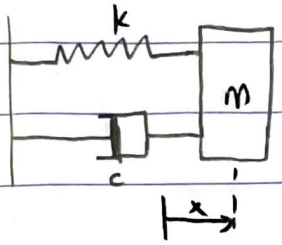
1. $m\ddot{x} + kx + c\dot{x} = 0$



$x(0) = x_0$

$\dot{x}(0) = v_0$

a) State variables:

The state variables are position (x) and velocity ($\dot{x}$).

b) Let $\dot{y} = \ddot{x}$, $\dot{x} = y$

i)

$$m\dot{y} + kx + cy = 0$$

$$m\dot{y} = -kx - cy$$

$$\dot{y} = -k/m\, x - \frac{c}{m} y$$

$$\dot{x} = y$$

(1) $\dot{x} = y$

(2) $\dot{y} = -k/m\, x - \frac{c}{m} y$

ii)

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} y \\ -k/m\, x - c/m\, y \end{bmatrix}$$

iii) $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

2. $x(t) = e^{\lambda t}$

$m\lambda^2 + c\lambda + k = 0$

$\lambda = \dfrac{-c \pm \sqrt{c^2 - 4mk}}{2m}$ since λ is complex $c^2 - 4mk < 0$

$\lambda_1 = \dfrac{-c}{2m} + \dfrac{\sqrt{|c^2 - 4mk|}}{2m} i \qquad \lambda_2 = \dfrac{-c}{2m} - \dfrac{\sqrt{|c^2 - 4mk|}}{2m} i$

let $\boxed{\alpha = \dfrac{-c}{2m}}$ $\boxed{\omega_d = \dfrac{\sqrt{|c^2 - 4mk|}}{2m}}$

Position

$x(t) = A e^{\alpha t} \cos \omega_d t + B e^{\alpha t} \sin \omega_d t = e^{\alpha t}(A \cos \omega_d t + B \sin \omega_d t)$

$\boxed{x(0) = x_0 = A}$

Velocity

$\dot{x}(t) = \alpha e^{\alpha t}(A \cos \omega_d t + B \sin \omega_d t) + e^{\alpha t}(\omega_d(-A \sin \omega_d t + B \cos \omega_d t))$

$\dot{x}(t) = v_0 = \alpha A + B \omega_d$

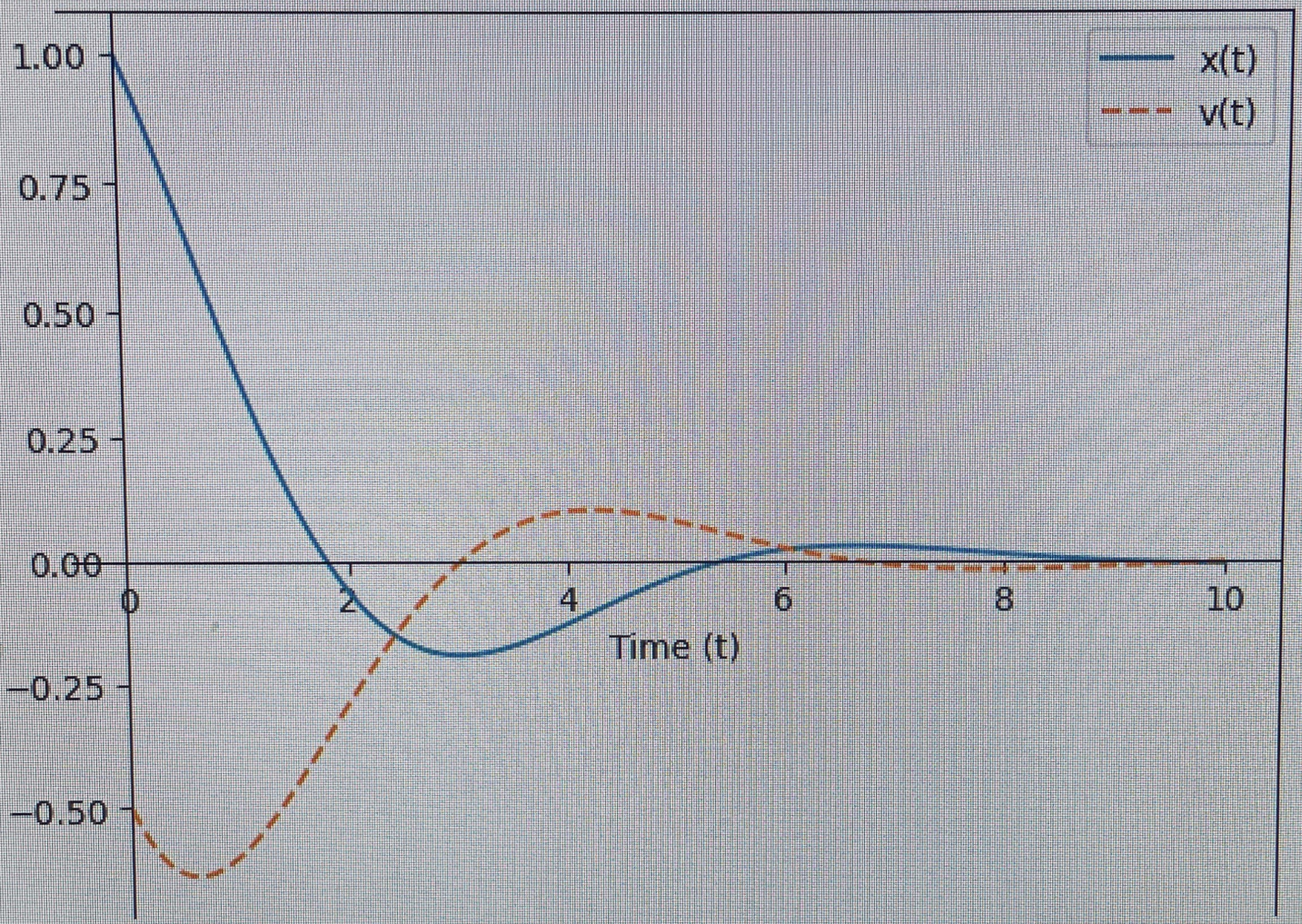
$v_0 - \alpha A = B \omega_d$

$B = \dfrac{v_0 - \alpha A}{\omega_d} = \dfrac{v_0 - \alpha x_0}{\omega_d}$

$\boxed{B = \dfrac{v_0 + c/2m\, x_0}{\sqrt{|c^2 - 4mk|}} * 2m}$

```
ax.spines['bottom'].set_position('zero')


plt.plot(pos[0],pos[1], label = 'x(t)')
plt.plot(vel[0],vel[1], label = 'v(t)', linestyle='--')
plt.xlabel('Time (t)')
plt.legend()
plt.show()
```



[ ]:

```
#Problem 4: Numerical Solution of ODE

from scipy.integrate import solve_ivp
import matplotlib.pyplot as plt
from numpy import array, exp, sin, pi, linspace

k=1
m=1
c=1

#case [position, velocity]
case_1 = [1,-0.5]
case_2 = [0.5,0.5]
```

```
ax =plt.gca()
ax.axhline(0,color='black', linewidth=0.7)
ax.axvline(0,color='black', linewidth=0.7)
ax.margins(0)
ax.set_xlim([0,10])
ax.set_ylim([-1,1])
plt.plot(sol_1.t,sol_1.y[0], color='red', label='case 1')
plt.plot(sol_1.t,sol_1.y[1], linestyle='--',color='red')

plt.plot(sol_2.t,sol_2.y[0], color='blue', label='case 2')
plt.plot(sol_2.t,sol_2.y[1], linestyle='--', color='blue')
plt.xlabel('Time (t)')
plt.plot([],[],'k-', label='Position $x(t)$')
plt.plot([],[],'k--',label='Velocity $\dot{x}(t)$')
plt.legend()
plt.show()
```

```
<>:48: SyntaxWarning: invalid escape sequence '\d'
<>:48: SyntaxWarning: invalid escape sequence '\d'
/tmp/ipykernel_6657/1673307733.py:48: SyntaxWarning: invalid escape sequence '\d'
  plt.plot([],[],'k--',label='Velocity $\dot{x}(t)$')
```