The No-Haircut Condition

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This analysis examines under what conditions, in the presence of both system surplus and underwater Vaults, DAI holders receive a haircut in an Emergency Shutdown (ES).

NB: this analysis neglects active auctions of all types at shutdown time. Generally, these don't affect the analysis too much, unless they represent a significant portion of system collateral/debt. An active collateral auction can be treated as an underwater Vault if its assigned DAI collection target (including penalty!) exceeds the market value of its associated collateral.

A general note on notation: when a subscript i is added to a variable, it indicates a value that is defined per-ilk (an "ilk" is a collateral type).

The version of the code referenced in this document can be found here. The End contract implements ES.

The DAI system is said to be in a surplus, or to have a surplus, when the DAI balance of the Vow exceeds its assigned bad debt, or sin: vat.dai(vow) > vat.sin(vow).

A "haircut to DAI holders during ES" obtains when the dollar value of the collateral that one DAI can be redeemed for during ES is less than the system's defined dollar value of one DAI (the par parameter).

Looking at the End.cash function, we can write down an expression for the dollar value of one DAI during ES:

$$(\$ \text{ per DAI in ES}) = \text{par} \sum_i \text{fix}_i / \text{tag}_i \tag{1}$$

Here, fix_i is the amount of a given collateral that a single DAI is entitled to and tag_i is the quantity of the given collateral valued at one DAI according to the oracle price used during shutdown. To procede further, we must define more values. Let debt be the total DAI supply at the time of shutdown, and let surplus denote the difference between the DAI assigned to the Vow and the bad debt assigned to the Vow at the time of shutdown. Since the End.thaw function must be called prior to End.cash and requires that vat.dai(vow) = 0, the actual dai supply (vat.debt) post-thaw will be (debt − surplus). Then, studying End.flow to see how fix_i is calculated, we can substitute the definition of fix_i into the equation above:

$$(\$ \text{ per DAI in ES}) = \text{par} \sum_i \left(\frac{\text{Art}_i \cdot \text{rate}_i \cdot \text{tag}_i - \text{gap}_i}{\text{debt} - \text{surplus}} \cdot \frac{1}{\text{tag}_i} \right) \tag{2}$$

The value gap_i is the collateral shortfall for a given ilk. This is the difference between the collateral required to cover all debt from that collateral (according to ES pricing) minus the quantity of that collateral actually retained for disbursal to DAI holders by the End's logic. It can also be expressed as the total unbacked DAI from a given ilk multiplied by tag_i. The product $\text{Art}_i \cdot \text{rate}_i$ is of course just the total debt drawn against a given ilk. We can rearrange the expression above as follows:

$$(\$ \text{ per DAI in ES}) = \frac{\text{par}}{\text{debt} - \text{surplus}} \sum_i \left(\text{Art}_i \cdot \text{rate}_i - \text{gap}_i/\text{tag}_i \right) \qquad (3)$$

The first term can be summed to yield the total DAI supply at the time of ES, which we recall was defined as debt. The expression finally becomes:

$$(\$ \text{ per DAI in ES}) = \text{par} \frac{\text{debt} - \sum_i \text{gap}_i/\text{tag}_i}{\text{debt} - \text{surplus}} \qquad (4)$$

The remaining summation term has a natural interpretation as the total unbacked DAI due to underwater Vaults, or "Vault shortfall". We can see that DAI holders receive at least par dollars of collateral if and only if the surplus is at least the total Vault shortfall. Hence this can be called the "no-haircut condition":

$$\text{vat.dai(vow)} - \text{vat.sin(vow)} \geq \sum_i \text{gap}_i/\text{tag}_i \qquad (5)$$

Note that all values in the equation above are to be taken immediately prior to calling End.cage.

For completeness, we write down a formula for the total Vault shortfall given particular oracle prices. Define P_i to be the current oracle price of a given ilk (dollars / collateral, e.g. \$200 / ETH). The expression $\delta(\text{logical condition})$ is 1 if the logical condition is true, 0 if it is false. A subscript u refers to a Vault address. The product $\text{art}_{iu} \cdot \text{rate}_i$ is a Vault's total DAI debt, and ink_{iu} denotes a Vault's collateral.

$$\text{tag}_i = \text{par}/P_i \qquad (6)$$

$$\sum_i \text{gap}_i/\text{tag}_i = \sum_{i,u} \delta(\text{art}_{iu} \cdot \text{rate}_i > \text{ink}_{iu}/\text{tag}_i) \left(\text{art}_{iu} \cdot \text{rate}_i - \text{ink}_{iu}/\text{tag}_i \right) \qquad (7)$$

Alternatively, one can express the right-hand side by dropping the δ expression and restricting the sum to be over only underwater Vaults.