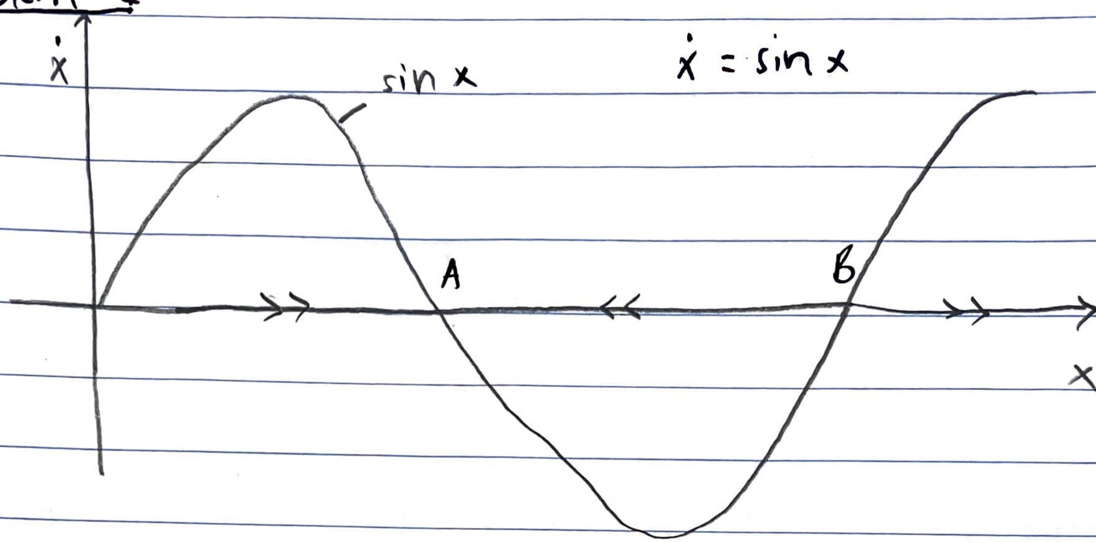
Problem 1

a)

$\dot{x} = \sin x$



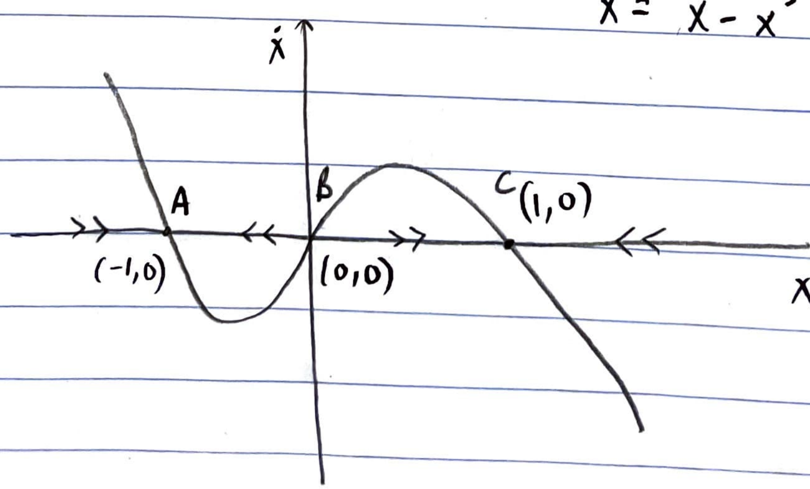
Point A - Attractor
Point B - Repeller

A - 180°
B - 360°

The system has two fixed points A and B that appear in every cycle of the sin function. An attractor at A and a repeller at B. If x is initialized between 0 and A, it will increase until it reaches A. If x is initialized between A and B, it will decrease until it reaches B

b)

$\dot{x} = x - x^3$

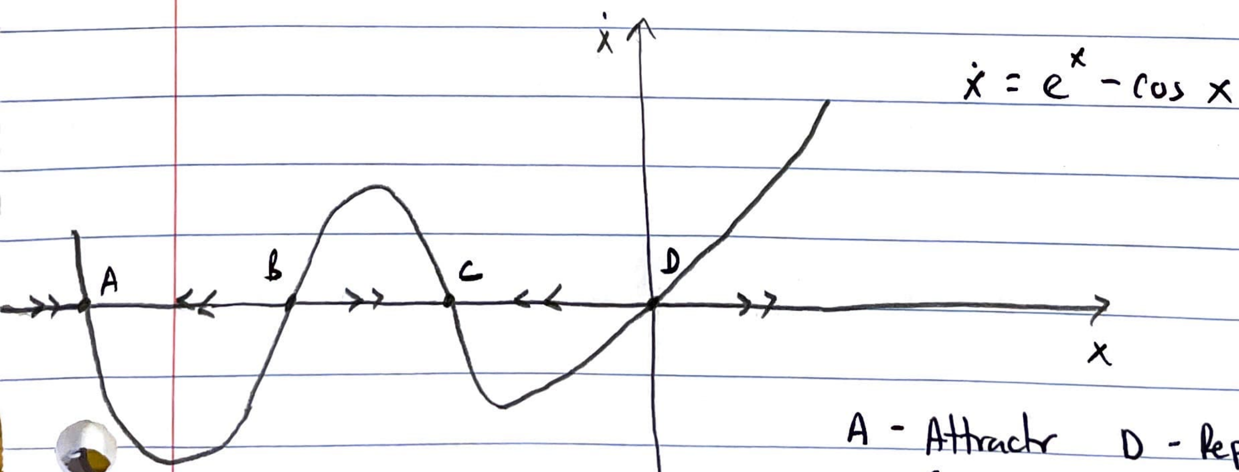


Point A : Attractor

B : Repeller

C : Attractor

The system has 3 fixed points A, B and C. An attractor at A, C and a ~~repp~~ repeller at B. If ~~the~~ x is initialized between $-\infty$ and B, it will tend towards A, where between $-\infty$ and A it will increase and between A and B it will decrease.
If the system is initialized between B and $+\infty$, it will tend towards C, with it increasing between B and C and decreasing between C and $+\infty$.
In all cases between A and C, the system moves away from C ~~with~~ by decreasing between A and B and increasing between B and C



A - Attractor

B - Repeller

C - Attractor

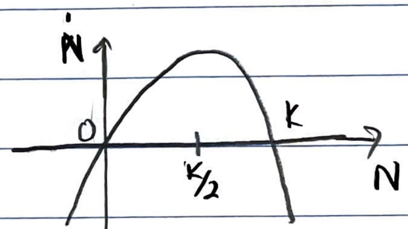
D - Repeller

The system has 4 fixed points A, B, C and D where A and C are attractors and B and D are repellers. For $x \in [A, B]$, the system will (decrease) tend towards A. For $x \in [B, C]$ the system will (increase) tend towards C, (decrease) likewise for $x \in [C, D]$. For $x \in [D, \infty]$, the system will move away from D and increase indefinitely

Problem 2

2.

$$\dot{N} = rN\left(1 - \frac{N}{K}\right)$$



@ $N = 0$, $\dot{N} = 0$ Mid point b/w $N=0$, $N=K$

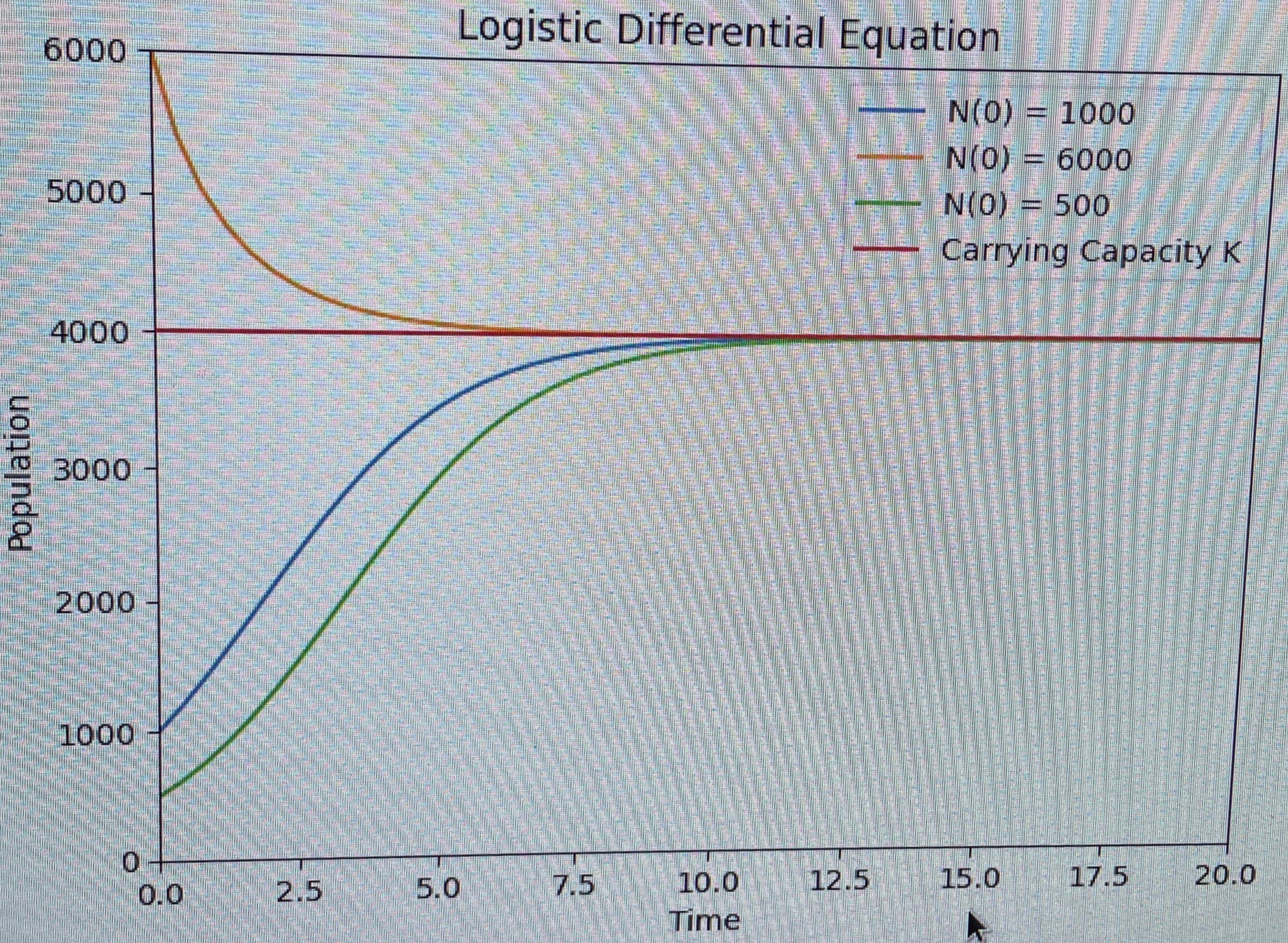
@ $N = K$, $\dot{N} = 0$

$$1 - \frac{N}{K} = 0 \qquad N = K$$

@ $N = K/2$, N increases the fastest

b) Code uploaded as jupyter notebook

3/4



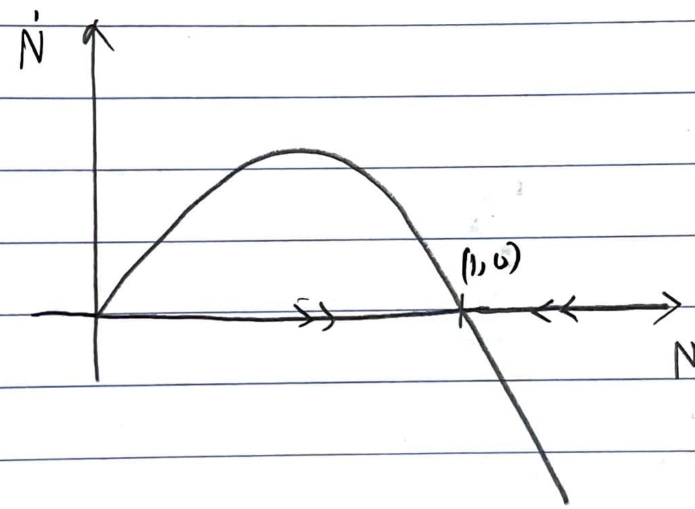
Logistic Differential Equation
N(0) = 1000
N(0) = 6000
N(0) = 500
Carrying Capacity K
Population
Time
6000
5000
4000
3000
2000
1000
0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

Problem 3

3. a - external factors such as white blood cells or chemo
that controls the growth response.

$\frac{1}{N}$ b - tumor size where no changes occur with time.

b)

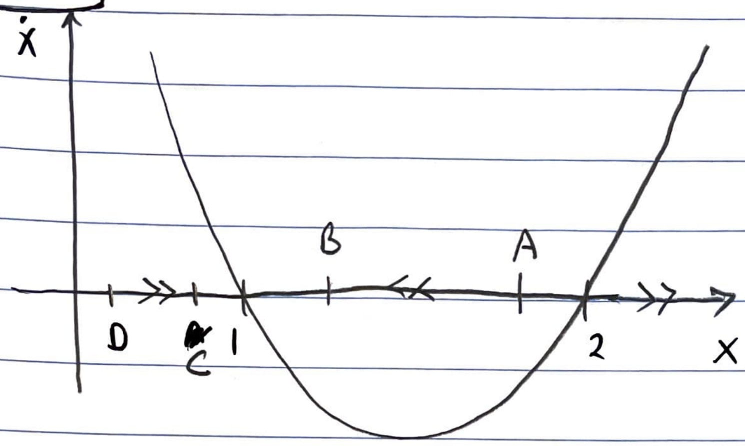


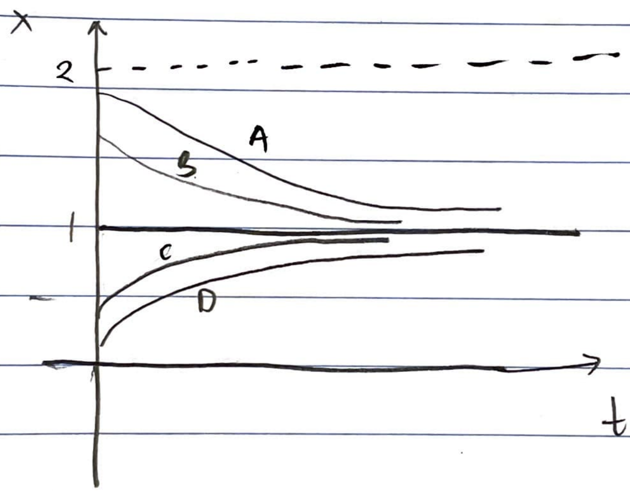
Initial conditions Plot in next page, code in jupyter notebook

c) A stable equilibrium is at 1. Therefore for $N(0) = 0.1$,
the number of tumor cells grows, and maintains the
value 1, likewise for when $N(0) = 0.6$.
For $N(0) = 1.6$, the value is above the stable equilibrium, therefore
the number of tumor cells rapidly decreases and attains
the equilibrium value 1 as t approaches ∞.

Problem 4

4.

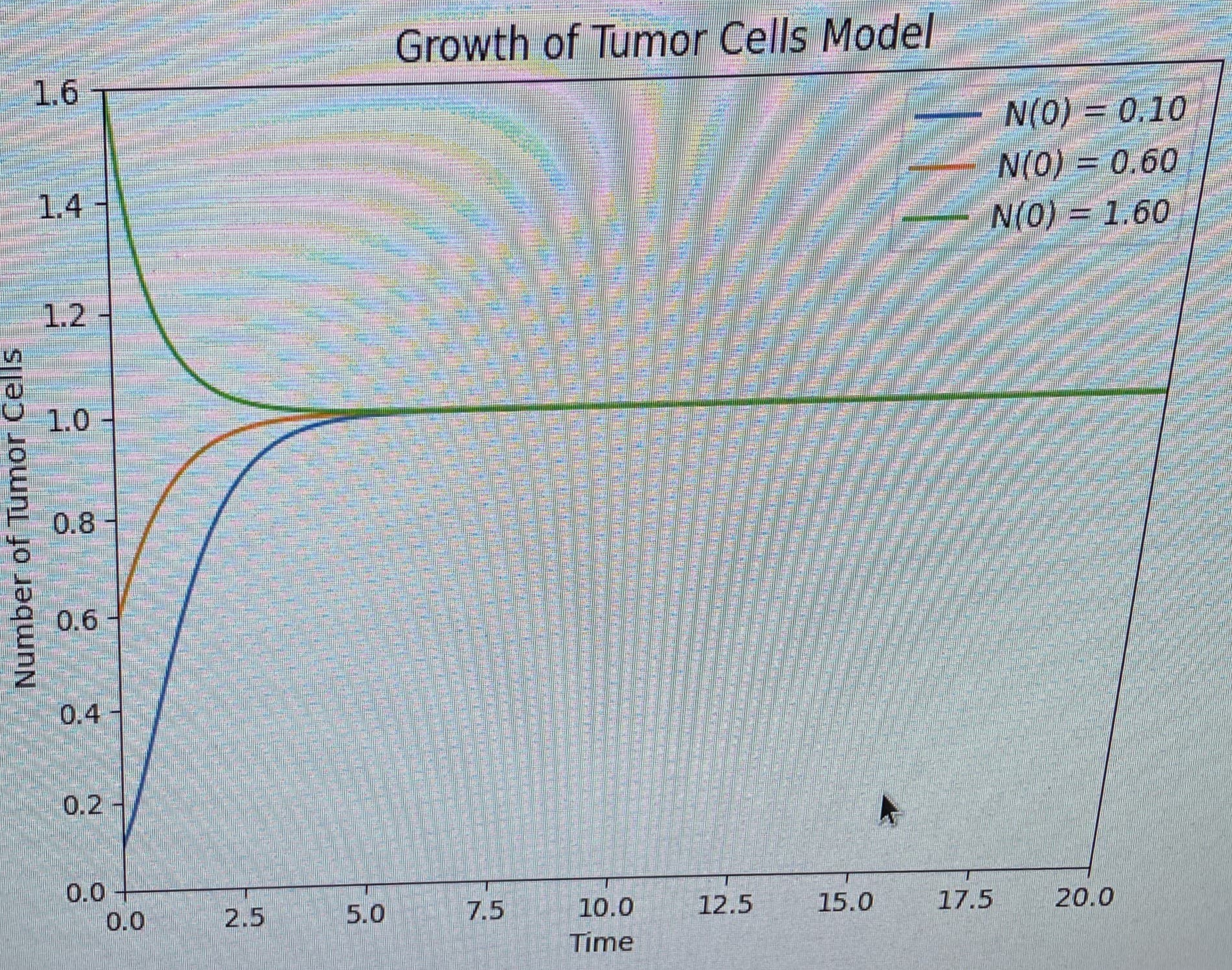




All appear to be possible trajectories arising from the dynamical system. ~~They are initialized~~

D and C increase to approach 1, and they are initialized where $\dot{x} > 0$.

B and A decrease to approach 1, and they are initialized when $\dot{x} < 0$.


Growth of Tumor Cells Model
N(0) = 0.10
N(0) = 0.60
N(0) = 1.60
Number of Tumor Cells
Time
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0