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Synthesis of Binary Sequences with Good Auto- and cross-correlation Properties by GA

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ABSTRACT

In this letter we employ genetic algorithm (GA) to synthesize binary sequences with good autocorrelation and crosscorrelation properties. The feasibility and robustness of this method are proved by the sequences obtained in simulation, which are better in auto- and cross-correlation properties than those had been reported.

Keywords: Genetic Algorithm, Binary sequences

I. INTRODUCTION

Binary codes have been widely used in radar and communication areas for years [5][6].

As it is well known, in CDMA systems each user is assigned a sequence—code—that serves as its address. In order for the receiver, in this case a correlator, to correctly recognize the active users, the employed codes must be quasi-orthogonal to one another. This means the following conditions must be satisfied:

1.The peak of the autocorrelation function

$$r_u(\tau) = \int_{-\infty}^{\infty} u(t)u(t-\tau)dt$$

of u(t) should be maximized.

2.The sidelobes of the autocorrelation function $r_u(\tau)$

of u(t) should be minimized.

3.The cross-correlation function

$$r_{u_\alpha, u_\beta}(\tau) = \int_{-\infty}^{\infty} u_\alpha(t)u_\beta(t-\tau)dt$$

of any two signals $u_\alpha(t)$ and $u_\beta(t)$ in the system should be minimized.

As discrete systems are concerned, the above conditions translate into the ones described in next section.

The synthesis of binary codes with good properties is a nonlinear multivariable optimization problem. In [5] [6], the exhaustive searching method was used to perform this task and best binary sequences up to length N=48 obtained. However, the computation cost makes it quite impossible to find longer ones.

Recently, some other effective approaches, such as Hopfield Neural Network (HNN) and Simulated Annealing (SA), have been employed to generate binary sequences of practical value.

On the other hand, Genetic Algorithm (GA) is now considered an efficient optimization algorithm. Compared with other conventional methods, GA shows high feasibility. It doesn't require gradient information, which is often unavailable or too much time-consumed. Also it is unlikely to be trapped into local minimum, with carefully selected parameters of its genetic operation. Actually, GA has been successfully applied in various fields. In this letter, we propose to use the algorithm to tackle the difficult problem mentioned above. Better results are obtained with acceptable costs of time.

II. GENETIC ALGORITHM FOR DESIGN OF BINARY SEQUENCES

To synthesize a set of binary codes with good autocorrelation and crosscorrelation properties, consider X to be a set of K binary sequences of period N:

$$u^{(p)} \in X, p = 0,1,\cdots,K-1$$

where

$$u_i^{(p)} \in (-1,1), i = 0,1,\cdots,N-1$$

The autocorrelation function for sequence $u^{(p)} \in X$ is defined by:

$$C_a(u^{(p)})(j) = \sum_{i=0}^{N-1-|j|} u_i^{(p)} u_{i+|j|}^{(p)}$$

$$j = 0,\pm1,\pm2,\cdots,\pm(N-1)$$

The crosscorrelation function for sequence $u^{(p)}, u^{(q)} \in X$ is defined by:

$$C_c(u^{(p)},u^{(q)})(j) = \sum_{i=0}^{N-1-|j|} u_i^{(p)} u_{i+|j|}^{(q)}$$

$$j = 0,\pm1,\pm2,\cdots,\pm(N-1)$$

Thus we have the optimization criterion based on peak sidelobe as follows:

$$E = \sum_{p=0}^{k-1} \max_{i\neq0} |C_a(u^{(p)})(i)| + \lambda \sum_{p=0}^{k-2} \sum_{q=p+1}^{k-1} \max_{i} |C_a(u^{(p)},u^{(q)})(i)|$$

Notice that it is slightly different from the energy function defined in [4], as there only the one sequence generation problem was concerned. Thus, if k=1, the two are actually the same. Furthermore, also the peak sidelobe criterion is adopted here, due to its privilege in detection, other criteria are also possible [3].

In solving optimization, Genetic Algorithm (GA) encodes the potential solution and then performs the genetic operators on the genetic representations. It is proved to be very powerful in global searching [7]. As to the synthesis of binary sequences, it searches directly in the space of potential solutions by doing crossover and mutation of sequences. The overall operation of the GA to be used is as follows:

(i) Set the parameters: the population, probability of crossover and probability of mutation;
(ii) Randomly generate an initial generation including a lot of individuals;
(iii) Evaluate each individual by computing its fitness function and select the best one;
(iv) If the termination condition is met, return with the best solution;
(v) Selection: select each individual from the whole generation with a probability that equals its normalized fitness, to take part in the genetic process. Thus it encourages better solution naturally;
(vi) Perform crossover and mutation and produce a new generation. Go back to step (iii).

As one can see, the algorithm described above is rather a simple version of GA. However, to improve the performance, some modified GA should be adopted.

III. SIMULATION

In the simulation, we chose to generate binary sequences of various lengths. First, examined the case of one sequence generation, when only the autocorrelation function was computed. Table II and III show the result in comparison with that using HNN [4] and SA [3] method, separately. For the length=150, no results of HNN is available.

Most of the results are obtained within 500~600 generations only, and what is more, it almost converges to the global minimum in every run.

The population was set 100, and the probability of crossover 0.85 and the probabilities of mutation were between 0.001 and 0.0058.

From the simulation results we found indeed improvements in correlation property over other methods. It indicates that GA is powerful in searching the required binary sequences.

Table I: Minimum Maximum sidelobe of binary sequences generated by GA

LENGTH OF SEQUENCES (L)	50～63	64～77	78～92	93 ～ 100	～150
MAXIMUM SIDELOBE (S)	5	6	7	8	～10

Table II: Various length binary codes generated: (In upper line are the results of HNN, below are that of GA)

	S	EXAMPLE CODES

56	6	00000,10001,01111,01111,00001,11011,01001,10001,10110,00101 01010,0
	5	00010,11110,10010,00110,11011,01111,00111,11100,11000,11000 10101,0
62	6	10010,10101,10101,01110,00101,11110,11001,10110,11110,11000 00010,00001,01
	5	11011,00011,10010,00011,11010,01011,00111,10100,11001,01110 11101,01010,00
76	7	01000,01011,10100,10100,11011,10001,00101,10011,00101,01000 10101,01011,01100,11000,11101,1
	6	00000,01110,01110,01011,11000,01011,01011,00110,00001,10101, 10110,01001,01010,00100,00010,0
80	8	01111,01001,00010,00001,10100,10010,10100,01110,10011,00000 11110,11000,11101,11001,10010,11010
	7	00010,10001,10110,01111,01000,01101,10110,11100,11101,10100, 10111,11110,00110,01101,01010,10000
90	8	10111,01000,01010,01001,10100,00101,00100,11111,01101,01111 10111,01000,00110,01110,00011,11110,01000,11001
	7	01000,01111,11111,00100,11100,01011,10111,10111,10101,10100 10110,01001,00010,00101,10011,00011,10101,10111
97	9	01001,00011,11110,11101,00000,11101,01110,11100,00101,00110 00001,11001,11000,11011,10011,10010,01000,10010,01001,00
	8	01011,01100,11010,01111,00111,00001,10011,10101,00010,10100, 01011,00010,01000,00010,10010,11111,00000,01010,10000,10
150	10	00010,01111,11110,11111,11000,01000,00111,10110,00011,00100, 10111,01000,01010,10001,01000,01000,00110,11010,10110,00010, 00011,01110,01100,11011,00110,01010,01100,00101,00001,11001

Table III: Example binary sequence pairs by GA:

L	E(GA)	E(SA)	EXAMPLES
31	16	17	1100101100100111111101010011100 0110111101111000011011001000101
37	20	21	0011110111111111001100010111001001011 1010010011111000000100100010101111 0011
53	24	24	00000010100111011000010110010000110101011101101 10011 00101011100010100111001101110111111011110000010110 10

IV. CONCLUSION

In this letter, genetic algorithm is used to generate binary sequences with minimum sidelobe. The robustness and theoretical global convergence enable GA to produce better binary sequences in almost every run,

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