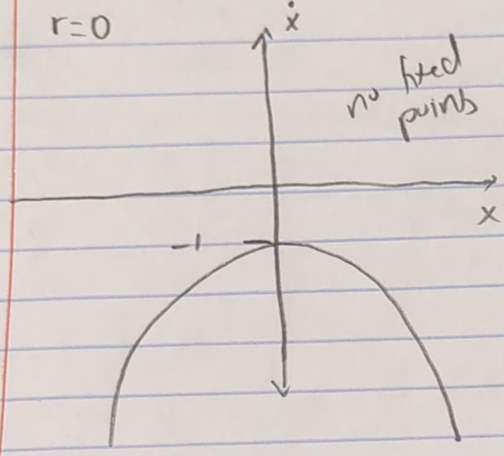
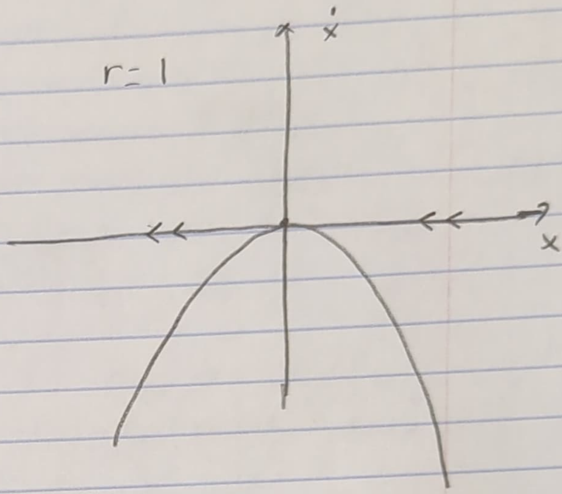
1. $\dot{x} = f(x; r) = r - \cosh x$

a) $r = 0$

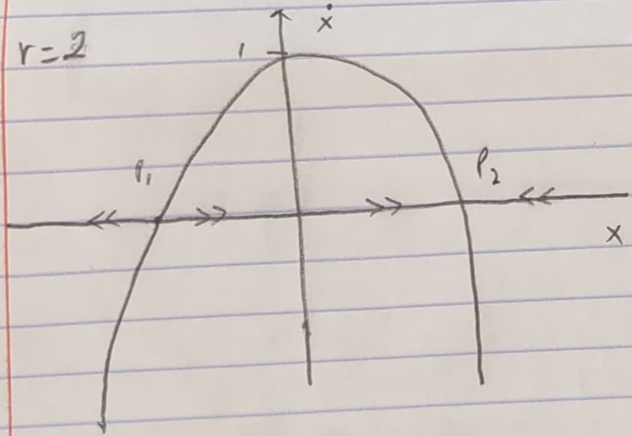


$r = 1$



$x = 0$, semistable

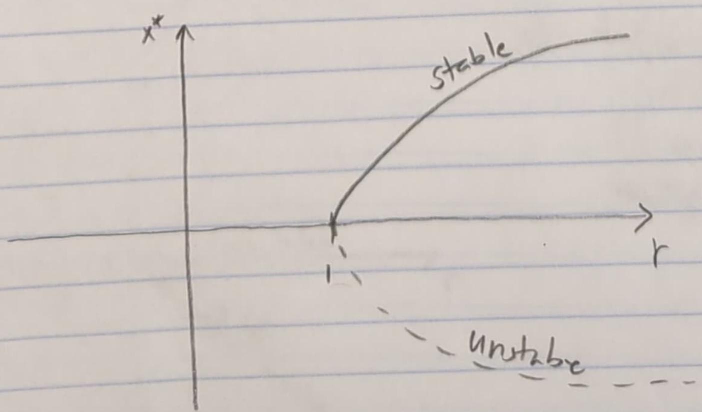
$r = 2$



$x = p_1 = -1.31696$, unstable

$x = p_2 = 1.31696$, stable

b)

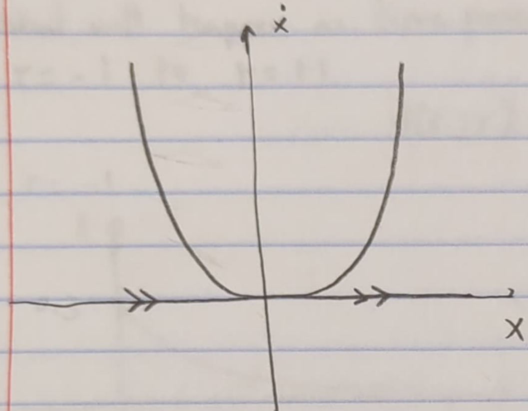


c) bifurcation occurs at $r = 1$. It is saddle node bifurcation

2. $\dot{x} = f(x; r) = rx^3 + x^4$

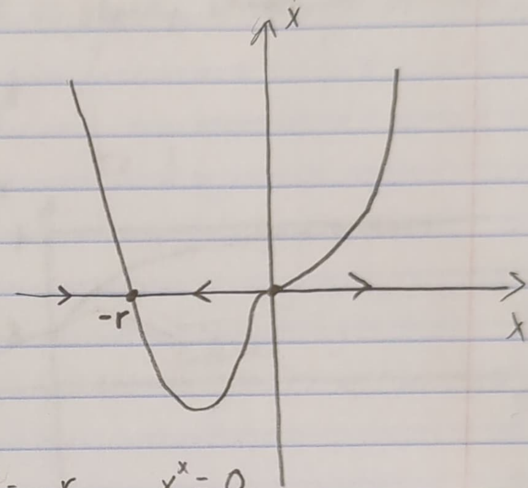
a)

$r = 0$



$x^* = 0$, semistable

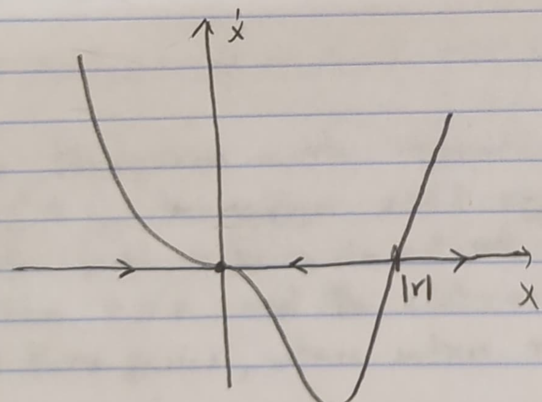
$r > 0$



$x^* = -r$ stable

$x^* = 0$ unstable

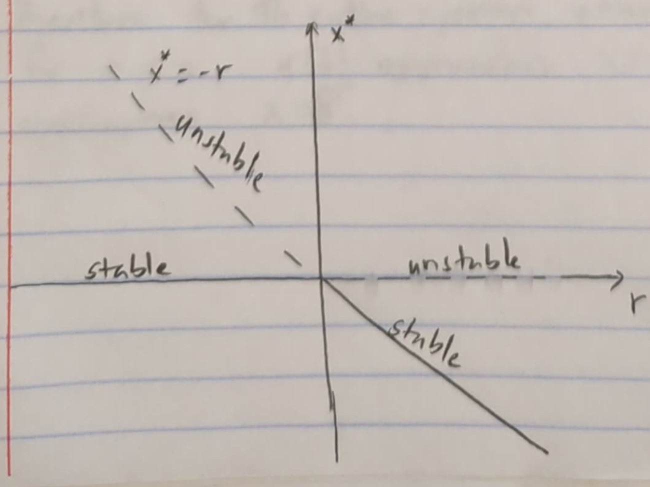
$r < 0$



$x^* = |r|$ unstable

$x^* = 0$ stable

b)



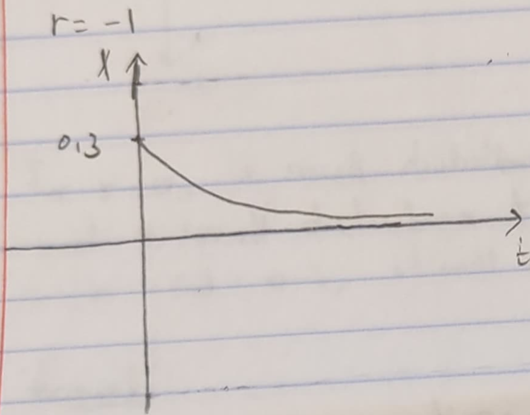
c) bifurcation occurs at $r = 0$.
It appears to be transcritical bifurcation.

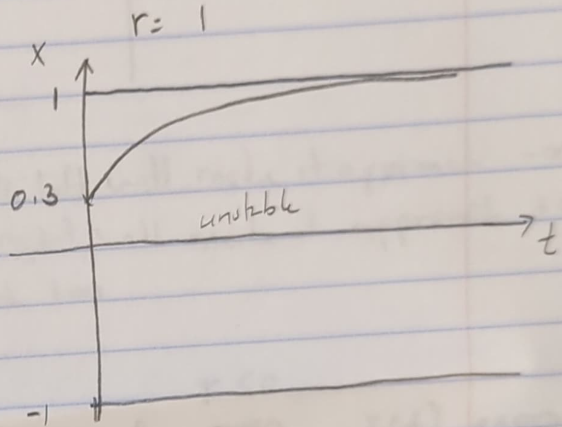
3. $\dot{x} = f(x;r) = rx \pm x^3$

$x(0) = 0.3$

What will happen as time passes and r is slowly increased from $r=-1$ to $r=+1$.

$f(x;r) = rx - x^3$





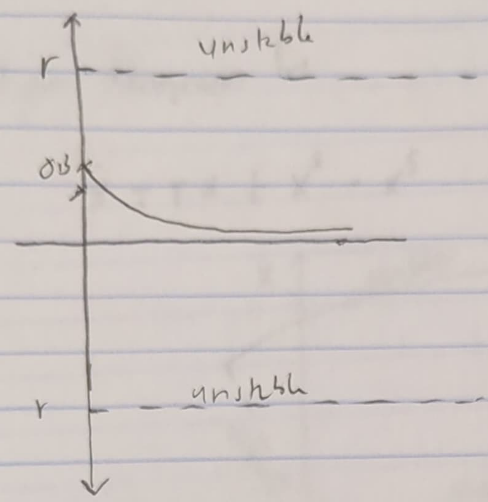
for $r=0$, same as $r=-1$ but approaches 0 much slower.

For the system with supercritical pitchfork bifurcation, when $r \le 0$, ~~the system~~ $x(t)$ approaches $x=0$. It does so at varying rates with it being slowest ~~at~~ when $r=0$.
when $r>0$, and ~~the~~ $x(0)=0.3$, the system will approach $x=r$ as time passes, where when $r=1$, $x(t)$ will approach $x=1$.

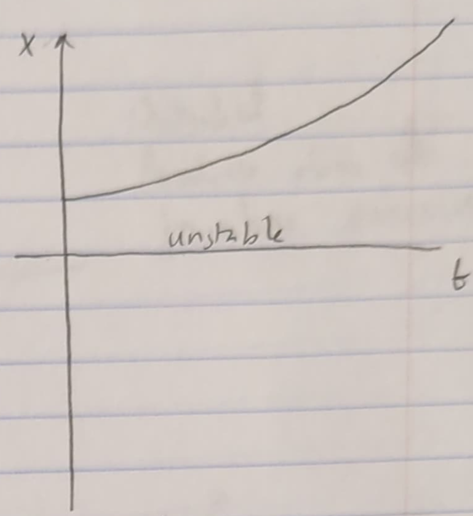
* Therefore for the entire system, when r increases as time progresses for $r \le 0$, $x(t)$ approaches $x=0$ and for $r>0$, $x(t)$ approaches $x=\sqrt{r}$.

$$f(x;r) = rx + x^3$$

$r = -1$



$r = +1$



For $r=0$, a small disturbance to the left will make it approach $-\infty$ and a small disturbance to the right will make it approach $+\infty$, with $x(0) = 0.3$, it will approach $+\infty$.

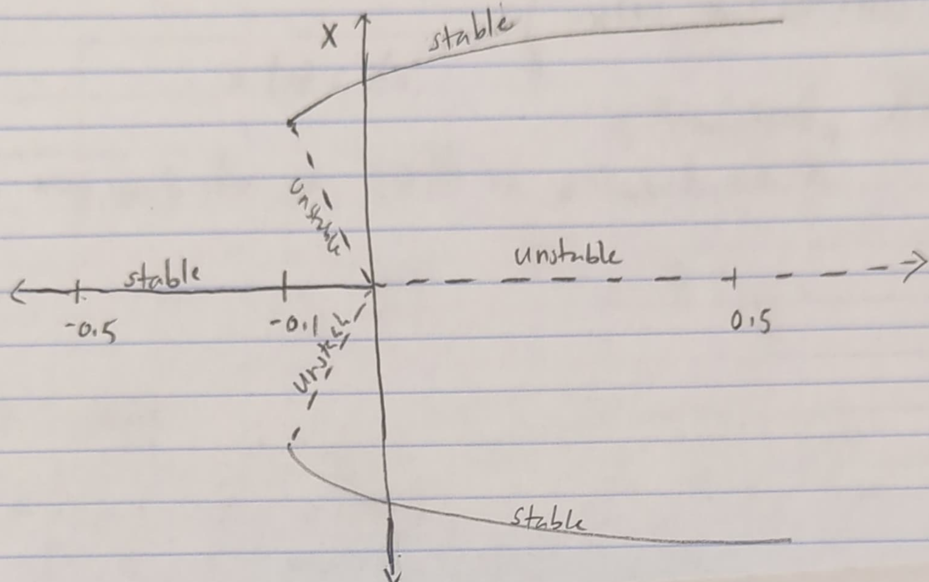
Generally:
When r increases as time progresses, for $r<0$, $x(t)$ approaches $x=0$ and for $r \geq 0$, $x(t)$ explodes and tends towards $+\infty$ since $x(0) > 0$.

Taking into account our initial condition, $x(0)=0.3$, we need to modify our observation for $r<0$. ~~when~~ $\cancel{\sqrt{|r|} \leq 0.3}$
When $\sqrt{|r|} \leq 0.3 = x(0)$, $x(t)$ explodes and tends towards $+\infty$.

4 Bifurcation diagram for

$$\dot{x} = rx + x^3 - x^5$$

Detailed Analysis done ~~a)~~ in-class exercise



bifurcation occurs @ $r = 0$ - subcritical pitchfork bifurcation
$r = -0.1$ - saddle-node bifurcation