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Best-Known Autocorrelation Peak Sidelobe Levels for Binary Codes of Length 71 to 105

Best-known binary code autocorrelation peak sidelobe levels (PSLs) are updated for lengths 71 to 105. For lengths 71 to 82, codes with PSL 4 are found, establishing 4 as almost certainly the optimal value for these lengths. PSL-5 codes are produced for all lengths from 83 to 105, in many cases improving on best-known values.

I. INTRODUCTION

Pulse compression coding is used in radar applications to gain the signal-to-noise ratio (SNR) benefits of a long pulse along with the range resolution of a short pulse. An important figure of merit used to describe pulse compression codes is the peak sidelobe level (PSL). Codes with low PSL can be used to discriminate returns of interest from close-in unwanted discrete returns.

Knowledge of the best PSL for given lengths is useful when performing random searches for low-PSL codes. It is hoped that as the tallies of best, or lowest achievable, PSL values extend to longer code length, a pattern will emerge to help predict optimal values for lengths where direct determinination of the optimal value is impractical.

Despite the computational challenges, progress has been made over time due to improvements in both computational resources and search methods. Lindner [9] in 1975 compiled optimal PSL for lengths up to $N = 40$. Cohen et al. [10] in 1990 continued up to $N = 48$. Coxson and Russo [11] pushed the frontier for highly probable optima to length $N = 70$ in 2004. Levanon and Mozeson [1] provide a summary of optimal PSLs for lengths up to 70.

The task of establishing the optimal PSL for a given length typically requires exhaustive search. However, an enormous amount of effort has been invested for years in searching for low-PSL codes. In particular, considerable evidence exists that there exist no PSL-3 codes longer than 51 (see, e.g., [3]). On the basis of this conjecture, it has been possible to

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establish, with near certainty, that the optimal PSL for lengths 52 to 70 is 4 by searching until a single PSL-4 code is discovered for each of these lengths. One case where an exhaustive search has been done is $N = 64$ [11]; the search established that the best PSL is 4.

This paper extends beyond 70 the list of lengths for which PSL-4 codes are known to exist, by exhibiting examples for each length from 71 to 82. In addition, PSL-5 codes are produced for all lengths from 83 to 105, improving best-known PSL in most cases.

Much effort has been made in progressively lowering best-known PSL for some lengths above 82. Ferrara [2] has compiled a list of the lowest PSL values up to $N = 100$, found by a variety of search methods. In the range from $N = 83$ to $N = 100$, best-known PSL values have been, up until now either 5, 6, or 7. We show that for all these lengths, as well as lengths from 101 to $N = 105$, the best PSL is no higher than 5.

The searches involved a combination of several global optimization methods. The earlier searches were run on a 2.8 GHz Pentium machine, but the later ones were transferred to a Beowulf cluster with eighteen 2.2 GHz AMD single-core Athlon processors.

II. MATHEMATICAL NOTATIONS AND DEFINITIONS

Let x be a binary code of length N, considered as a sequence $\{x_1, x_2, \ldots, x_N\}$, where x_i can be either 1 or -1. In this paper, the term "binary code" will refer to this specific set of element choices. The aperiodic autocorrelation of x is a sequence of length $2N - 1$ defined by

$$\mathrm{ACF}_x = x * \bar{x}$$

where $*$ is aperiodic convolution, $\bar{x}$ means the left-right transposition, or reversal, of x, and ACF is the autocorrelation function. Element k is

$$\mathrm{ACF}_x(k) = \sum_{i=1}^{k} x_i x_{N+i-k}$$

for $k = 1, \ldots, 2N - 1$.

The peak of the autocorrelation is $\mathrm{ACF}(N)$ and always has value N. The other elements are referred to as sidelobes. The PSL for a given binary code x is defined as

$$\mathrm{PSL}_x = \max_{k \neq N} |\mathrm{ACF}(k)|.$$

Another measure of sidelobe level is integrated sidelobe levels (ISLs). It is defined in decibels as

$$\mathrm{ISL}_x = 10 * \log_{10}\left[\sum_{k \neq N}\left(\frac{\mathrm{ACF}_x(k)}{N}\right)^2\right].$$

Where low PSL relates to rejecting unwanted point returns near a return of interest, ISL relates to ignoring unwanted distributed returns near a return of interest.

TABLE I
Codes with PSL = 4

		Comparison
71	63383AB6B452ED93FE	5 [12]
72	E4CD5AF0D054433D82	5 [6]
73	1B66B26359C3E2BC00A	6 [6]
74	36DDBED681F98C70EAE	5 [2]
75	6399C983D03EFDB556D	6 [6]
76	DB69891118E2C2A1FA0	5 [6]
77	1961AE251DC950FDDBF4	6 [7]
78	328B457F0461E4ED7B73	5 [6]
79	76CF68F327438AC6FA80	6 [6]
80	CE43C8D986ED429F7D75	6 [7]
81	0E3C32FA1FEFD2519AB32	6 [6]
82	3CB25D380CE3B7765695F	5 [6]

III. RESULTS

We were able to find PSL-4 codes for lengths 71 up to 82. These are shown in Table I. It has long been presumed that there are no binary codes beyond length 51 with PSL less than 4; so these are likely to be optimal-PSL codes.

Table I lists codes in hexadecimal, identifying 1 and -1 in the code with 1 and 0 in the hexadecimal representation. For lengths that are not multiples of 4, zeros are added at the left side of the code before converting to hexadecimal.

To illustrate both the scope of the search effort and the increasing difficulty of finding PSL-4 codes as the code length reaches the low 80s, consider three lengths for which a comparable number of PSL-5 codes were collected. For length 64, we found 563,512 codes with PSL 5 or better; of these, 1,151 actually achieve a PSL of 4 and another 2,010 would achieve PSL of 4 if not for a single sidelobe on each side with size 5. At length 78, the number of codes of PSL 5 or better was 421,643; of these, only one achieves PSL 4 and 47 have a single size-5 sidelobe on each side. At length 83, despite finding 498,461 codes of PSL 5 or better, none of them achieve PSL 4 and 17 have a single size-5 sidelobe on each side.

Table II gives search results for lengths 83 to 105. For each of these lengths the searches found a PSL-5 code, in most cases improving on the best-known values, which are listed for comparison.

Due to the difficulty of low-PSL code searches, most of the effort to date has concentrated on code lengths below 100. Hence, for lengths above 100, it is hard to find results to compare against. The comparisons in Table II for codes of these lengths are against the PSL values of codes of the same length having best-known ISL. These best-known ISL values come from tables in [5] and from Knauer's website [6], which at one time maintained an up-to-date status for best-known merit factors for code lengths into the

TABLE II
Codes with PSL = 5

		Comparison
83	711763AE7DBB8482D3A5A	6 [7]
84	CE79CCCDB6003C1E95AAA	6 [7]
85	19900199463E51E8B4B574	5 [6]
86	3603FB659181A2A52A38C7	6 [6]
87	7F7184F04F4E5E4D9B56AA	6 [6]
88	D54A9326C2C686F86F3880	5 [3]
89	180E09434E1BBC44ACDAC8A	6 [6]
90	3326D87C3A91DA8AFA84211	5 [6]
91	77F80E632661C3459492A55	7 [4]
92	CC6181859D9244A5EAA87F0	7 [6]
93	187B2ECB802FB4F56BCCECE5	6 [6]
94	319D9676CAFEADD68825F878	6 [6]
95	69566B2ACCC8BC3CE0DE0005	7 [7]
96	CF963FD09B1381657A8A098E	6 [6]
97	1A843DC410898B2D3AE8FC362	7 [4]
98	30E05C18A1525596DCCE600DF	7 [7]
99	72E6DB6A75E6A9E81F0846777	7 [7]
100	DF490FFB1F8390A54E3CD9AAE	7 [7]
101	1A5048216CCF18F83E910DD4C5	7 [8]
102	2945A4F11CE44FF664850D182A	7 [6]
103	77FAAB2C6E065AC4BE18F274CB	7 [5]
104	E568ED4982F9660EBA2F611184	8 [6]
105	1C6387FF5DA4FA325C895958DC5	11 [4]

200s. Merit factor is inversely proportional to ISL (see [13]). The tables in [5] list best values for most odd lengths up to 201 and the best merit factors for many even lengths, including 102 and 104. Values from Knauer's website are from August, 2003; the website was removed sometime thereafter.

The results in Table I and Table II support conclusions in the 1986 paper by Kerdock et al. [3]. For PSL values 1 to 5, their paper listed the longest lengths for which each PSL was known to be achievable. They identified length 69 for PSL 4 and length 88 for PSL 5 but added that there were likely numerous PSL-4 codes above 69 and numerous codes with PSL 5 above 88. It is the authors' opinion that it is likely that many PSL-5 codes exist above 105. This opinion is based on the fact that much more search time was devoted to searching for PSL-4 codes for lengths between 80 and 90 than for PSL-5 codes with lengths above 105.

Note that at length 105, a PSL of 5 represents a peak-to-peak-sidelobe voltage ratio of 21 to 1. The longest PSL-4 code found, at length 82, achieves a peak-to-peak-sidelobe voltage ratio of 20.5.

Significant effort was made to find PSL-5 codes above 105 with no success. Ultimately, the decision was made to concentrate effort instead on finding better codes for lengths in the 80s, where further progress was then made.

The ISL values associated with the results of Table I and II are given in Table III, where they are represented in terms of dB below the peak. The PSL values are given as well, this time in dB below the peak.

TABLE III

N	PSL (dB)	ISL (dB)
71	−24.9840	−7.7927
72	−25.1055	−7.7815
73	−25.2253	−7.4077
74	−25.3434	−8.3864
75	−25.4600	−8.9642
76	−25.5751	−8.3528
77	−25.6886	−7.4117
78	−25.8007	−8.7785
79	−25.9113	−8.5176
80	−26.0206	−7.9588
81	−26.1285	−8.0329
82	−26.2351	−8.1648
83	−24.4022	−8.1367
84	−24.5062	−7.8261
85	−24.6090	−7.6402
86	−24.7106	−8.1438
87	−24.8110	−8.2137
88	−24.9103	−8.0118
89	−25.0084	−7.7821
90	−25.1055	−7.5681
91	−25.2014	−7.3113
92	−25.2964	−8.1097
93	−25.3903	−7.4945
94	−25.4832	−8.3701
95	−25.5751	−6.7260
96	−25.6660	−7.0543
97	−25.7560	−7.1827
98	−25.8451	−8.1869
99	−25.9333	−7.8661
100	−26.0206	−7.5746
101	−26.1070	−7.2989
102	−26.1926	−7.5285
103	−26.2773	−7.9471
104	−26.3613	−7.8073
105	−26.4444	−7.7007

IV. CODES FOUND WITH LOW INTEGRATED SIDELOBE LEVEL

Although low-ISL codes were not the goal, good ISL codes were retained when found during the search for low-PSL codes. The best ISL values achieved for each code length are given in Table IV, alongside the current best-known ISL values for each length. These current bests were based on [5] and the last known results at Knauer's website [6]. This comparison shows that up to length 89, and for lengths 93 through 95 codes were found that are comparable in ISL to the best now known.

The second column of Table IV indicates whether the current best-known ISL belongs to a code with Golay's skew symmetry (see [13]). Often searches for odd-length codes of higher lengths exploit skew symmetry to reduce the search effort by a significant amount. That is, given an odd length N, the full space of binary codes has size 2^N, while the subspace of skew-symmetric codes has size $2^{(N-1)/2}$. Searches over skew-symmetric codes can often yield excellent ISL values but cannot, in general, be relied upon to find optimal-ISL codes.

TABLE IV

		Best ISL (dB)	
N	Skew	Current	Found
71	yes	−9.6216	−9.6215
72	no	−9.3651	−9.3651
73	no	−9.3707	−9.3707
74	no	−9.0468	−9.0468
75	no	−9.3189	−9.3190
76	no	−9.3685	−9.3685
77	yes	−9.1807	−9.1807
78	no	−9.4283	−9.4283
79	no	−9.6402	−9.6402
80	no	−9.5861	−9.5861
81	no	−9.4539	−9.4540
82	no	−9.5026	−9.5026
83	yes	−9.6078	−9.6078
84	no	−9.1406	−9.1406
85	no	−9.4081	−9.4081
86	no	−9.2550	−9.2550
87	no	−9.4353	−9.4353
88	no	−9.3666	−9.3666
89	no	−9.6277	−9.6227
90	no	−9.5136	−9.3628
91	yes	−9.3853	−8.9689
92	no	−9.2932	−9.2932
93	no	−9.4930	−9.4930
94	no	−9.4712	−9.4713
95	yes	−9.7408	−9.7047
96	no	−9.4751	−8.5733
97	yes	−9.4335	−8.3746
98	no	−9.4503	−8.3819
99	yes	−9.2907	−8.2990
100	no	−9.3704	−8.2623
101	yes	−9.4569	−8.1887
102	no	−9.6259	−8.1799
103	yes	−9.8035	−8.4440
104	no	−9.4629	−9.0309
105	yes	−9.4896	−8.9378

Skew symmetry can only occur at odd code lengths. However, for some of the even lengths, especially those around 100 and above, the best-known ISL values seem to belong to codes found by local search around skew-symmetric codes having best-known ISL for an adjacent odd length.

The best-known ISL listed in Table IV for odd lengths 95 and above all belong to skew-symmetric codes. That the currently known best ISL for each of these lengths is for a skew-symmetric code suggests the difficulty of searching over the full search space when lengths reach this level.

V. SUMMARY

Best-known binary code autocorrelation PSLs are updated for lengths 71 to 105. Under the generally accepted assumption that no PSL-3 binary codes exist for lengths greater than 51, an optimal value of 4 is established with high probability for lengths 71 to 82 by exhibiting PSL-4 codes in each case. PSL-5 codes are found for all lengths from 83 to 105, in most cases improving on the best-known PSL. It is the authors' opinion that many of these codes are themselves optimal for PSL.

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