New Binary Sequences with Good Aperiodic Autocorrelations Obtained by Evolutionary Algorithm

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***Abstract*—In this letter, new binary sequences for lengths up to 100 with good autocorrelation function properties are presented. The results obtained by an evolutionary algorithm are better than other known results in most cases.**

***Index Terms*—Evolutionary algorithm, sequences.**

I. Introduction

SEQUENCES with low aperiodic autocorrelation function (ACF) sidelobes have found extensive applications in radar and communication systems. Barker sequences are the only known binary sequences with the lowest possible peak sidelobe level (PSL) of unity. Unfortunately, Barker sequences are few in number, and the longest known Barker sequence is of length 13. Efforts have been made to find longer Barker sequences or to prove that there are no others. What is known to date is that further binary can only exist for length $N = 4C^2$ (with C an integer) and that no such sequence exists up to length 12 100. Therefore, it is generally believed, although not completely proved, that no other Barker sequence exists.

Since most practical applications require peak-to-sidelobe ratios much greater than 13, a compilation of sequences with the lowest possible sidelobes at the longer length is needed. Due to the fact that no known analytical technique to construct sequences with minimum PSL, only time-consuming and money-consuming exhaustive computer search program have been used to generate the best possible sequences. By this method, Lindner [1] found all binary sequences up to length 40 with minimum PSL in 1975. In 1990, with an improved algorithm, Cohen *et al.* [2] extended those results to length 48. It was also noted that there are no length 49 or 50 binary sequences with PSL of three or less. However, even with the most powerful computers, enumeration algorithms are only able to globally search for the best sequences with rather small length within a reasonable amount of time. Therefore, for longer length effective optimization method should be adopted to search sequences with good rather than the best aperiodic ACF properties. Using a neural network approach, Hu *et al.* [3] obtained useful binary sequences for lengths up to 100. In this letter, an evolutionary algorithm is applied to generate sequences with low PSL. It is shown that our results are better than that of [3].

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II. Merit Measures

For general complex-valued sequence $\{a_n\}$ of length N, the aperiodic ACF, for $\tau \geq 0$, is given by

$$C_a(\tau) = \sum_{n=0}^{N-\tau-1} a_n a_{n+\tau}^*. \tag{1}$$

One of the most commonly used merit figures of aperiodic ACF is peak sidelobe level (PSL), which is the maximum magnitude of the out-of-phase ($\tau \neq 0$) ACF, i.e.,

$$\mathrm{PSL} = \max_{\tau \neq o} |C_a(\tau)|. \tag{2}$$

Another important measure of aperiodic ACF, merit factor (MF), which specifies the ratio of the energy of ACF mainlobe to the energy of the ACF sidelobes, is defined as

$$\mathrm{MF} = \frac{C_a(0)^2}{2\sum_{\tau=1}^{N-1} |C_a(\tau)|^2}. \tag{3}$$

When the merit factor is expressed in decibels, it is called the integrated sidelobe level (ISL), which is widely used in radar literature.

It is desirable, in many applications, to make the PSL as small as possible and the MF as great as possible [4].

III. Optimization Procedure

Natural evolution is a population-based optimization process. Simulation of this process on a computer results in stochastic optimization techniques that can often outperform classical methods of optimization when applied to difficult real-world problems [5]. Evolution algorithms abstract the basic principles of replication, variation and selection from Darwinian biological evolutionary theory. Those algorithms start from a randomly generated (rather poor quality) initial population of search space positions. During each search cycle, or generation, the population members are assessed according to a fitness function, and those with poor fitness are excluded. New population members are created by some specific reproduction operators (mutation and crossover).

TABLE I
BINARY SEQUENCES WITH IMPROVED APERIODIC ACF

Seq Len	Min PSL	Min PSL in[3]	MF	Example Sequence
49	4	5	6.0025	0010011100100100001111011000001001000001101010101
50	4	5	4.5788	10000111111111110001101110001101000100110101011011 0
51	4	5	6.1056	010101101001110100111000101110110000110111101111101
52	4	5	4.5369	0110100110110011001110001010000000010111100001010110
53	4	5	4.0128	10010100110000111000010100100110010000000101111101010 1
54	4	5	5.0801	101100001000000011111110001111010110101010110010011001
55	5	5	5.5	0011011101001101110011101000001011011110001010000100001
56	5	6	5.0909	10011100000011001110001111111101110100101011101101101010
57	5	6	4.8348	101101110111011001010111100000001110110101110000110000110
58	5	6	4.9326	0001001000110011111101011001011011101010000111101000111100
59	5	5	4.5208	0000100101011000100110111100000101100100111100011000010001 0
60	5	6	4.2654	001101001111001110100100011001111111101110010101110100000101
61	5	6	5.0284	0001111000011110011001011001100100101001000000011011110101010
62	5	6	4.4184	10001100110111100111000001001011101010001111101101111010101101
63	5	5	4.4002	100110101101111000010000000111010000010111001010001101010110011
64	5	6	5.0693	1000101001001111011101110100101000010100011001000000110111000011
65	5	6	4.0009	01111110110011010110111011010010101110101000011001111000111110000
66	5	6	4.3823	101011010011010110111001100110100100010110000000111110111110001010
67	5	6	4.4446	1101100110100111001001111000101011000100010110000100011111111110010
68	5	6	3.5137	10101101000000000100001101011000001110011101000011011011001101101010 1
69	5	6	4.1185	110010011001101000110001100110101000010110100000100000100101011111000
70	5	6	4.1455	0011010011110000101000101111110001101111100010001100001000100101001000
71	6	7	4.8942	11011010010001001001010111011111110000111111001110001110010111001100 10
72	6	7	5.0233	011000111100111111001010111100001000100010001011110101000100101101101100
73	6	6	4.0617	0100010101101110011001010001111100000111110010100001100010110111111110110
74	6	7	4.2450	01110011100101000100101010001011001000001111100001101101111010000100100011
75	6	6	4.2040	001101100110010101011110101000001111101111101111111010010110100100111000110
76	6	7	4.9283	0111000001100000100011100011011100000100111111100100110101110110101101010011
77	6	6	4.2471	11001011001110010110000110001000010001000101100010111100101010100100000111111
78	6	7	4.4023	101110011111000011010000001001110011001011101000000011011101010100101101111111011
79	6	7	3.8861	10111110000010010100010111000110000101110011011111111010011001101101100001010111
80	6	8	3.4335	00111000001100110100001110110000011010011101010001100011001010010010111111111010
81	6	7	4.2494	001011111110010001100101111000011000000000001100101011011000011001010000101010 1001
82	6	7	3.5880	0011101000000111000010000111010010100000110011011111001010001000101001001110010011
83	6	7	3.4138	10110111001110001100000000111110001011010011011010110001010100010001001100000010010
84	6	7	3.6522	001010011100101111001110101000110101100010110000000100000001111101100000111011001001
85	6	7	4.1145	0000101111110101010000100110000000011000110001100101110001110100011011010110100110110
86	6	7	3.3285	11100001011011101100110010010111000010101011101011011011100001010001000010011100010000
87	6	8	3.4688	101000110010010010101000101001011110000000111101010010001110011010000000110001110110 1100
88	6	7	3.5328	1111110100011110100101000101011000011001000101101000110010101001100100000100001100111110
89	7	7	4.3049	01111000001111101011010001010011111011000110001100101011011011101001100000100001000000110
90	7	8	3.7886	001101101000010001011010111001011011000111111001100100000100011110001000101010000010000110
91	7	7	3.9774	1110000000000110011111111111001110111110010011101011001011010010111000101010100101011001110 10
92	7	7	4.0268	00001100011110101100010011111010110010001001001010111110111011000011011100010111110110101111
93	7	9	4.1343	100101001111001011001111010001100111011011011111111101001000100101010100000001110010111110001
94	7	8	3.3600	0001100011001011100011000100100111110000111010000001001110010001011011110110111101011010101010
95	7	8	4.3939	01010000110101100001001011101001100100111100110001101100000111001001111111111111010101010010111
96	7	10	3.8658	001010110111111010101111111000001001011001101100110000100101011001110010001010001110010110100001 1
97	7	9	3.6300	1011101000010000010000101000111100001011110010101001100010011010000100011101110001100100101110110
98	7	8	4.0251	00111011101101000010101010000100111110010010011010010000100000001111010001110001011011110110111000
99	7	8	4.4188	111100011001110001100001111100011001100110010101001011010100000000000001011001001011101010110110010
100	7	8	3.3829	0110001110010001001010110111111110111101101100000110101101100010011010001111101010101000110101110001

When applied to sequence search, this method can be implemented as follows.

1) Generate an initial population of P parent sequences at random.
2) Evaluate all initial parent sequences according to a predefined fitness function.
3) Mutation is applied to each parent sequence and creates a new sequence by randomly flip one or two bits.
4) Crossover is applied to two randomly chosen parent sequences and creates two offspring sequences by selecting a random position and splicing the section that appears before the selected position in the first sequence with the section that appears after the selected position in the

second sequence, and vice versa. This operation repeats until certain number of population is produced.

5) Evaluate all new candidate sequences according to the fitness function.
6) Competition is applied and the P best sequences (with the highest fitness) become the new parents in the next generation.
7) This process is terminated if no further improvement is possible within the predetermined time, or if the available computation is exhausted; otherwise go to step 3) and the cycle is repeated.

In our simulation, the fitness is selected as

$$F = \frac{\alpha}{\mathrm{PSL}} + \beta \cdot \mathrm{MF} \tag{4}$$

where PSL and MF are the ACF peak sidelobe level and merit factor of the sequences, respectively. α and β are empirical weight coefficients, which determine the importance of PSL and MF in the process of optimization. Since we refer to the "good sequences" mainly as those with very low PSL throughout this letter, therefore, α is chosen to be much greater than β.

IV. Simulation Results

By employing the optimization procedure given above, a list of sequences of lengths 49–100 has been obtained, as shown in Table I. In this table, the sequence length, the achievable PSL for each length, the MF achieved with the given PSL and a sequence example obtained are given in columns 1, 2, 4, and 5, respectively, where 0's are used to represent -1's to conserve space. In column 3 of the table, the minimum PSL obtained in reference [3] is also listed for comparison.

V. Concluding Remarks

In this letter, binary sequences with good aperiodic ACF, for lengths up to 100, is obtained by an evolutionary algorithm. Results show that this method outperforms the approach in [3]. Not surprisingly, we observed that it is quite efficient to find a good local optimum with the evolutionary algorithm, and some of the results are optimal with respect to PSL, say sequences of lengths 49 and 50. But in general, the search for global optimum probably depends on a lucky selection of the initial population of parent sequences. As the sequence length increases, it becomes easier to leave a local optimum, so the search procedure requires more time to find a good solution. Also, the solution quality improves with the number of generations. The more number of trials, the more confident we are to say about the optimality of the results. Although it is intended to search binary sequences, the same procedure can also be extended to nonbinary cases with slight modification.

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