



The Efficient Frontier in Modern Portfolio Theory

Weaknesses and How to Overcome Them

By Marcus Schulmerich, PhD, CFA®, FRM

In 1952, Harry Markowitz set a milestone in financial theory.¹ By introducing mean-variance optimization (MVO), often referred to as “Markowitz optimization,” he originated modern portfolio theory (MPT). In doing so, he provided a comprehensive theoretical framework for investment analysis and guidelines for optimal portfolio construction. Even 60 years later his model is still widely used among private and professional investors, despite its shortcomings.

This article presents the theory of Markowitz, taking a closer look at the assumptions and the issues associated with the model. It closes with answers to some of the most critical shortcomings in the theory of the efficient frontier.

Modern Portfolio Theory: A Review

Let’s consider the following situation: You have won \$100,000 in a lottery and want to invest this amount in global equity securities. What stocks should you pick? What should your decision be based on? Thousands of stocks are available from which to choose, so you need a systematic approach to investing.

Naturally, you’d start looking at the expected return of each of these securities. On the other hand, you know that equity investing is associated with risk. These are the two dimensions you need to pay attention to: return versus risk.

While return is easy to understand, the term “risk” needs clarification. For a layperson, risk is mainly associated with losing money. This is even more true after all the recent events in Greece. Events such as the March 2012 Greek debt haircut, where investors had to

give up on their bond values, are associated with credit risk, i.e., the risk of losing money.

This is, however, not what Markowitz meant when he talked about risk. In MPT, risk refers to market risk. Specifically, risk refers to the fluctuations in the security price. Markowitz defined risk as the standard deviation of the percentage returns of a security—be they daily, weekly, or monthly returns in practice. This standard deviation is known as the volatility of the security.

Therefore, Markowitz characterizes each investment opportunity by:

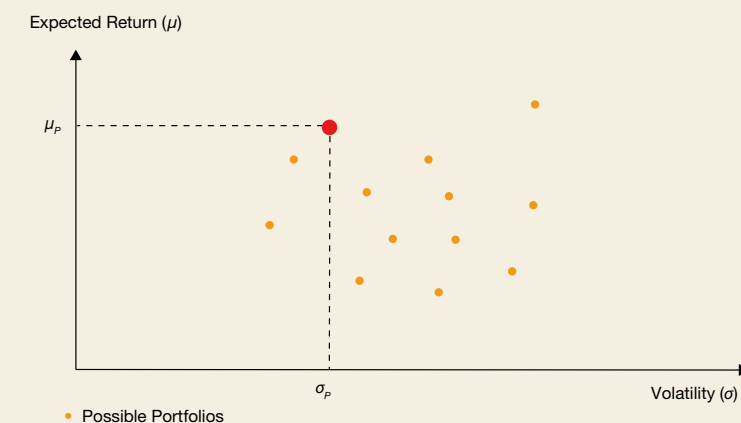
- its expected return (μ), and
- its volatility (σ).

Let’s go back to our example and try to construct a portfolio comprising three stocks: A, B, and C. The question now arises what the weights of each stock ought to be. Theoretically we have

to look at all possible combinations of weights and calculate for each of these combinations the portfolio return and the volatility. To show this idea graphically, let’s look at figure 1, which shows what is known as the μ - σ (mu-sigma) space: Each dot represents a different portfolio as a result of a certain combination of weights for stocks A, B, and C. The portfolio’s expected return, μ_p , is shown on the vertical μ -axis and the associated volatility, σ_p , can be found on the horizontal σ -axis.

In figure 1, we have shown only a dozen different combinations of weights, so the picture is not striking yet. However, if we look at substantially more combinations of weights for securities A, B, and C, such as we do in figure 2, the picture gets clearer: The dots do not seem to be evenly spread in the μ - σ space. Even if you look at all possible combinations of weights for stocks A, B, and C, the resulting dots are not evenly

FIGURE 1: PORTFOLIOS IN THE μ - σ SPACE



Source: SSgA, for illustrative purposes only.

spread in the μ - σ space. As a matter of fact, there are areas in the μ - σ space where no dots lie. They are concentrated in a certain area that seems to be bounded. This boundary leads to what is known as the efficient frontier, as indicated in figure 3.

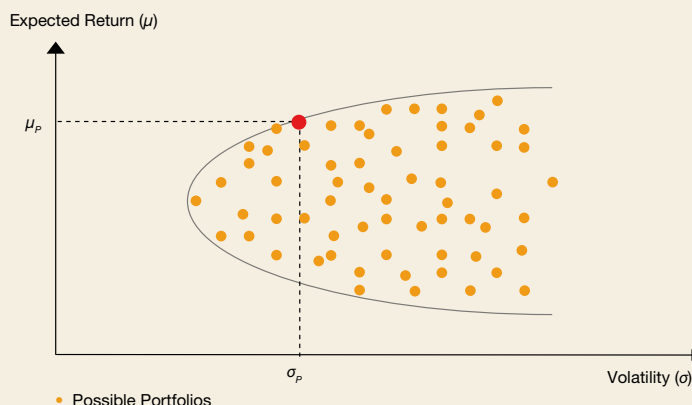
Markowitz looks at an efficient portfolio. A portfolio is efficient if no other portfolio has the same expected return with a lower volatility. Graphically, this is explained in figure 3. On the green horizontal line lie all portfolios with an expected return of μ_p . The efficient portfolio of all of those dots on the green line is the one with the lowest volatility σ . If we undertake this procedure with all expected returns we get the curved line in figure 3, comprising a blue part and a gray part. While all portfolios located on the blue and gray line are efficient in the sense of the definition above, obviously only the blue line is beneficial to the investor because it provides increased returns. This blue line is called the efficient frontier. The point of the line with the lowest volatility is called minimum-variance portfolio (MVP).

So far no lending or borrowing of money is included in the model. But in reality you can also deposit your money in the bank or borrow some more. The interest rate associated with these activities is called the risk-free rate (R_f), i.e., it is the return on investment with no volatility. Please note that this concept already assumes that lending and borrowing is possible at the same risk-free rate.

Let's come back to our original investment problem: How do I invest \$100,000 in three stocks A, B, and C? We learned that the optimal combination of these three stocks is an efficient portfolio on the efficient frontier. However, it is not clear yet which portfolio on the efficient frontier to choose. The higher return I demand the higher my associated risk will be, as figure 4 shows.

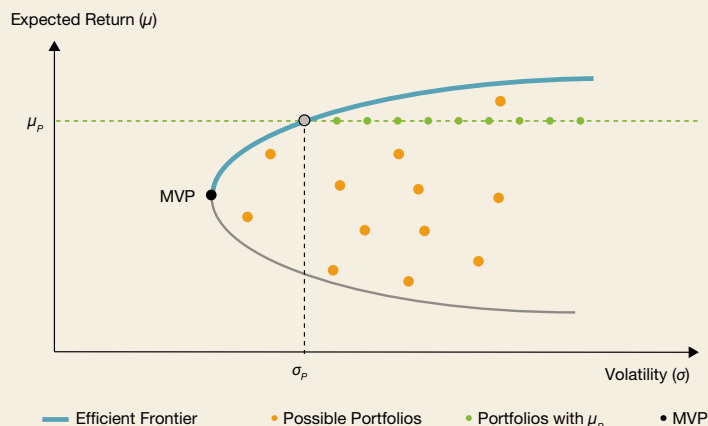
The key idea is now to include the risk-free asset as a possible investment in your investment problem: Invest a portion of your \$100,000 in an efficient

FIGURE 2: PORTFOLIOS AND THEIR BOUNDARY IN THE μ - σ SPACE



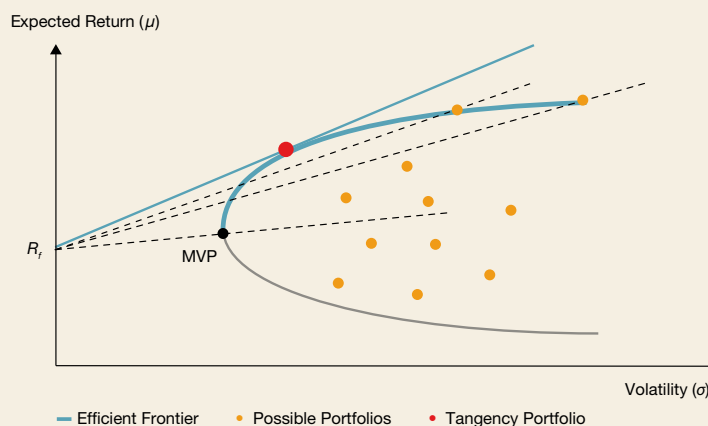
Source: SSgA, for illustrative purposes only.

FIGURE 3: EFFICIENT FRONTIER IN THE μ - σ SPACE



Source: SSgA, for illustrative purposes only.

FIGURE 4: EFFICIENT FRONTIER WITH RISK-FREE ASSET



Source: SSgA, for illustrative purposes only.

**TABLE 1: ASSUMPTIONS REQUIRED FOR THE MARKOWITZ OPTIMIZATION**

Assets	Investors	Markets
<ul style="list-style-type: none"> • Asset returns are normally distributed. • Everyone agrees on their distribution. • Assets are highly divisible into small parcels. • There is a risk-free asset. 	<ul style="list-style-type: none"> • Are rational and behave in a manner as to maximize their utility. • Base decisions on expected returns and standard deviations of the returns. • Are risk averse and try to minimize the risk and maximize return. • Perceive risk as the standard deviation of returns. • Maximize the single-horizon utility. • Are price takers, i.e., cannot influence prices. 	<ul style="list-style-type: none"> • Guarantee free access to fair and correct information on the returns and risk. • Are efficient and absorb the information quickly and perfectly. • Have no transaction costs or taxes. • Allow unrestricted short selling. • Every asset is tradable at any point of time.

Sources: Markowitz (1952) and Füss (2008)

portfolio on the efficient frontier, invest the remaining in the risk-free asset. To give an example, let's assume the risk-free rate is 3 percent and the chosen efficient portfolio on the efficient frontier has an expected return of 10 percent and a volatility of 12 percent. If I invest half of my money in the efficient portfolio and the other half in the risk-free asset, my overall investment has an expected return of $0.5 \times 10\% + 0.5 \times 3\% = 6.5\%$. Because the volatility of the risk-free asset is 0, the volatility of the overall investment can be calculated as $0.5 \times 12\% = 6\%$.²

The efficient portfolio I should choose is the portfolio that delivers the best relationship of return versus risk, i.e., portfolio return (over risk-free rate) divided by portfolio volatility. This ratio is known as the Sharpe ratio and is also graphically shown in figure 4: It is the slope of a straight line that begins at the risk-free rate on the μ -axis and goes through a portfolio on the efficient frontier.

Hence, to maximize the Sharpe ratio we seek the portfolio with the steepest straight line. The efficient portfolio to choose is the tangency portfolio. In MPT this portfolio is also called the market portfolio and in practice it is replicated by using a capitalization-weighted index.

For this model to be derived, several assumptions have to be made about the asset returns, the investors, and the markets. All these assumptions are listed in table 1. In fact, most of these assumptions have to be questioned (Füss 2008). For instance, there certainly are

TABLE 2: HISTORICAL MEAN RETURNS IN US\$ AND VOLATILITIES OF THE MSCI WORLD INDEX (NET DIVIDENDS REINVESTED)

Period ending December 31, 2011	Return Annualized (%)	Risk Annualized (%)
Past 3 years	8.36	20.41
Past 5 years	-4.53	20.90
Past 10 years	1.64	17.27

Source: MSCI, as of December 31, 2011. For all calculations, monthly data are used. Past performance is not a guarantee of future results.

transaction costs or taxes, and the investors' perception of risk isn't exactly the same as the volatility of returns. Also, the existence of a true risk-free asset is debatable, but this can be circumvented, which eventually led to the zero-beta capital asset pricing model.

Some problems associated with MVO are even less obvious. As hinted above, investors don't necessarily perceive risk as the standard deviation of returns. The common perception of risk is that risk means losing money or doing worse than expected. Therefore many people, especially after the subprime crisis, use (in addition) downside risk measures such as drawdown or Value-at-Risk (VaR) when looking at investments.³ In light of the events in the stock markets in 2008 and 2011, these asymmetrical risk measures that look primarily at the downside of returns might seem more intuitive to us or have a higher practical relevance to investment decisions.

Before we continue on the relevance of volatility as an appropriate risk measure, let's take a closer look at MVO as it is, and the relationship between the input and the output of the portfolio optimization. Obviously, the first ques-

tion is where shall we get the required input data? As shown above, the data required to calculate a portfolio return and volatility are the expected returns for all securities and the covariances between the percentage returns of all assets. The next question would be, how does the choice of the input parameters affect the output, i.e., how sensitive is the output versus the input to MVO?

Below, we explore the problems arising from these questions and advise how to overcome them.

The Input Data to MVO and Their Output

There are three sources for expected returns and covariances for MVO: We derive predictions, or we use historical data to calculate historical returns and covariances that we use as future expected returns and covariances, or we use a combination of both.

When using historical data, the volatility of the markets is a crucial aspect to consider. Obviously, the impact of the chosen historical time period is important for the MVO output. In table 2, the historical mean returns

(in US\$) and volatilities of the MSCI World Equity Index (net dividends reinvested) were calculated for different time periods as of December 31, 2011. As table 2 shows, the choice of the historical time period for MVO input has a significant impact on the resulting data. This clearly affects the optimal portfolio weights in MVO and the efficient frontier calculation itself.

Another important factor to consider when historical data are used is the data frequency: Should daily, weekly, or monthly data be used to, e.g., calculate the covariances? This has direct implications on MVO. If, for example, daily data are used, returns and covariance have to be daily as well such that MVO optimizes on a daily basis.

When using forward-looking estimations from fundamental analysts we face other issues. Stocks of financial institutions, for example, had great years before the subprime crisis but suffered a lot in 2008 and early 2009. In mid-2008, most of the estimation for forward earnings per share or stock prices prepared by fundamental analysts was still very positive while the market had already started to head southward. Here, the lag in analysts adjusting their estimations

**FIGURE 5: DEUTSCHE BANK STOCK PRICES
MARCH 31, 2007–OCTOBER 31, 2009**



Source: Bloomberg, as of April 30, 2012

was detrimental. As the market rallied upward, starting in March 2009, likewise the analysts' estimations rose, but with a time lag. Figure 5 shows, as an example, the stock price of Deutsche Bank in US\$, as quoted at the New York Stock Exchange.

So how can an investor attain the right expectation for return and covariance at any point in time? The importance of that objective was reinforced by Michael Best⁴ and Robert Grauer.⁵

They showed in a detailed analysis that the weights of efficient port-

folios are very sensitive to changes in expected returns (Best and Grauer 1991). Accordingly, a wrong choice can lead to serious misallocations. Table 3 gives an overview of important research work on how sensitive the MVO outcome is to its input.

So far we have looked at the issues of MVO in terms of the determination of input and how this affects the output. This leads us to the question of how we can improve the problems detected. Because everything starts with the input parameters, such as the return

TABLE 3: LITERATURE OVERVIEW: EFFECT OF ESTIMATION ERRORS ON OPTIMAL PORTFOLIOS

Authors (Year)	Title	Published	Content
R. Klein and V. Bawa (1976)	The Effect of Risk on Optimal Portfolio Choice	<i>Journal of Financial Economics</i> 3: 215–231	The effect of taking estimation risk into account for portfolio optimization is examined and possible differences in portfolio choice are illustrated.
J. Jobson and B. Korkie (1980)	Estimation for Markowitz Efficient Portfolios	<i>Journal of the American Statistical Association</i> 75, no. 371: 544–554	The performance of a portfolio with estimated expectations returns is compared to an equally weighted and the true optimal portfolio. The results reinforce the hypothesis that estimation errors are maximized throughout the optimization process.
M. Best and R. Grauer (1991)	On the Sensitivity of Mean-Variance-Efficient Portfolios to Changes in Asset Means: Some Analytical and Computational Results	<i>The Review of Financial Studies</i> 4, no. 2: 315–342	Properties of efficient portfolios are analyzed under changes in expected asset returns. A surprisingly small increase in assets' means substantially restructures portfolio composition. The effect is shown to intensify by increasing the number of assets in the universe.
V. Chopra and W. Ziemba (1993)	The Effect of Errors in Means, Variances, and Covariances on Optimal Portfolio Choice	<i>Journal of Portfolio Management</i> 19, no. 2: 6–11	Empirical evidence indicates that errors in means are over ten times as damaging as errors in variances, and over twenty times as damaging as errors in covariances. Moreover, the relative impact of errors in means is even greater at higher risk tolerances.

Source: SSgA, as of April 30, 2012

estimations for each security, we take a closer look at how we can get better estimations that do not suffer from the problems described above. An important step in improving the quality of the input parameters is the Black-Litterman model.

The Black-Litterman Model

The Black-Litterman model is a portfolio-selection approach with a focus on meaningfully estimating asset returns. It was developed and published by Fischer Black⁶ and Robert Litterman⁷ in the early 1990s and has attracted a lot of attention ever since. In their model, subjective prognoses are combined with formally derived expected asset returns (Black and Litterman 1992). The decision-making process basically splits into three steps (see figure 6).

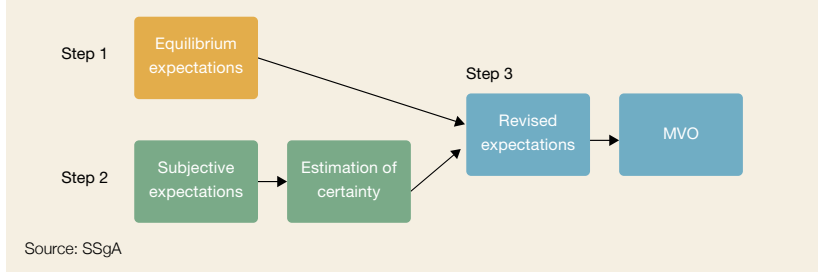
Step 1: Equilibrium return expectations are derived. A way to do so is to use the capital asset pricing model (CAPM), which states that

$$R = R_f + \beta(R_M - R_f)$$

That is, the asset's return (R) equals the risk-free interest rate (R_f) plus the market excess return ($R_M - R_f$) multiplied by the coefficient β (which measures the correlation of the asset with the market). Because not all components are known, we need to estimate the expected market return (R_M) and also β . For that, statistical tools such as the Fama-MacBeth regression provide good results. The asset is first regressed against risk factors (in CAPM: market risk) to obtain β . Then the asset return is regressed against the estimated β to obtain risk premia (in CAPM: market excess return). Finally, equilibrium expected returns can be calculated using the equation above.

Step 2: Subjective return prognoses are made and their degree of certainty is determined. A portfolio manager could directly determine both. But since it is not easy to classify the reliability of one's own prognoses, and to avoid exposure to individual misjudg-

FIGURE 6: THE BLACK-LITTERMAN MODEL OF PORTFOLIO CONSTRUCTION



ments, it's common to use third-party data. A wide range of analysts' estimates is publicly available from, e.g., I/B/E/S. This allows the level of certainty to be derived from a large sample of analysts' estimates.

Step 3: Equilibrium expectations are put together with the subjective prognoses and their certainty levels. Complex mathematical calculations deliver revised expected returns, which then can be used for classical MVO.

Because of the influence of equilibrium returns, it turns out that portfolios are much more broadly diversified and stable, too (He and Litterman 1999). Thus, even though the mathematics of the Black-Litterman framework are complex, its benefits for MVO are substantial.

The Risk Measure Choice

Let's come back to a key assumption that Markowitz starts with when developing his MVO: Risk is the standard deviation of the security's percentage returns. Although standard deviation is mathematically easy to handle, it is highly questionable—especially in light of the recent crisis with high annual losses (like in 2008) or high uncertainty (like in 2011)—that this symmetrical risk measure is really appropriate.

It would seem to be inappropriate for two reasons. First, as explained above, symmetrical risk measures fail to capture risk in times of crisis. Other risk measures such as semi-deviation, drawdown, VaR, or shortfall probability would be far more suited to capture investors' perception of risk—both for

retail investors and, increasingly in recent years, also institutional investors.

VaR, e.g., is nowadays a standard risk measure in the financial industry and is a foundation of key concepts in the Basel legal framework. VaR measures the risk of losing money in a certain period with a certain probability. For example, a 10-day 5-percent VaR of \$1,000 means that there is a 5-percent probability that the investment will fall in value by more than \$1,000 within the next 10 days. Using VaR as the risk measure, Markowitz's efficient frontier concept can be modified to a VaR efficient frontier concept in which the optimal portfolio is derived in μ -VaR space.

The main problem associated with MVO, however, is that MVO is a concept for absolute return portfolios. But in institutional asset management, portfolios are almost always managed against an index as benchmark. This means that the portfolio manager is not interested in absolute return and risk any more but has to focus on relative return versus the benchmark (the alpha) and relative risk known as tracking error. Therefore, the task for an active asset manager is to perform better than the benchmark (no matter how the benchmark performs) and stay within the given tracking error budget.


Traditional MVO cannot help in this situation. However, it is possible to transfer the analytical procedure of Harry Markowitz's efficient frontier such that a benchmark is included in the framework, using alpha as return and tracking error as the risk measure. Then, the active portfolio manager's

task becomes a solvable problem in the alpha-tracking error space: To a given target alpha, find portfolio weights that lead to the smallest tracking error possible. This is sometimes referred to as the tracking error efficient frontier. More on that concept can be found in Jorion (2003) or Roll (1992).

Summary

The Nobel prize-winning innovation of Harry Markowitz definitely changed investment thinking and established a more analytic and quantitative approach to portfolio construction. This article introduced this concept and examined its practical suitability.

It was shown that the aggregation of required data is a tough task to handle. Historical mean returns often are not very useful in that context, but the sophisticated Black-Litterman model might be a solution. Therein, equilibrium returns are combined with subjective views, leading to good results in practice.

Many asset managers focus on a benchmark when actively managing a portfolio. In this situation, the Markowitz concept can be transferred to the tracking error efficient frontier concept. Additionally, if VaR is required as the risk measure in MVO, this is also possible, leading to the VaR efficient frontier concept. 

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Endnotes

- 1 Harry M. Markowitz (1927–) is a world-renowned economist and professor of finance at the University of California, San

Diego. In his seminal article, “Portfolio Selection” (Markowitz 1952), he did some pioneering work for modern portfolio theory and formally showed that asset diversification lowers the risk of a portfolio. For his achievements in financial theory he was awarded the John von Neumann Theory Prize in 1989 and the Nobel Memorial Prize in Economic Sciences in 1990.

- 2 The universal formula for portfolio variance is $\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_{i \neq j} w_i w_j \sigma_i \sigma_j \rho_{ij}$, where w_i is the weight of asset i and ρ_{ij} the correlation between assets i and j . Their square root yields the volatility.
- 3 See also Schulmerich (2011).
- 4 Michael Best is a professor at the University of Waterloo and focuses on portfolio optimization and finance.
- 5 Robert Grauer is a finance researcher and professor at Simon Fraser University in British Columbia.
- 6 Fischer Sheffey Black (1938–1995) was an economist and is best-known for the famous Black-Scholes model and formula. After years of academic work at The University of Chicago and MIT Sloan School of Management, he joined Goldman Sachs in 1984.
- 7 Robert Litterman is an American economist. He was assistant vice president in the research department of the Federal Reserve Bank of Minneapolis and an assistant professor in the economics department at the MIT Sloan School of Management. In 1986 he joined Goldman Sachs and became chairman of the Quantitative Investment Strategies group of Goldman Sachs Asset Management.

References

- Best, M., and R. Grauer. 1991. On the Sensitivity of Mean-Variance-Efficient Portfolios to Changes in Asset Means: Some Analytical and Computational Results. *Review of Financial Studies* 4, no. 2: 315–342.
- Black, F., and R. Litterman. 1992. Global Portfolio Optimization. *Financial Analysts Journal* 48, no. 5: 28–43.
- Füss, R. 2008. Ansätze zur Optimierung eines Portfolios—Warum (nicht) Markowitz. *Financial Planning Praxis F*. http://www.treasuryworld.de/sites/default/files/H-1_Portfoliooptimierung_Adams_Fuess_Glueck_Lenz_Tilmes.pdf (in German).

He, G., and R. Litterman. 1999. The Intuition behind Black-Litterman Model Portfolios. Goldman Sachs Investment Management Research. http://papers.ssrn.com/sol3/papers.cfm?abstract_id=334304.

Jorion, P. 2003. Portfolio Optimization with Constraints on Tracking Error. *Financial Analysts Journal* 59, no. 5: 70–82.

Markowitz, H. 1952. Portfolio Selection. *Journal of Finance* 7, no. 1: 77–91.

Roll, R. 1992. A Mean/Variance Analysis of Tracking Error. *Journal of Portfolio Management* 18, no. 4: 13–22.

Schulmerich, M. 2011. How the Recent Crisis Has Changed EMEA Investors’ View of Risk. State Street Global Advisors. www.ssga.com.

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