

## FILTER

$u$  given  $\rightarrow$  see file

Helmholtz filtering

$$\begin{cases} \tilde{u} - \delta^2 \Delta \tilde{u} = u & \text{Here the only player is } \delta \\ \tilde{u} = 1 & \text{on } \partial\Omega \end{cases}$$

$$\alpha(u) = \|u - \Delta_h(\tilde{u})\| \quad D_N = \sum_{k=0}^N (I - F_H)^k$$

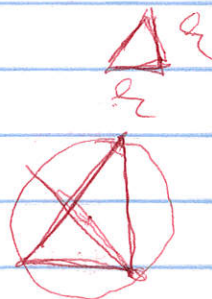
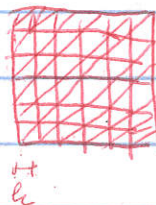
$$F_H = (I - \delta^2 \Delta)^{-1}$$

$$\bullet N=0: D_0 = I, \alpha = \|u - F_H u\| = \|u - \tilde{u}\|$$

Plot  $\alpha$  for  $\delta = h, \sqrt{2}h, 6h$

edge length  $\nearrow$  diameter  $\nwarrow$  quads et al

with a uniform mesh



$$\bullet N=1: D_1 = I + I - F_H = 2I - F_H$$

$$\alpha = \|u - (2I - F_H)\tilde{u}\| = \|u - 2\tilde{u} + \underbrace{F_H(\tilde{u})}_{\tilde{\tilde{u}}}\|$$

$$= \|u - 2\tilde{u} + \tilde{\tilde{u}}\|$$

To get  $\tilde{\tilde{u}}$ :

$$F_H(\tilde{\tilde{u}}) = (I - \delta^2 \Delta)^{-1} \tilde{\tilde{u}} = \underbrace{F_H(\tilde{\tilde{u}})}_{\tilde{\tilde{u}}} - \delta^2 \Delta \underbrace{F_H(\tilde{\tilde{u}})}_{\tilde{\tilde{u}}} = \tilde{\tilde{u}}$$

$$\rightarrow \tilde{\tilde{u}} - \delta^2 \Delta \tilde{\tilde{u}} = \tilde{\tilde{u}}$$

Same plots as before

$$\begin{aligned}
 \bullet N=2: D_2 &= I + (I - F_H) + (I + F_H)^2 \\
 &= I + I - F_H + I^2 + F_H^2 - 2F_H \\
 &= 3I - 3F_H + F_H^2
 \end{aligned}$$

$$\begin{aligned}
 \text{err} &= \|u - 3\tilde{u} - 3\underbrace{F_H(\tilde{u})}_{\tilde{\tilde{u}}} + \underbrace{F_H(F_H(\tilde{u}))}_{\tilde{\tilde{\tilde{u}}}}\| \\
 &= \|u - 3\tilde{u} - 3\tilde{\tilde{u}} + \underbrace{F_H(\tilde{\tilde{u}})}_{\tilde{\tilde{\tilde{u}}}}\|
 \end{aligned}$$

To get  $\tilde{\tilde{\tilde{u}}}$ :

$$\tilde{\tilde{\tilde{u}}} - \delta^2 \tilde{\tilde{\tilde{u}}} = \tilde{\tilde{\tilde{u}}}$$

Same plots as before. Continue with  $N=3,4$  and plots for  $\delta = h, \sqrt{h}$ , and  $h^2$ . After all of these tests we should be able to draw some conclusions ~~for~~ for uniform meshes.

$$F_H(u) = (I - \delta^2 \Delta)^{-1} u$$

$$(I - \delta^2 \Delta)$$

$$(I - \delta^2 \Delta) F_H(u) = u$$

$$\underbrace{F_H(u)}_{\tilde{u}} - \delta^2 \Delta \underbrace{F_H(u)}_{\tilde{u}} = u$$







$$\boxed{\tilde{u} - \delta^2 \Delta \tilde{u} = u}$$

$$\delta = h \quad \delta = \sqrt{2}h \quad \delta = 6h$$

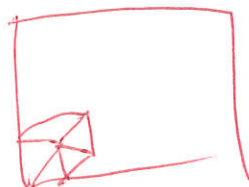
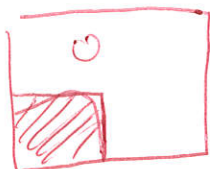
$$N=0$$

$$N=1$$

$$N=2$$

	$\delta = h$	$\delta = \sqrt{2}h$	$\delta = 6h$
$N=0$			
$N=1$			
$N=2$			

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hmm  
hang  
xong ?