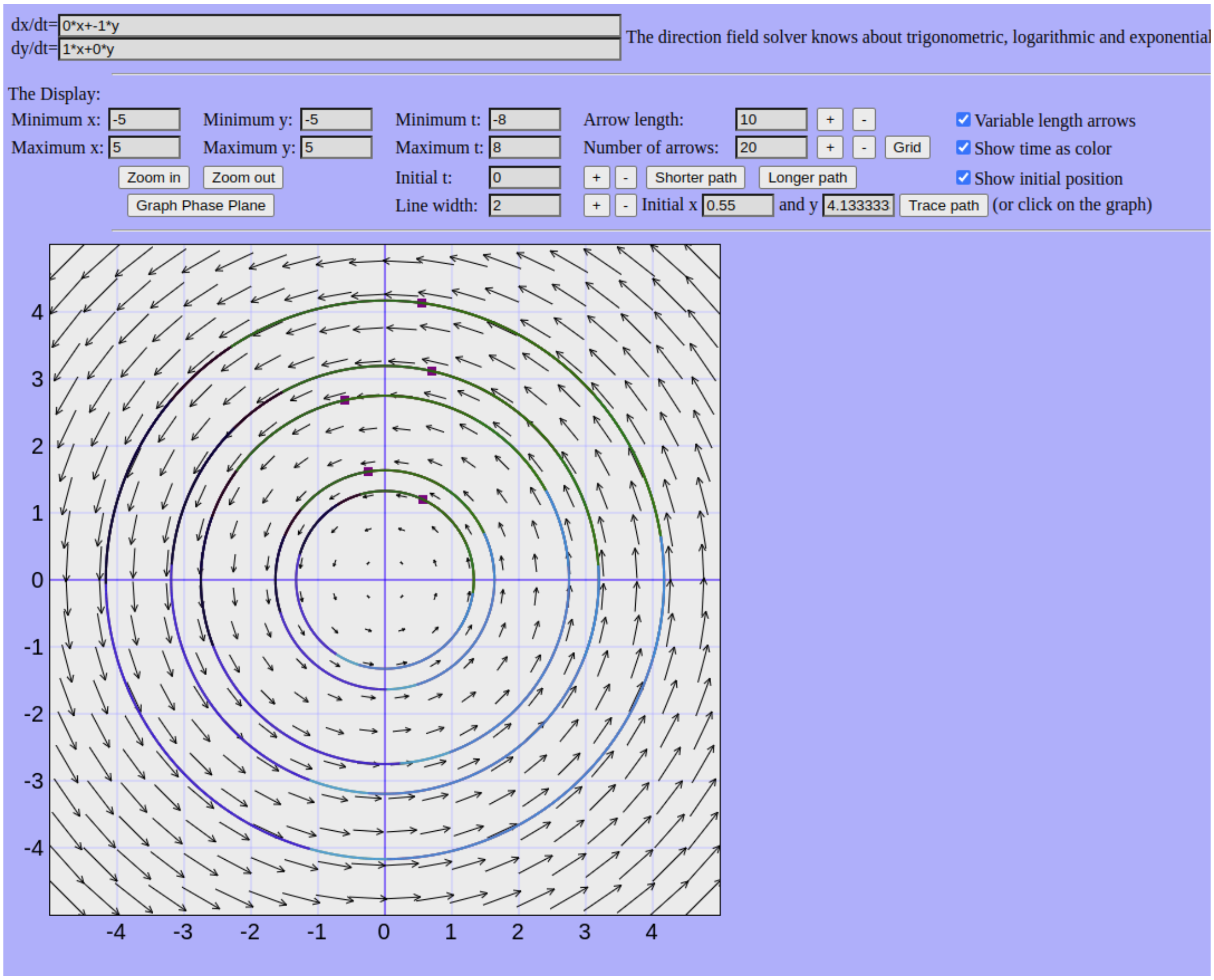


dx/dt= 1*x+1*y
dy/dt= 0*x+1*y
The direction field solver knows about trigonometric, logarithmic and exponential
The Display:
Minimum x: -5
Maximum x: 5
Minimum y: -5
Maximum y: 5
Minimum t: -8
Maximum t: 8
Initial t: 0
Line width: 2
Arrow length: 10
Number of arrows: 20
Grid
Variable length arrows
Show time as color
Show initial position
Zoom in
Zoom out
Graph Phase Plane
Shorter path
Longer path
Initial x 2.583333 and y -1.81666
Trace path
(or click on the graph)
4
3
2
1
0
-1
-2
-3
-4
-4
-3
-2
-1
0
1
2
3
4

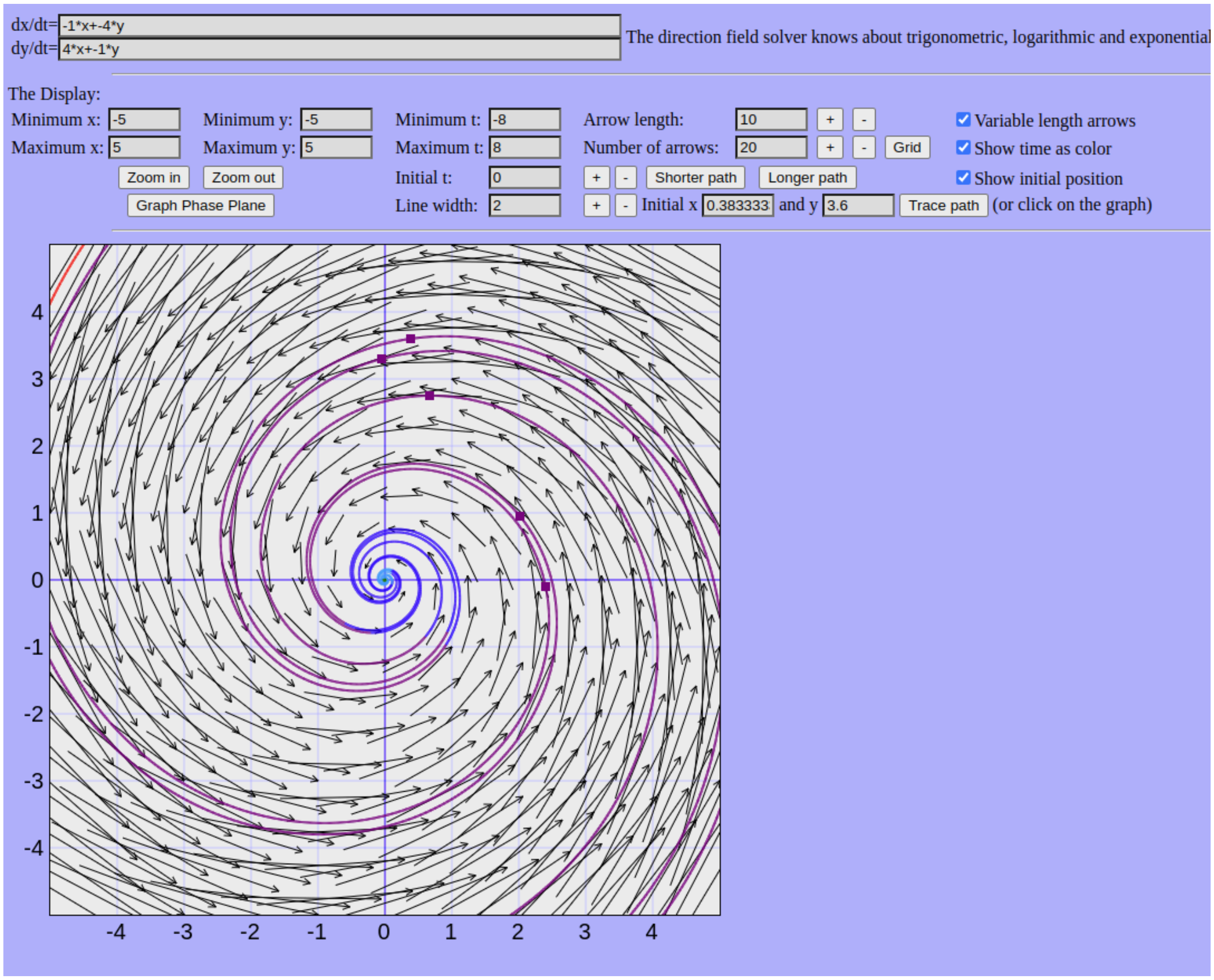
Point E


dx/dt= -1*x+-4*y
dy/dt= 4*x+-1*y
The direction field solver knows about trigonometric, logarithmic and exponential
The Display:
Minimum x: -5
Maximum x: 5
Minimum y: -5
Maximum y: 5
Zoom in
Zoom out
Graph Phase Plane
Minimum t: -8
Maximum t: 8
Initial t: 0
Line width: 2
Arrow length: 10
Number of arrows: 20
Grid
Shorter path
Longer path
Initial x -2.6 and y -1.63333
Trace path
(or click on the graph)
Variable length arrows
Show time as color
Show initial position
4
3
2
1
0
-1
-2
-3
-4

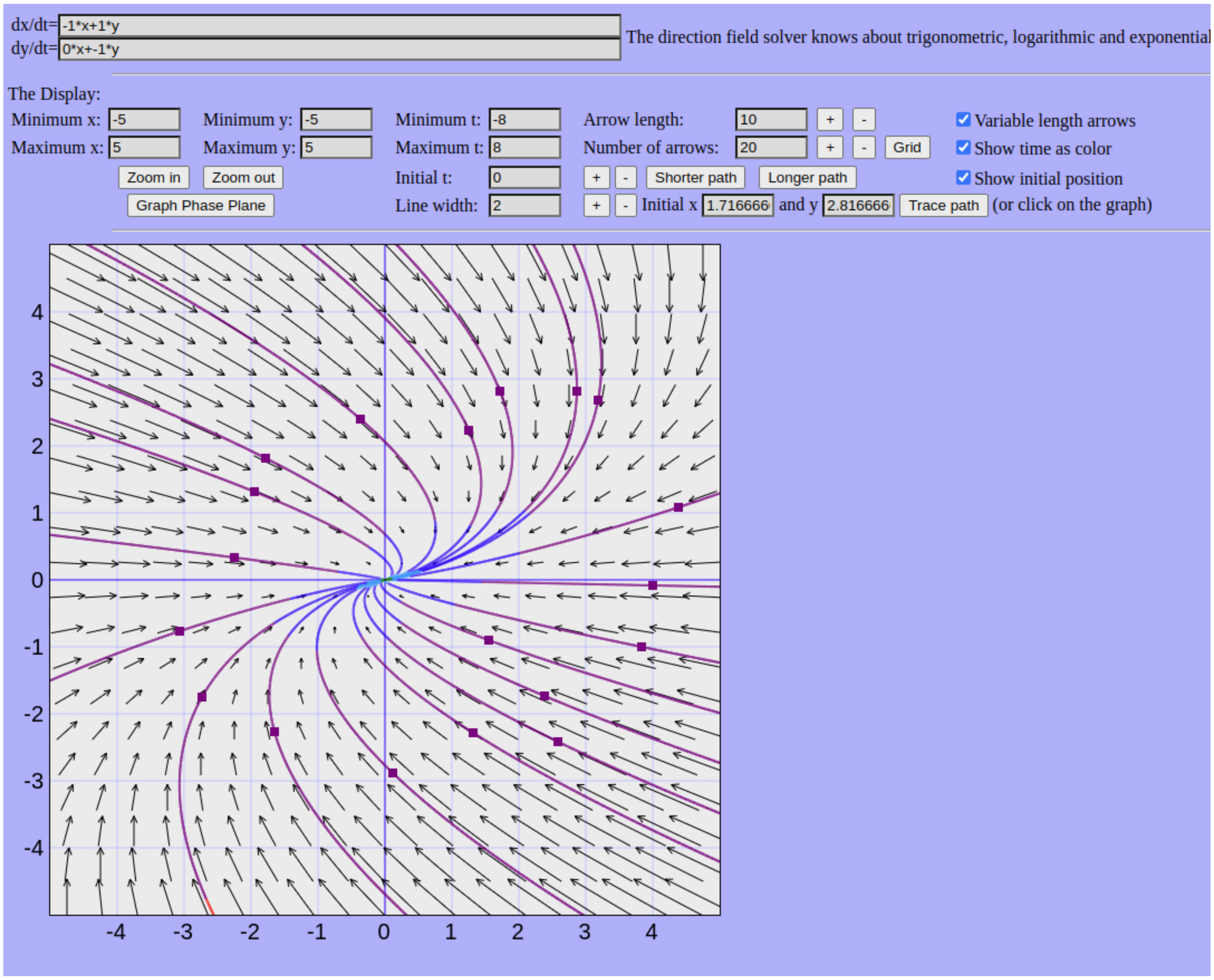
Point F


dx/dt= 0*x+-1*y
dy/dt= 1*x+0*y
The direction field solver knows about trigonometric, logarithmic and exponential
The Display:
Minimum x: -5
Maximum x: 5
Minimum y: -5
Maximum y: 5
Minimum t: -8
Maximum t: 8
Initial t: 0
Line width: 2
Arrow length: 10
Number of arrows: 20
Grid
Variable length arrows
Show time as color
Show initial position
Zoom in
Zoom out
Graph Phase Plane
Shorter path
Longer path
Initial x 0.55 and y 4.133333
Trace path
(or click on the graph)
4
3
2
1
0
-1
-2
-3
-4
-4
-3
-2
-1
0
1
2
3
4

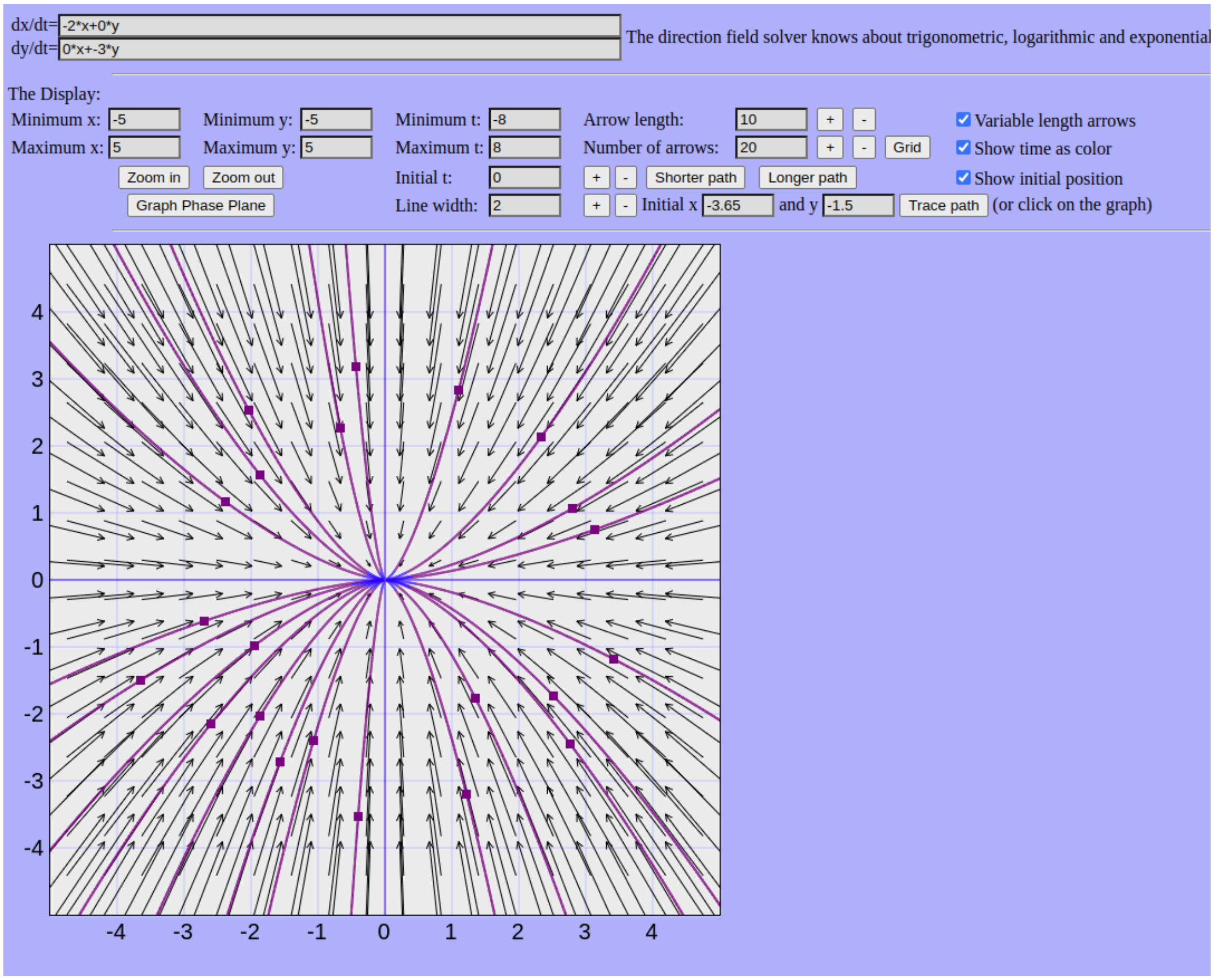
Point G


dx/dt= -1*x+-4*y
dy/dt= 4*x+-1*y
The direction field solver knows about trigonometric, logarithmic and exponential
The Display:
Minimum x: -5
Maximum x: 5
Minimum y: -5
Maximum y: 5
Minimum t: -8
Maximum t: 8
Initial t: 0
Line width: 2
Arrow length: 10
Number of arrows: 20
Grid
Zoom in
Zoom out
Graph Phase Plane
Shorter path
Longer path
Initial x 0.383333 and y 3.6
Trace path
(or click on the graph)
Variable length arrows
Show time as color
Show initial position
4
3
2
1
0
-1
-2
-3
-4
-4
-3
-2
-1
0
1
2
3
4

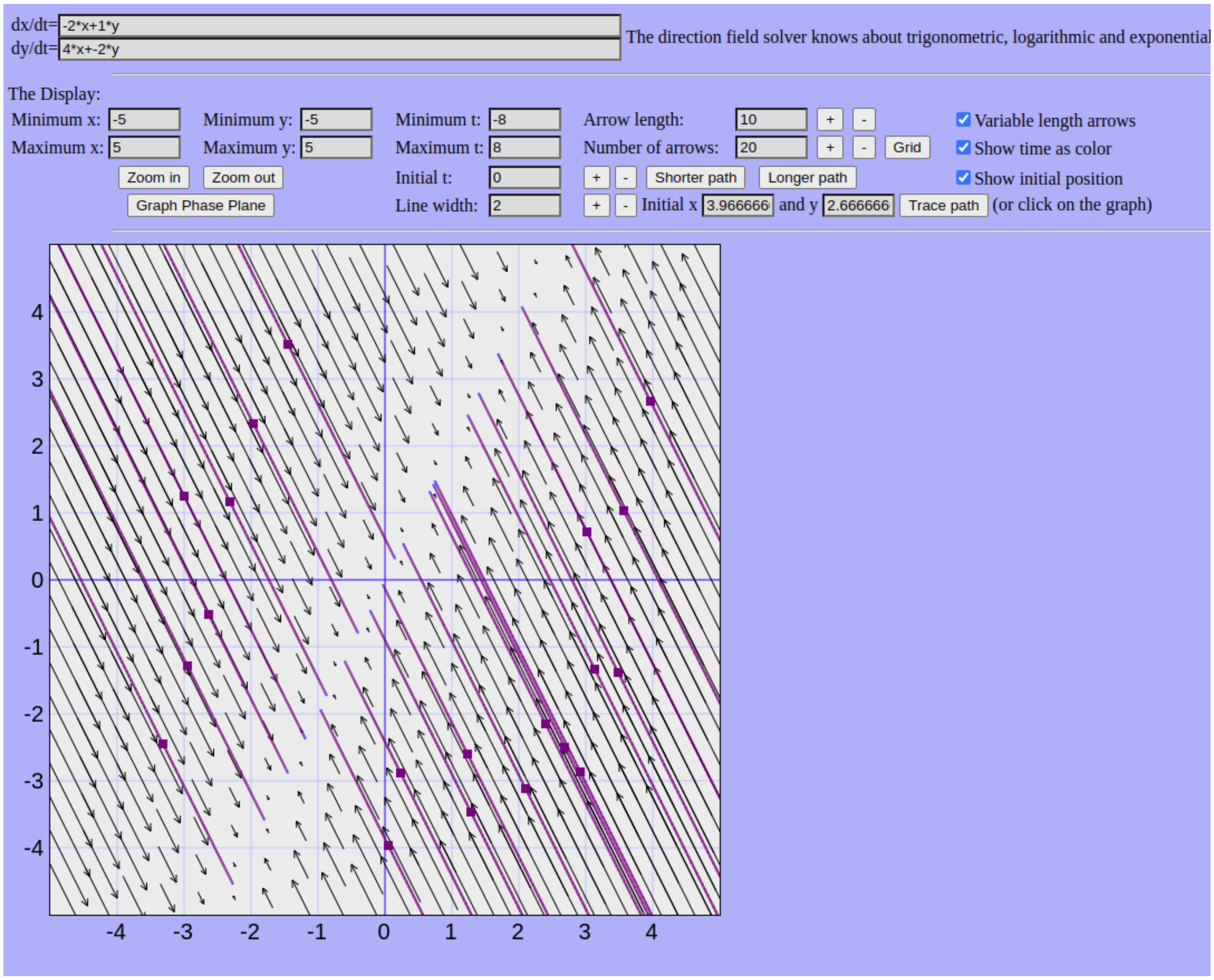
Point H


dx/dt= -1*x+1*y
dy/dt= 0*x+-1*y
The direction field solver knows about trigonometric, logarithmic and exponential
The Display:
Minimum x: -5
Maximum x: 5
Minimum y: -5
Maximum y: 5
Zoom in
Zoom out
Graph Phase Plane
Minimum t: -8
Maximum t: 8
Initial t: 0
Line width: 2
Arrow length: 10
Number of arrows: 20
Grid
Shorter path
Longer path
Initial x 1.716666 and y 2.816666
Trace path
(or click on the graph)
Variable length arrows
Show time as color
Show initial position
4
3
2
1
0
-1
-2
-3
-4
-4
-3
-2
-1
0
1
2
3
4

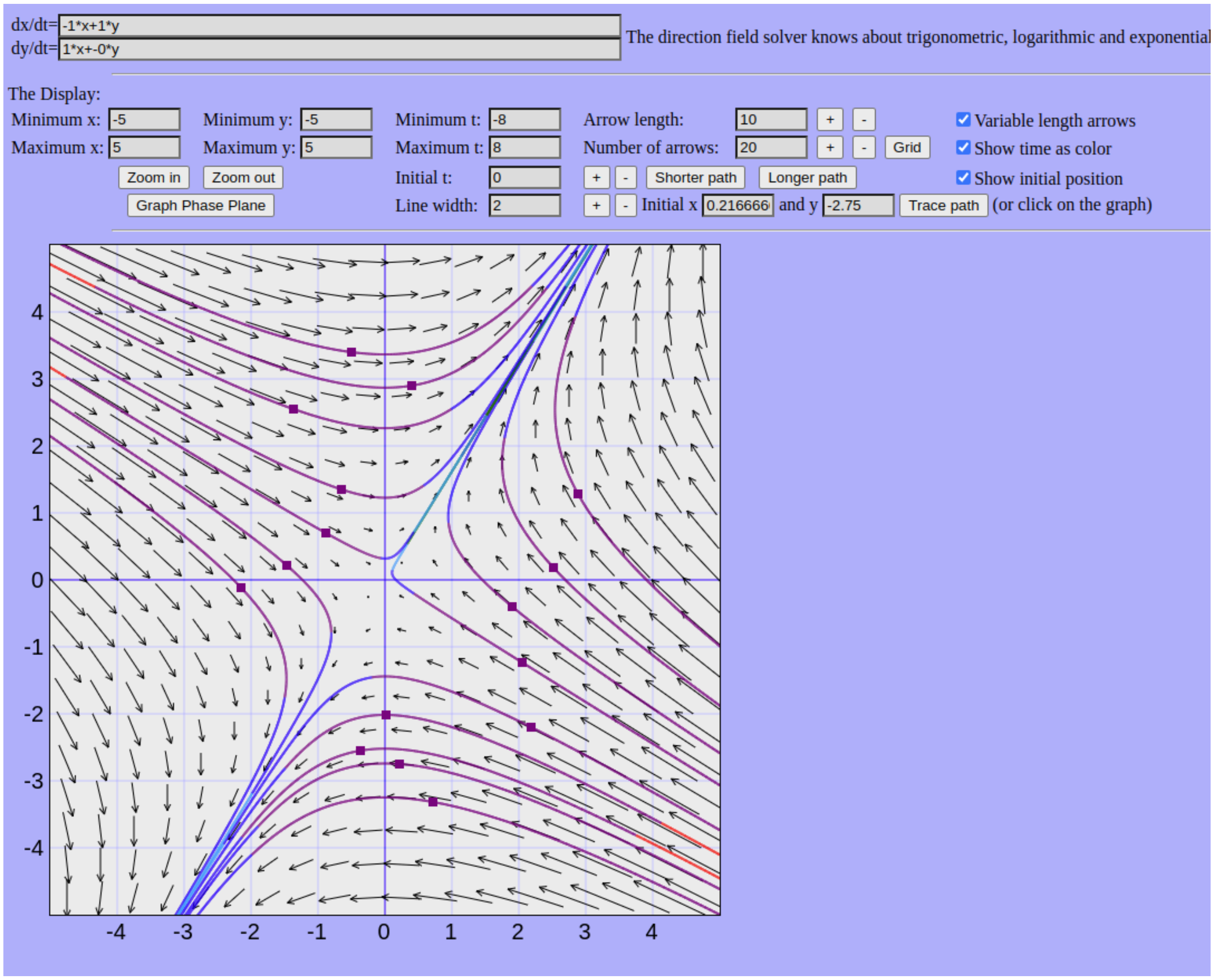
Point I


dx/dt= -2*x+0*y
dy/dt= 0*x+-3*y
The direction field solver knows about trigonometric, logarithmic and exponential
The Display:
Minimum x: -5
Maximum x: 5
Minimum y: -5
Maximum y: 5
Minimum t: -8
Maximum t: 8
Initial t: 0
Line width: 2
Arrow length: 10
Number of arrows: 20
Grid
Zoom in
Zoom out
Graph Phase Plane
Shorter path
Longer path
Initial x -3.65 and y -1.5
Trace path (or click on the graph)
Variable length arrows
Show time as color
Show initial position

Point J


dx/dt= -2*x+1*y
dy/dt= 4*x+-2*y
The direction field solver knows about trigonometric, logarithmic and exponential
The Display:
Minimum x: -5
Maximum x: 5
Minimum y: -5
Maximum y: 5
Minimum t: -8
Maximum t: 8
Initial t: 0
Line width: 2
Arrow length: 10
Number of arrows: 20
Grid
Variable length arrows
Show time as color
Show initial position
Zoom in
Zoom out
Graph Phase Plane
Shorter path
Longer path
Initial x 3.966666 and y 2.666666
Trace path
(or click on the graph)
4
3
2
1
0
-1
-2
-3
-4
-4
-3
-2
-1
0
1
2
3
4

Point K



dx/dt= -1*x+1*y
dy/dt= 1*x+-0*y
The direction field solver knows about trigonometric, logarithmic and exponential
The Display:
Minimum x: -5
Maximum x: 5
Minimum y: -5
Maximum y: 5
Minimum t: -8
Maximum t: 8
Initial t: 0
Line width: 2
Arrow length: 10
Number of arrows: 20
Grid
Zoom in
Zoom out
Graph Phase Plane
Shorter path
Longer path
Variable length arrows
Show time as color
Show initial position
Initial x 0.216666 and y -2.75
Trace path
(or click on the graph)
4
3
2
1
0
-1
-2
-3
-4
-4
-3
-2
-1
0
1
2
3
4

Point L

dx/dt= -1*x+0*y

dy/dt= 0*x+1*y

The direction field solver knows about trigonometric, logarithmic and exponential

The Display:

Minimum x: -5 Minimum y: -5 Minimum t: -8 Arrow length: 10 + - Variable length arrows

Maximum x: 5 Maximum y: 5 Maximum t: 8 Number of arrows: 20 + - Grid Show time as color

Zoom in Zoom out Initial t: 0 + - Shorter path Longer path Show initial position

Graph Phase Plane Line width: 2 + - Initial x -1.73333 and y -1.55 Trace path (or click on the graph)