EXAM — LIST OF TOPICS TO LEARN

Set 1 (Sobolev spaces):

(1) ($\star$) The fundamental theorem of calculus of variations;
(2) ($\star$) Sobolev spaces **(definition and basic properties)**;
(3) Sobolev embeddings: the case $1 < p < n$ and $p > n$, scaling argument;
(4) Poincaré's inequality, dependence of the domain of the constant in Poincaré's inequality in the case when the domain is a ball;
(5) Ladyzhenskaya's inequalities;
(6) Approximation **(formulation)**;
(7) Extension theorem (+)**(proof for a half-ball)**;
(8) Trace theorem (+)**(proof for a half-ball)**;
(9) Rellich–Kondrachov compactness theorem **(formulation and consequences)**.

Set 2 (Basic facts from the Calculus of variations):

(1) ($\star$) Derivation of the Euler–Lagrance equations for general Lagrangians (also for systems) and smooth minimizers;
(2) Uniqueness of minimizers for uniformly convex Lagrangians;
(3) Existence of the minimizers ($\star$ formulation);
(4) Minimizers are solutions to the Euler–Lagrange equation in the weak sense.

Set 3 (Harmonic maps):

(1) ($\star$) Definition of a (weakly) harmonic map, definition of a minimizing harmonic map;
(2) Examples of non-continuous solution, non-uniqueness of solutions, and non-minimizing solutions, exmaple of no density of smooth maps in Sobolev spaces into manifolds;
(3) Euler–Lagrange equation for the sphere valued harmonic maps, equivalent formulation;
(4) Unconstrained energy bounds constrained energy (for spheres in the target);
(5) Morrey's embedding theorem;
(6) Schauder estimates **(formulation)**;
(7) Regularity of minimizing harmonic maps in dimension 2 (+).

Not knowing a topic ($\star$) will result in not passing the course.