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The Impact of Network Topology on Banking Default Dynamics

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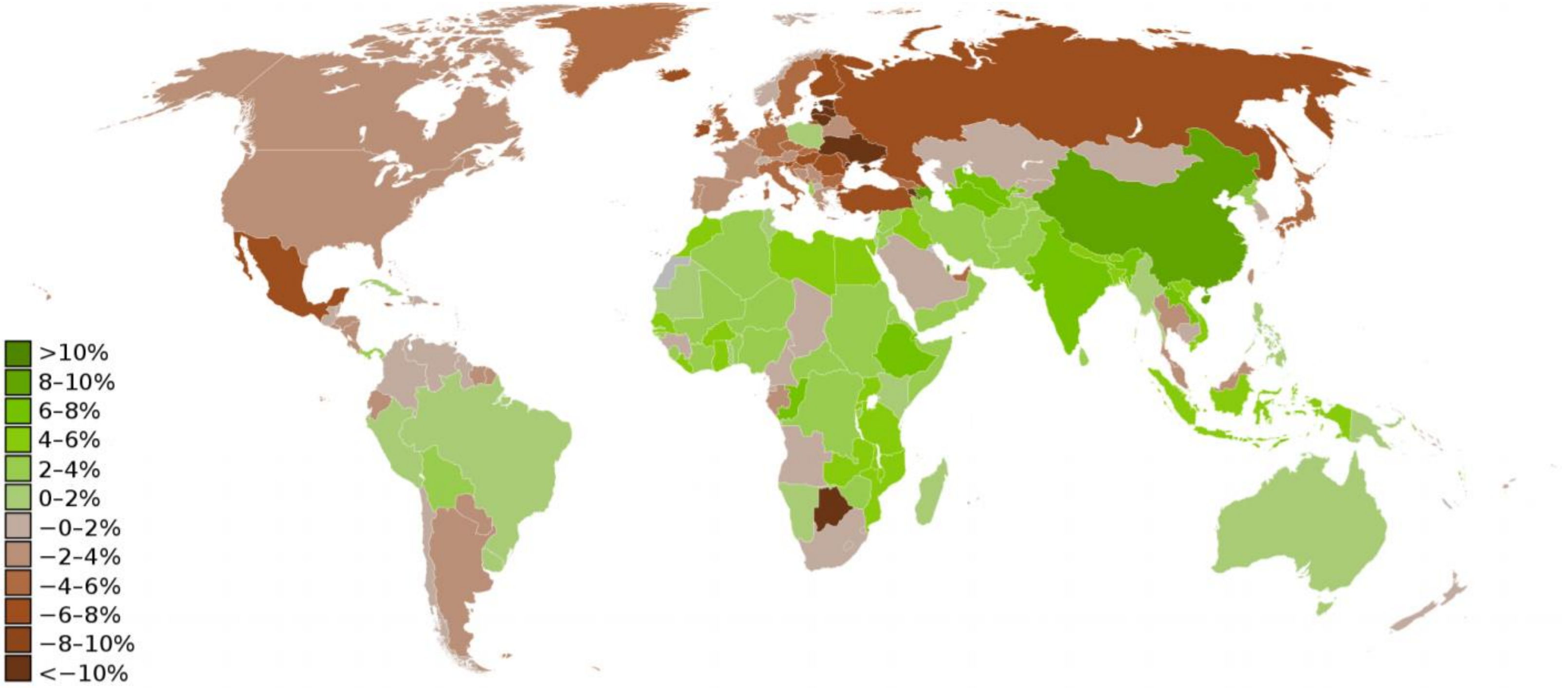
Financial crisis of 2008

- Drastic financial losses
- Decrease in the GDP growth rate
- Bankruptcy of the 4th largest investment bank in USA Lehman brothers



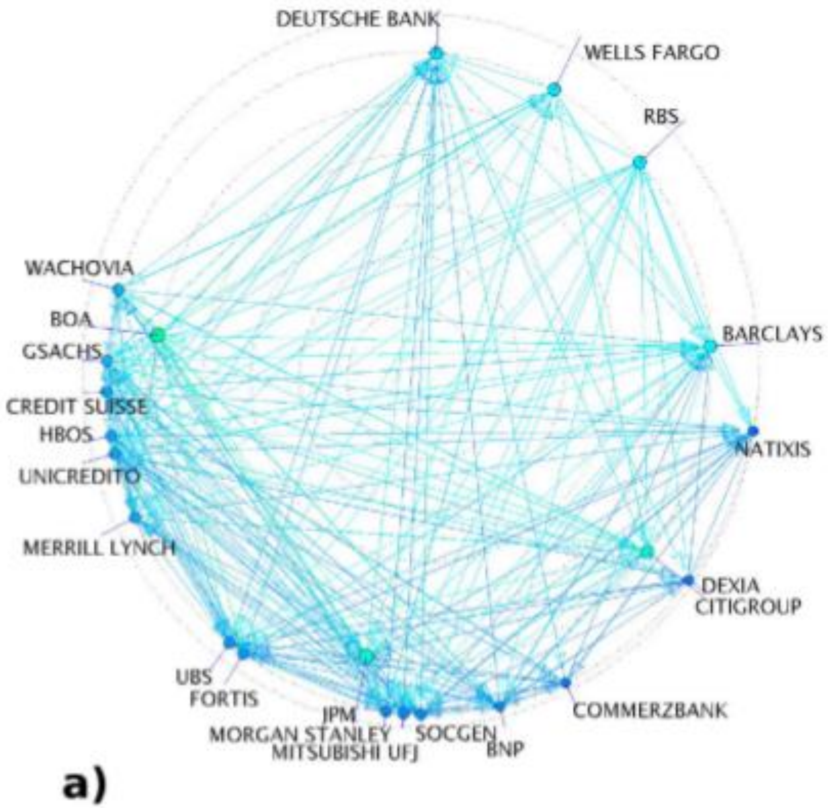
Networks matter

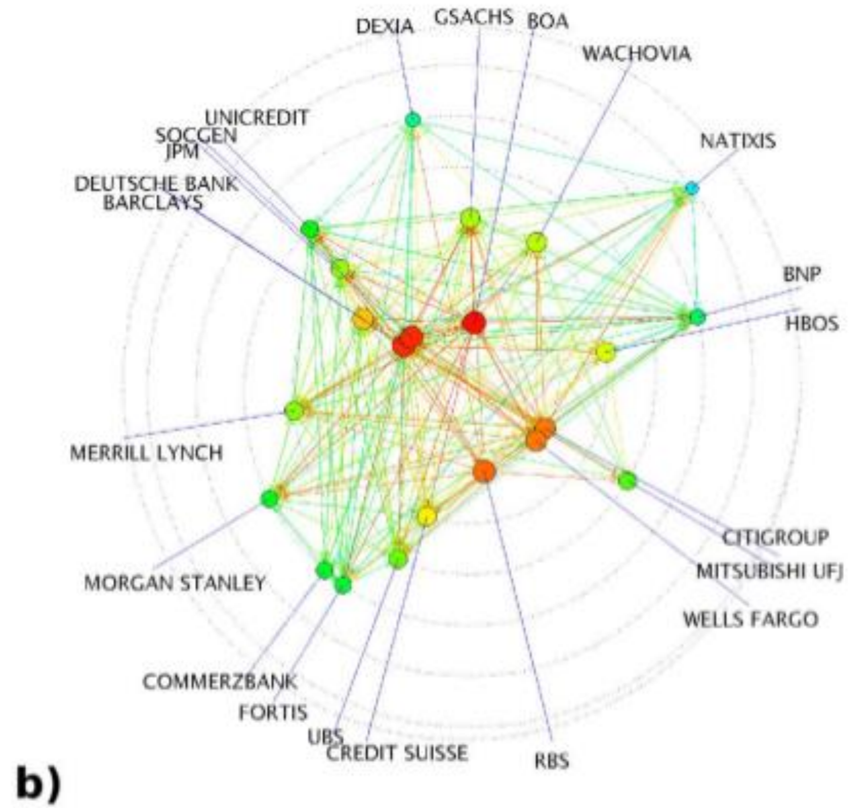
- **Networks matter.** The network structure is crucial to estimate systemic risk

- **Data scarce.** This is one of the main challenges for empirical studies of financial networks

- **Systemic risk can be estimated.** One measure of systemic risk is the DebtRank

Network matters





Debt rank

Bank at the beginning **(a)** and at the peak of the financial crisis **(b)**.

Project scope

- Nier E., Yang J., Yorulmazer T., Alentorn A. «Network Models and Financial Stability» (2008)
  - Simulation of default contagion
- Kashirin V. «Evolutionary Simulation of Complex Networks Structures with Specific Topological Properties» (2014)
  - Generation of complex networks

- Our project:
  - Implementation of simulation model
  - Implementation of network generation
  - **Observing the impact of network topology on default dynamics**

Generation of networks **G(n,p)**

- Network models
  - G(n, p) model
  - Stochastic Block Model (SBM)
  - Complex network structure simulation

- **G(n,p) Model.**
  - **n**: The number of nodes |V|
  - **p**: probability of connecting two nodes

Degree distribution of a node is given by:

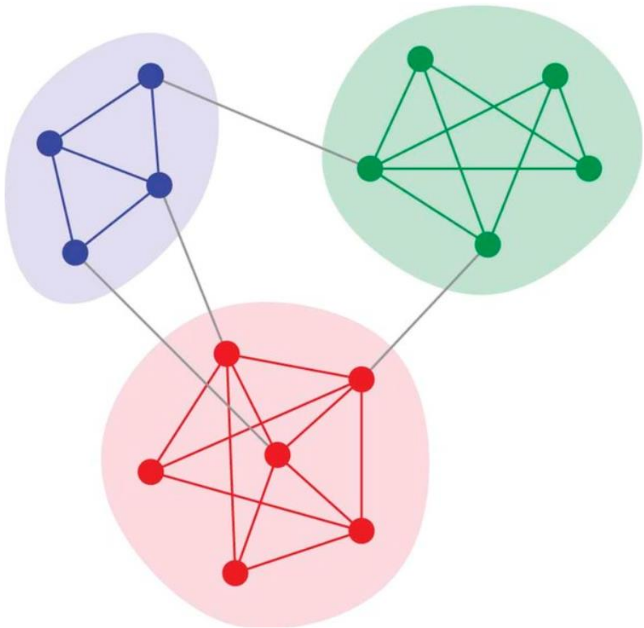
$$P(k) = \binom{n-1}{k} p^k (1-p)^{n-1-k}$$

The mean degree is given by:

$$\langle k \rangle = np$$

Generation of networks **SBM**

- Generate Community like structure



- Parameters
  - Number of communities C
  - Block assignment vector $\vec{z}$ with values in $\{0, \dots, C-1\}$
  - Stochastic block matrix
  - An example for 3 community stochastic block matrix

$$M_1 = \begin{pmatrix} 0.8 & 0.1 & 0.05 \\ 0.1 & 0.9 & 0.1 \\ 0.05 & 0.1 & 0.65 \end{pmatrix}$$

Complex network structure simulation

- Real-world complex networks are usually characterized by a set of specific topological characteristics:
  - High clustering coefficient
  - Short average path lengths
  - Communities
  - etc.

Simulated annealing

$$G^* = \underset{G \subseteq R}{\operatorname{argmin}}\, 1 - \psi(G)$$

Let G be the initial graph
Let $\psi(G)$ be the weighted sum of objective functions for graph G
$T \leftarrow T_0$
$t \leftarrow 0$
while $T > T_{min}$ **do**
  $E_{cur} \leftarrow 1 - \psi(G)$
  Mutate G to obtain G'
  $E_{new} \leftarrow 1 - \psi(G')$
  $G \leftarrow G'$ with probability $Pr = [-\frac{1}{T} * max\{0; E_{new} - E_{cur}\}]$
  $T \leftarrow T(t)$
  $t \leftarrow t + 1$
end while

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Graph mutation

- Connection of random pair of nodes
- Rewiring a pair of randomly selected edges

Global modifications

- Removal of random edge

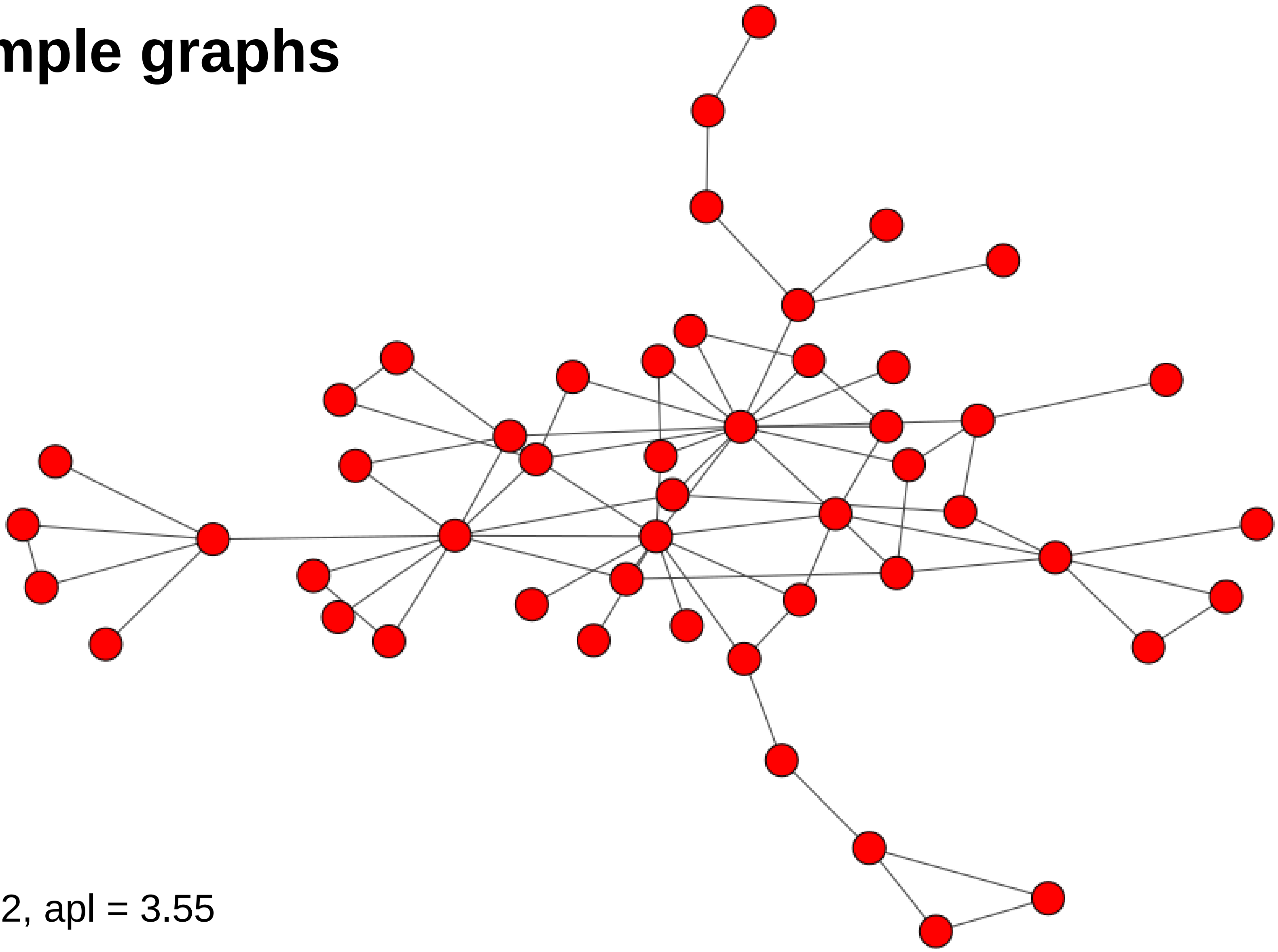
- Connecting local nodes
- Local rewiring a pair of random edges

Local modifications

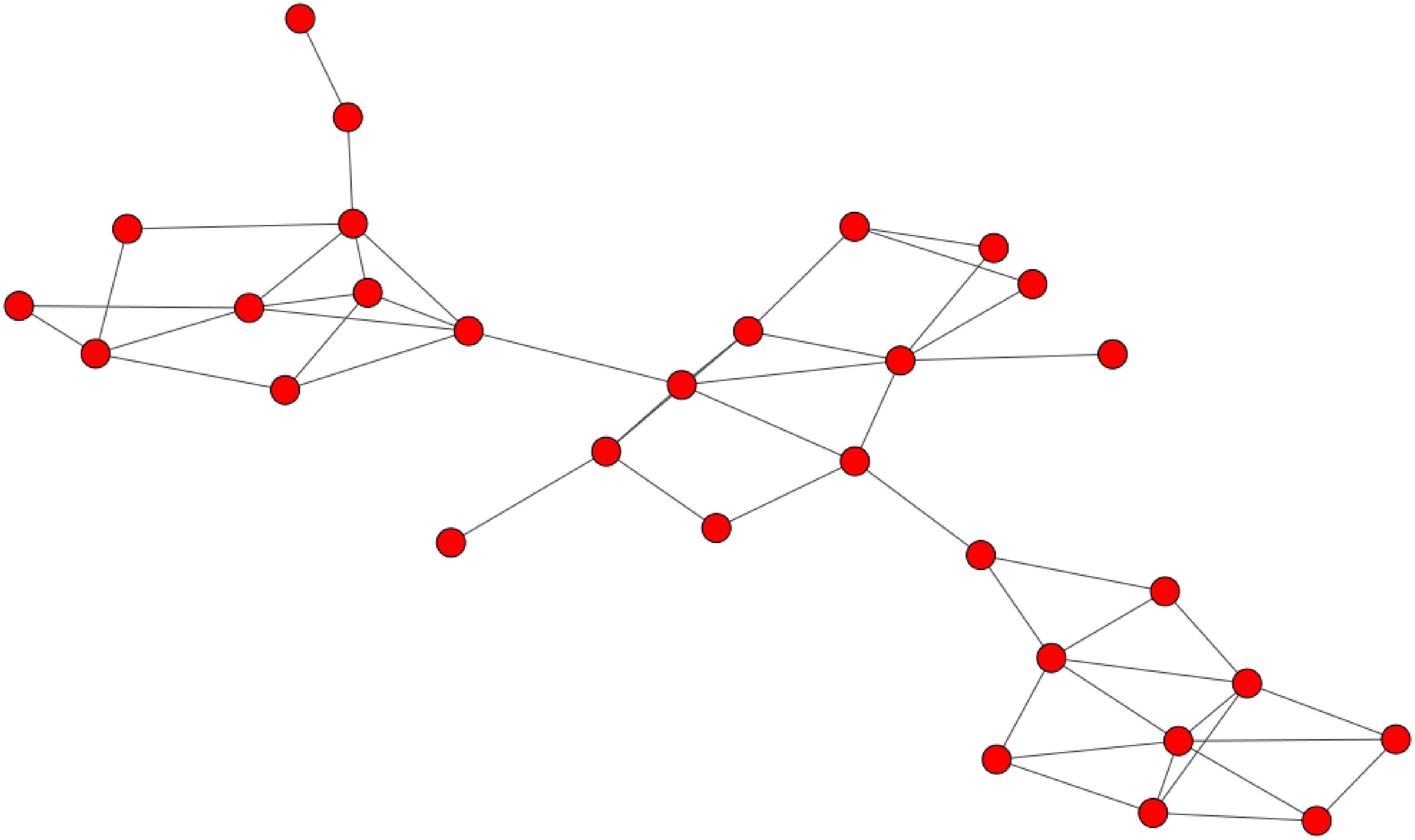
Compute-intensive task

- Graph generation could run up to 6 hours on ETH Euler’s 48 cores
- 5 days of CPU time (for 10`000 graphs)
- Must be parallelized!

Example graphs



C = 0.52, apl = 3.55



3 communities, 0.6 modularity

Shock and assets simulation model

Model parameters (p, n, γ, β, E)

- I, E Total internal/external aggregate assets A = E + I
- γ net worth percentage to total assets
- β, θ percentage of external/internal assets $\beta = \frac{E}{A}$
- a_i, e_i, i_i are total, external and internal assets for each bank
- Z is number of links

Algorithm 1: Algorithm distributing assets according to the aggregated values

$w \leftarrow \frac{I}{Z}$
for i $\in [1, n]$ **do**
  $i_i \leftarrow w \sum_{j=1}^{n} A_{ij}$
  $b_i \leftarrow w \sum_{j=1}^{n} A_{ji}$
  $\tilde{e}_i \leftarrow max(b_i - i_i, 0)$
end for
for i $\in [1, n]$ **do**
  $e_i \leftarrow \tilde{e}_i + [(E - \sum_{l=1}^{n} \tilde{e}_i)/n]$
  $a_i \leftarrow e_i + i_i$
  $c_i \leftarrow \gamma a_i$
  $d_i \leftarrow a_i - c_i - b_i$
end for

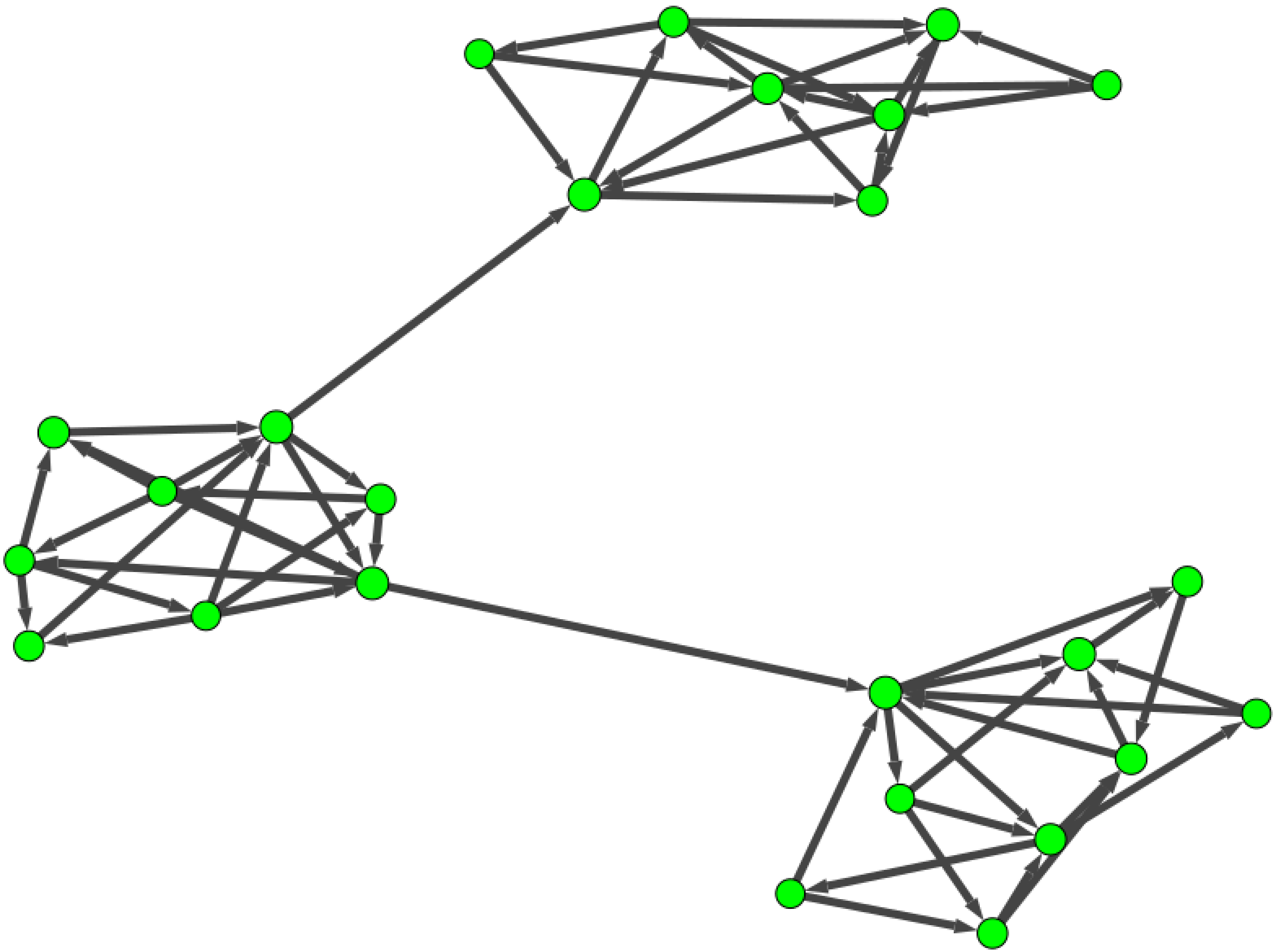
Shocks simulations

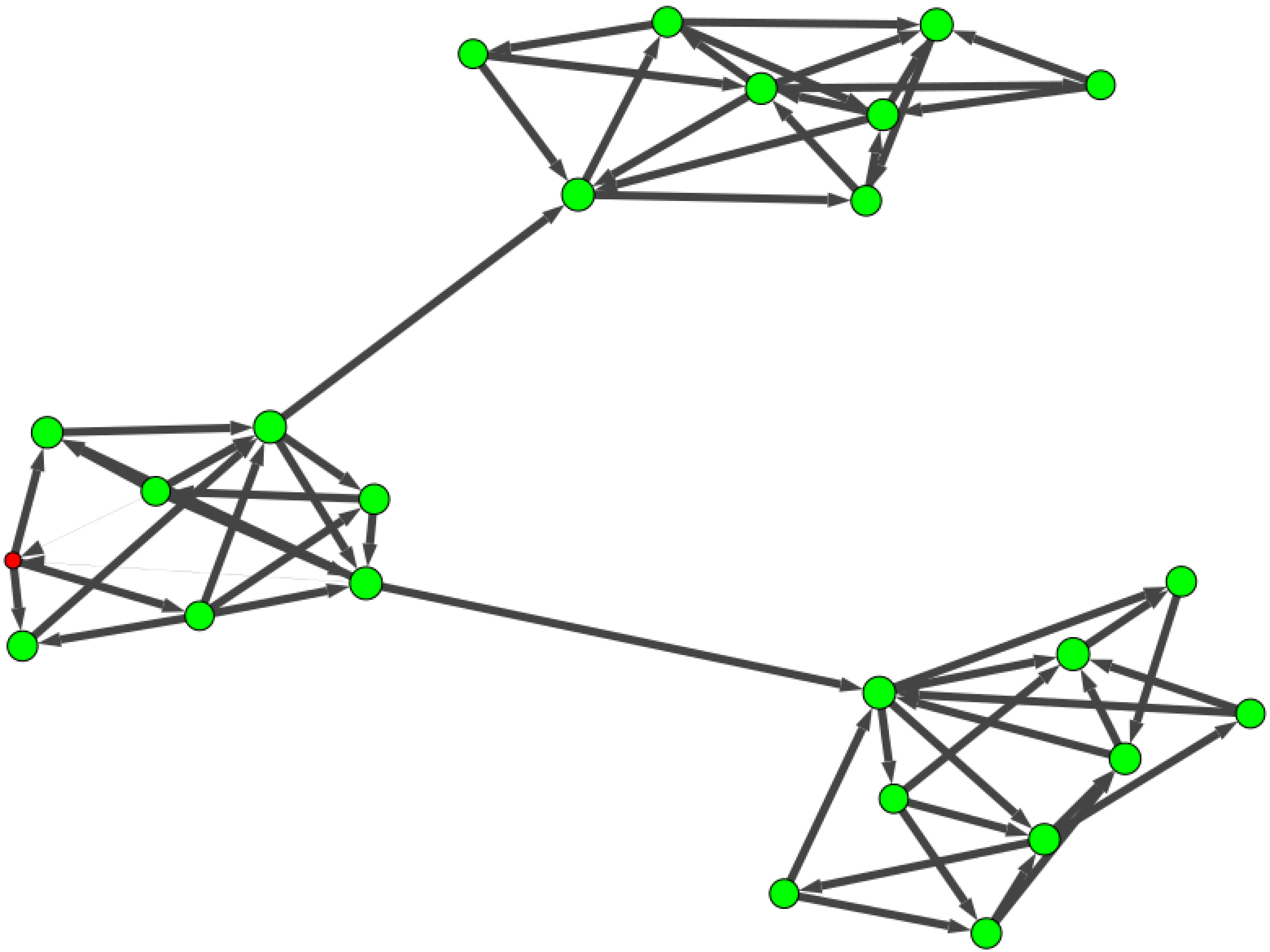
Model parameters (p, n, γ, β, E)

- c_i bank net worth $c_i = \gamma a_i$ (damping shocks)
- b_i inter-bank borrowing
- $a_i = e_i + i_i$

Algorithm 2: Algorithm of shocks and shock transmission

Let s_i be the size of initial shock
if $s_i > c_i$ **then**
  the bank i defaults
end if
if $s_i - c_i - b_i \leq 0$ **then**
  creditor banks lose $(s_i - c_i)$ distributed equally
else
  Depositor banks lose $s_i - c_i - b_i$
end if





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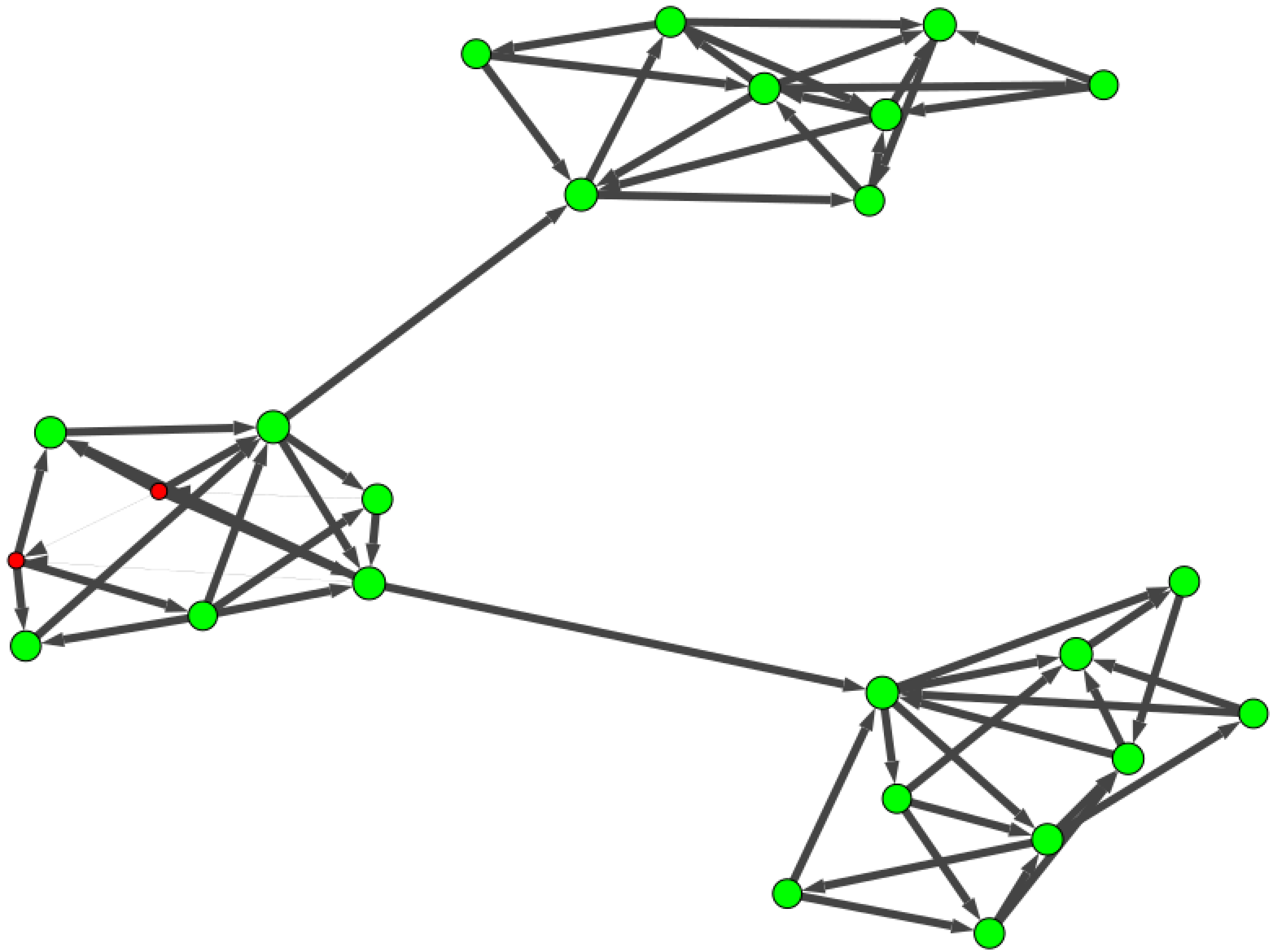

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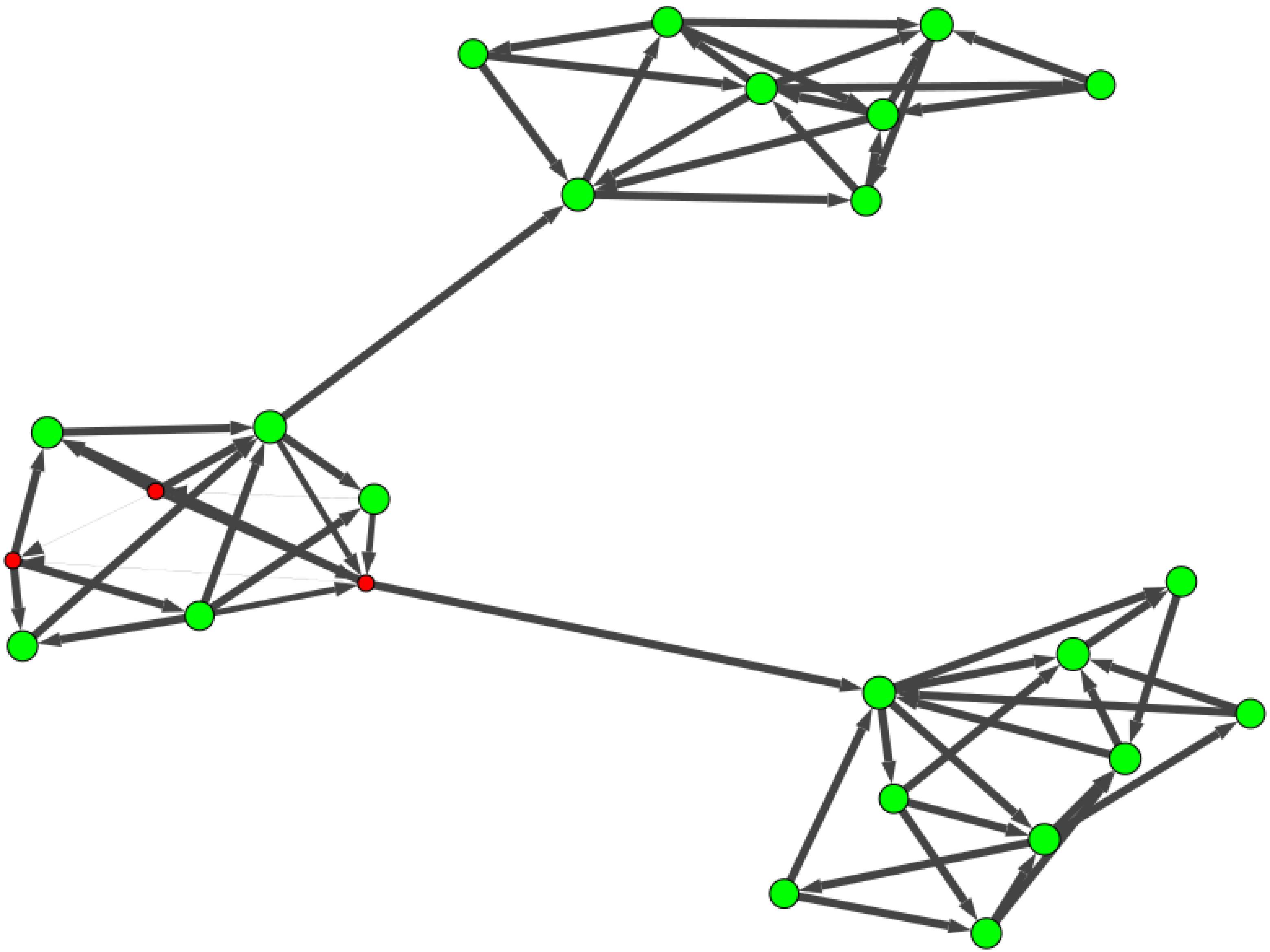

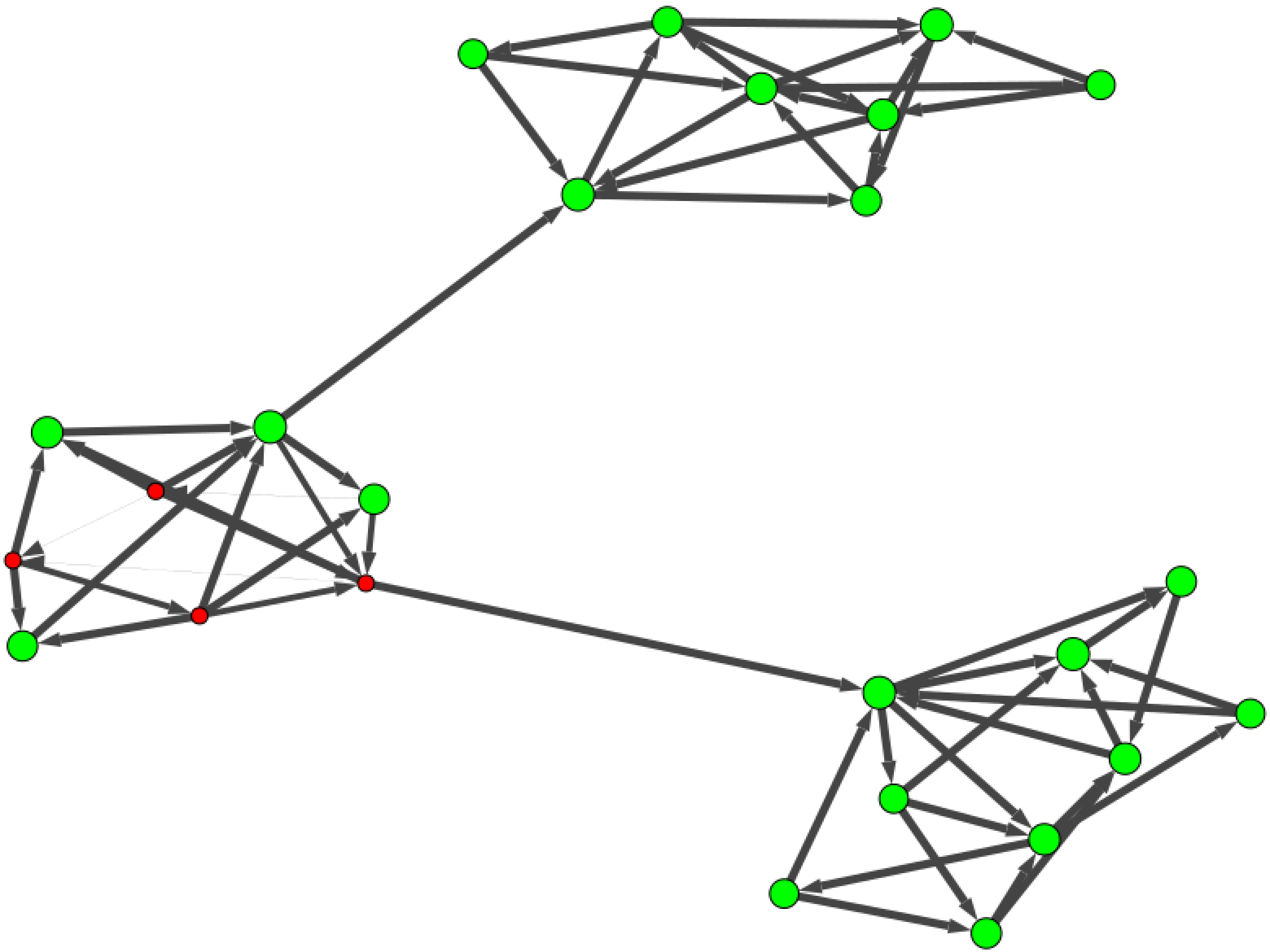

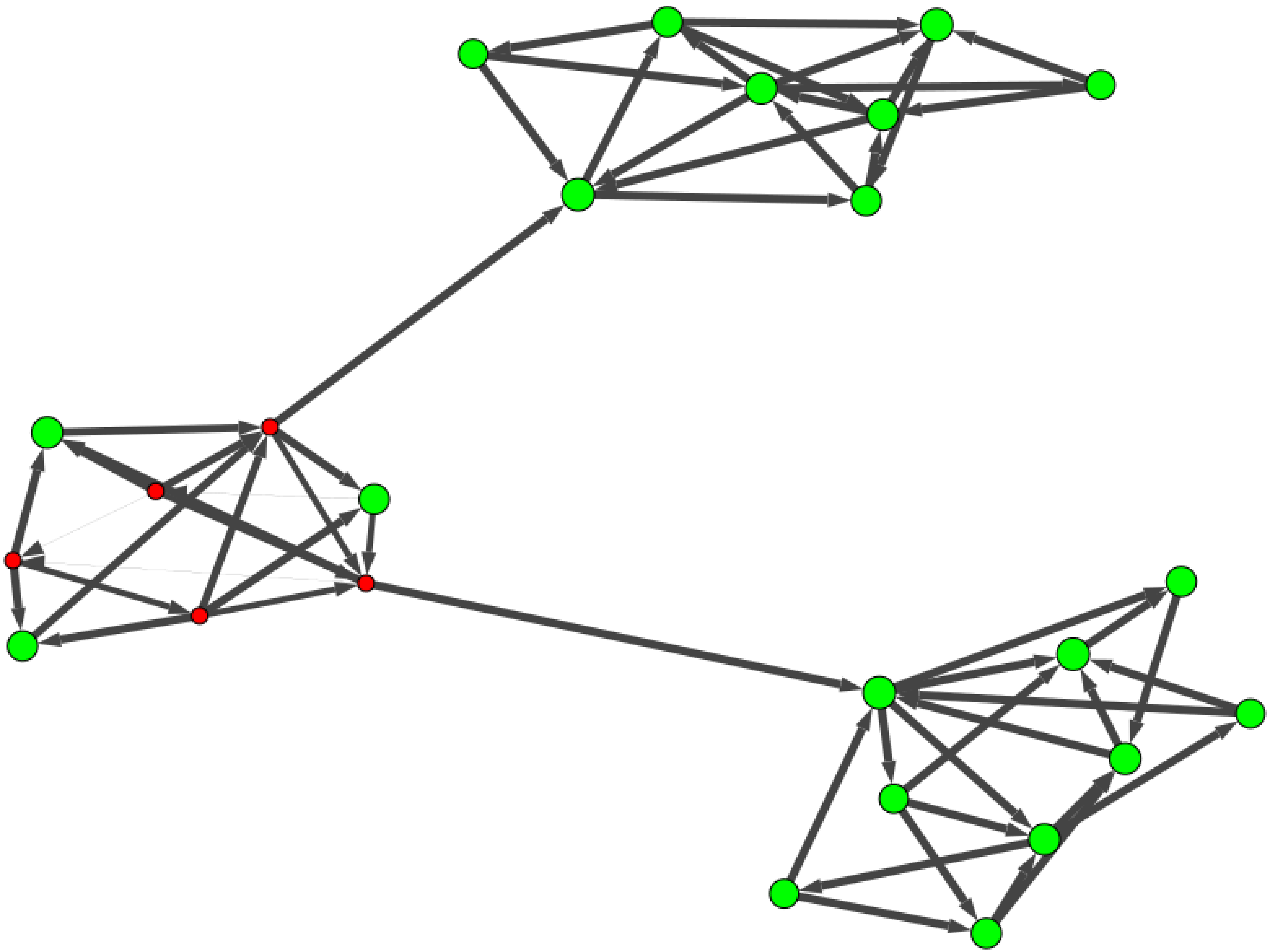

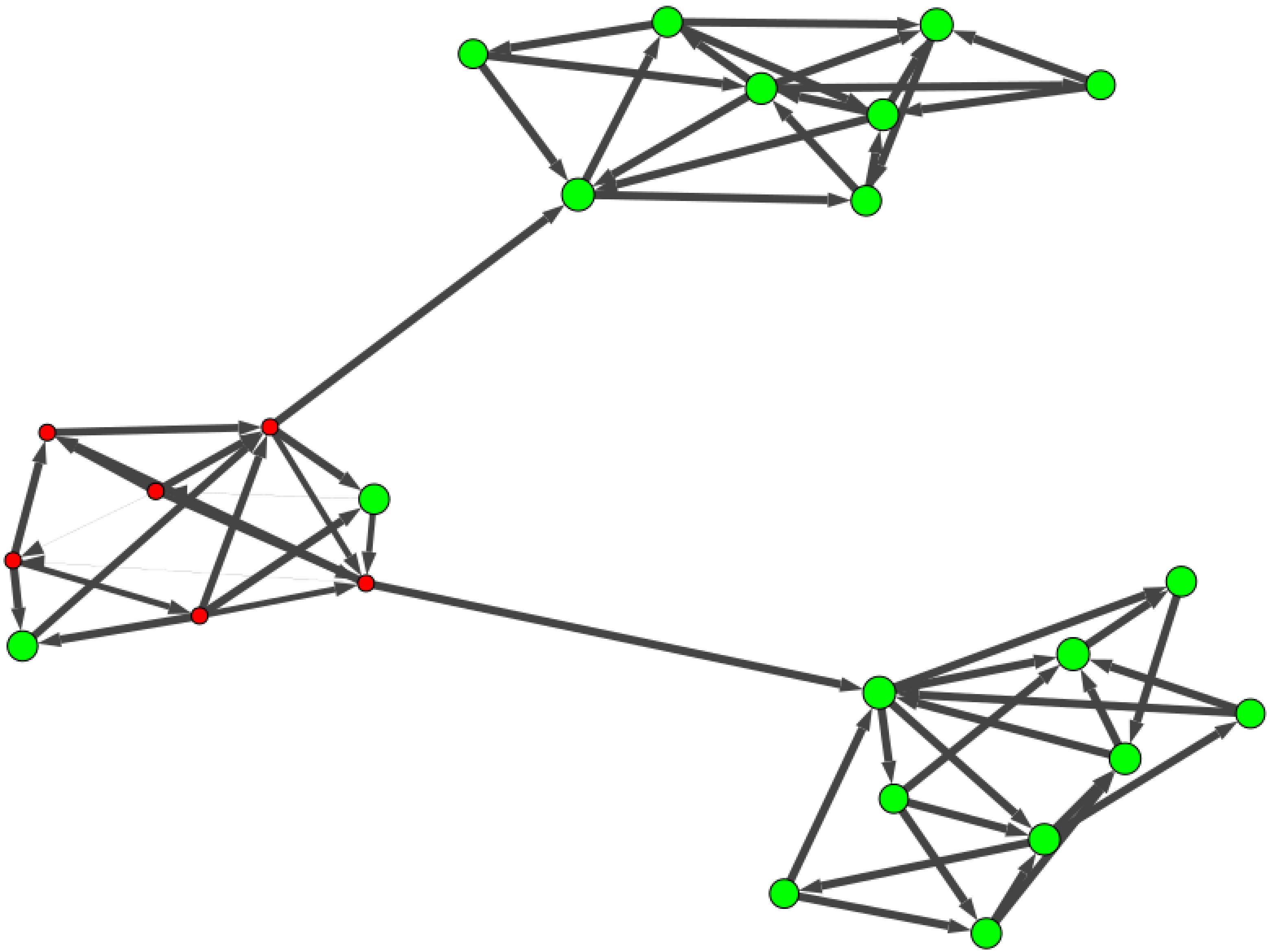

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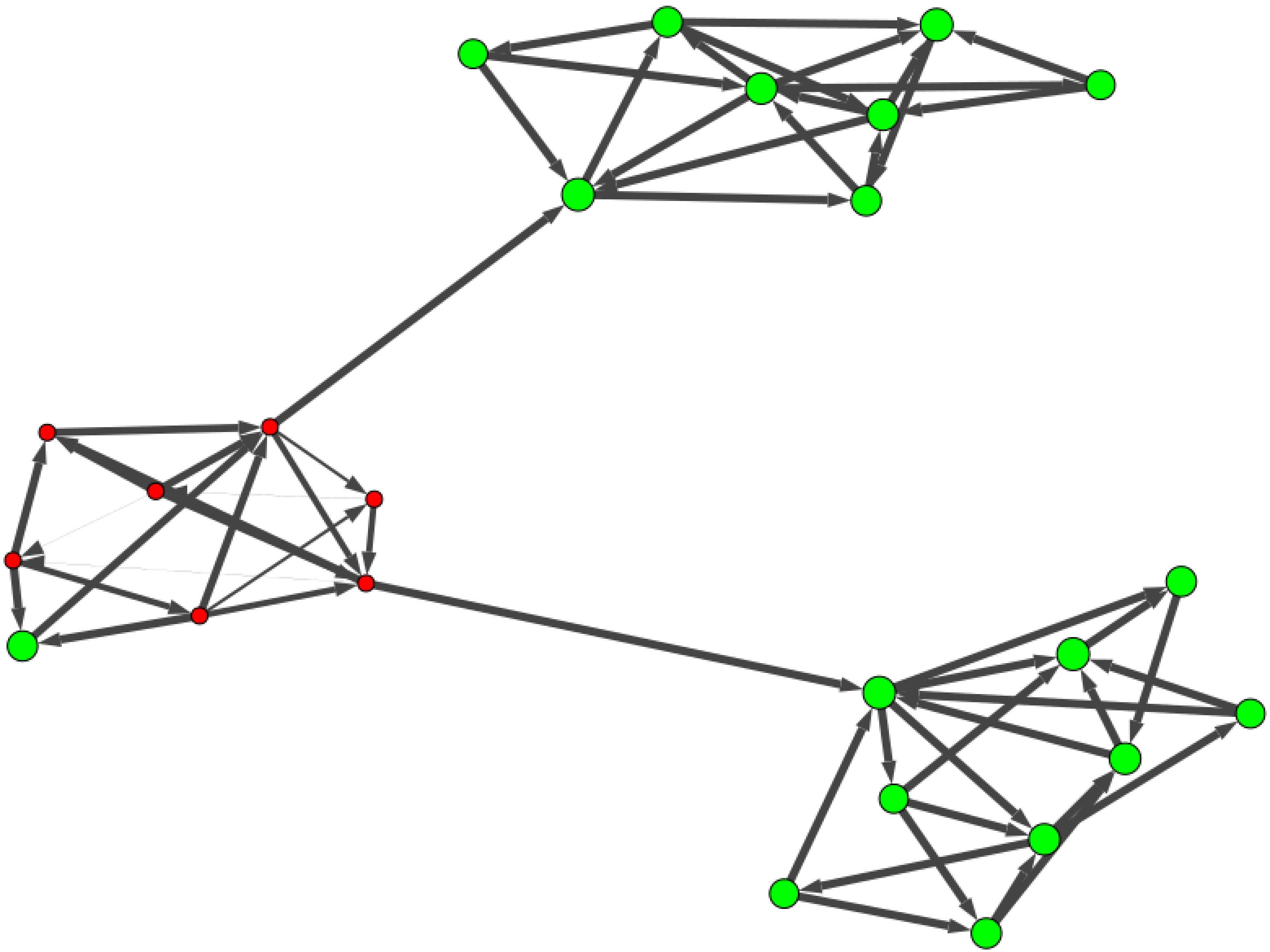
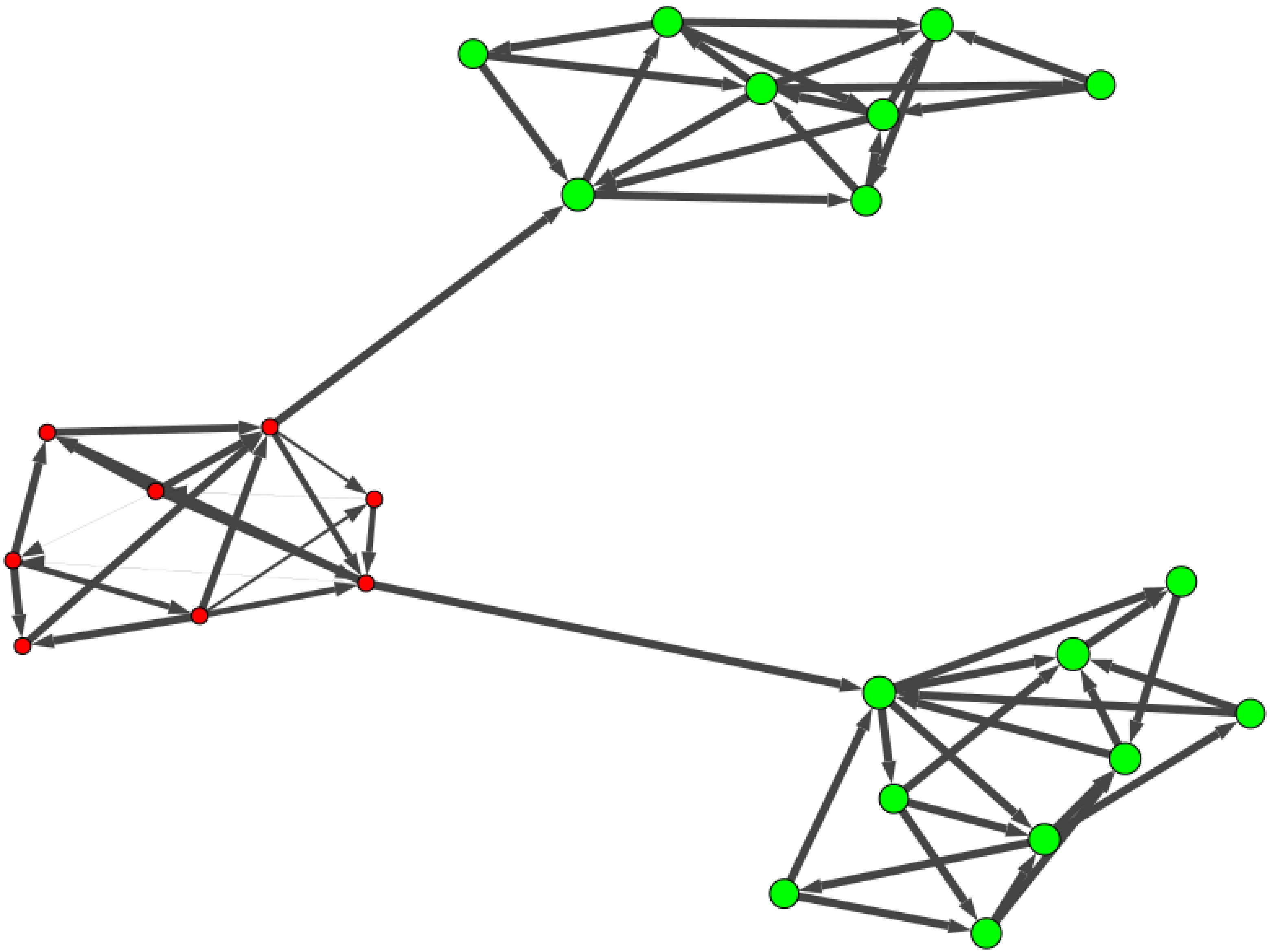
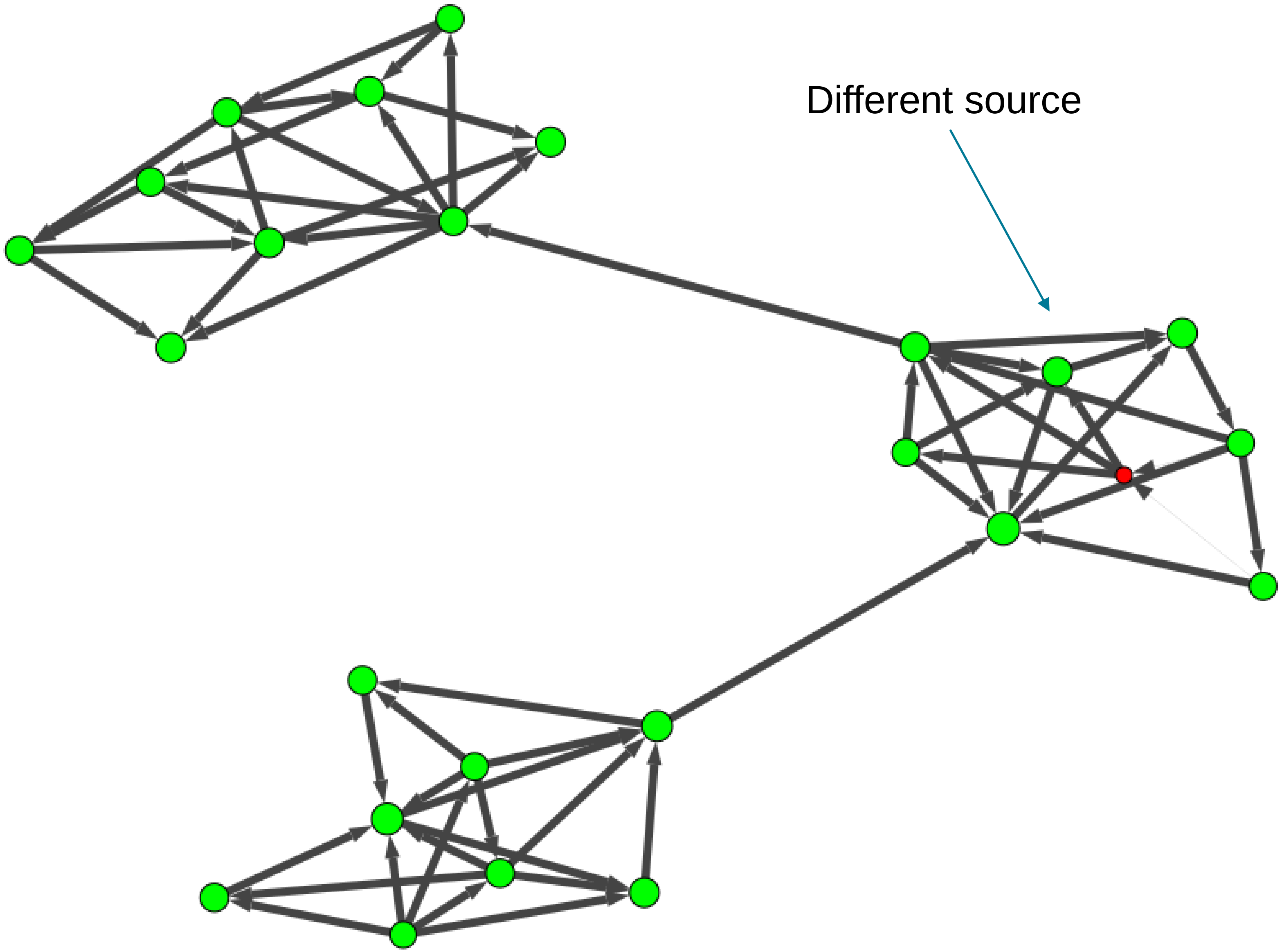
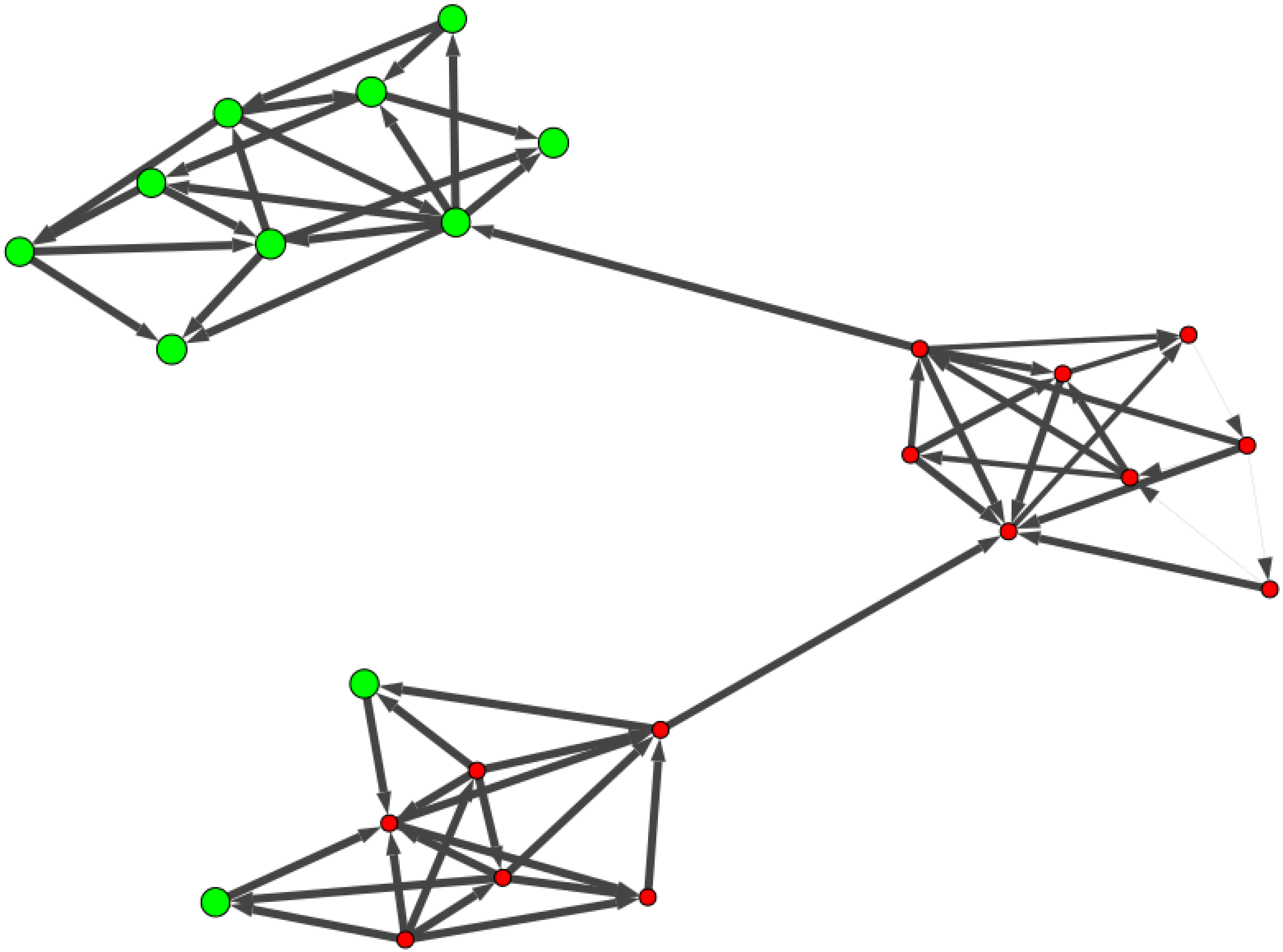


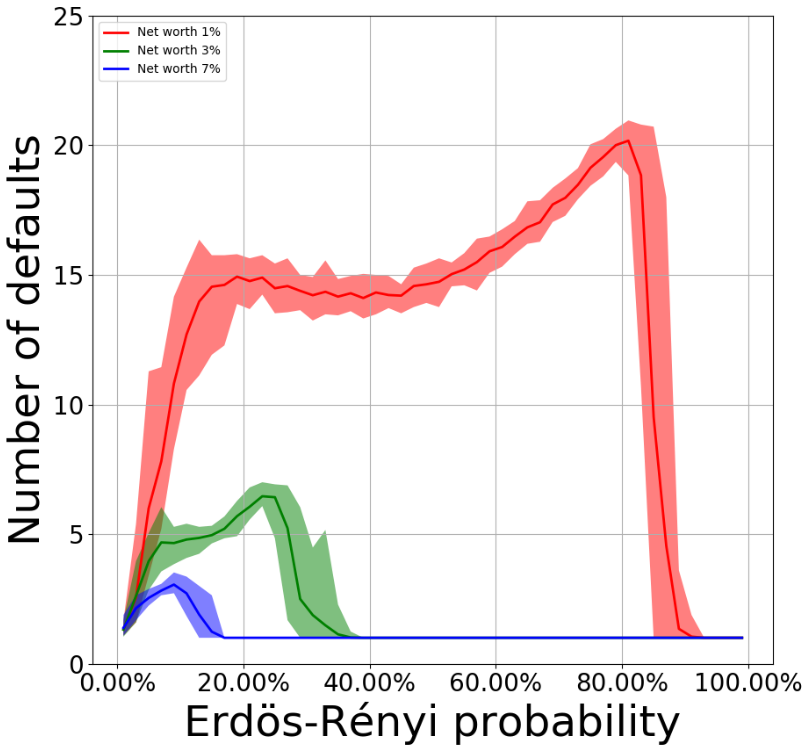



Different source

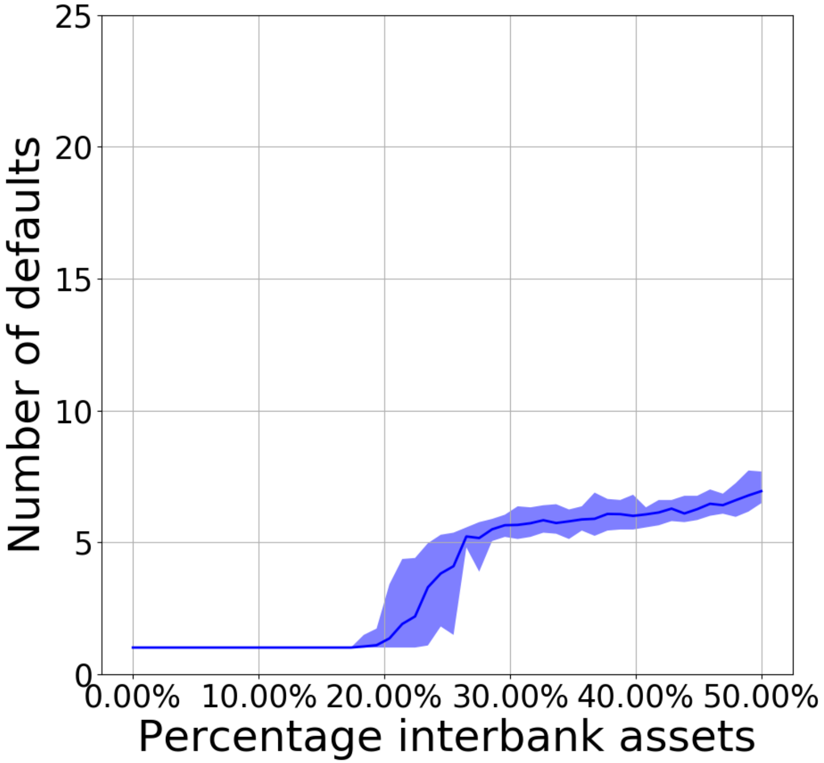


G(n, p) model results

- We scan parameters p and θ.



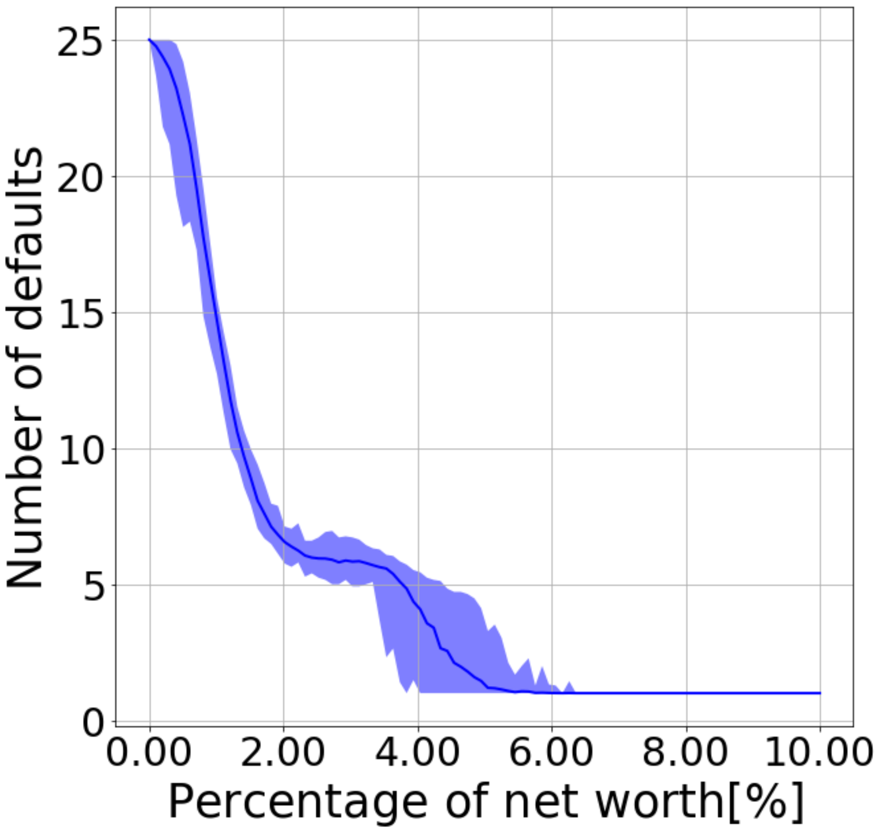
(a)



(b)

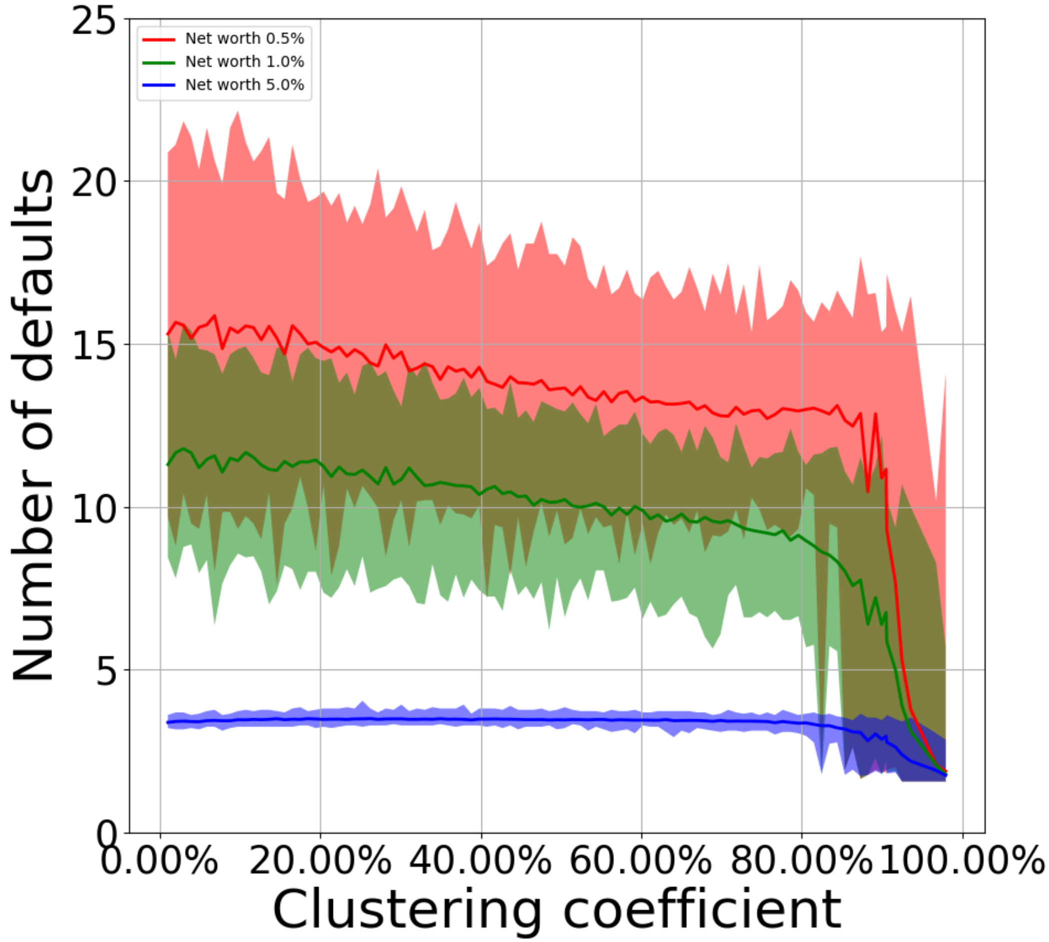
G(n, p) model results

- We scan the parameter γ (damping factor).

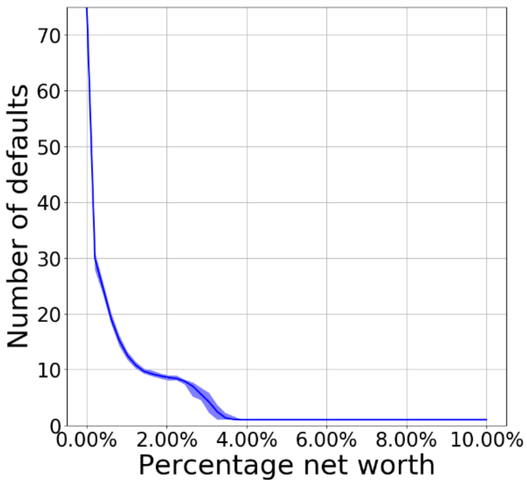


Recall : $\langle k \rangle = np = 25 \times 0.2 = 5.$

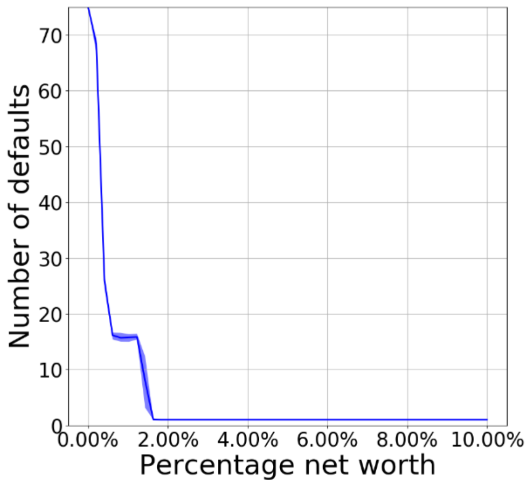
Impact of the clustering coefficient



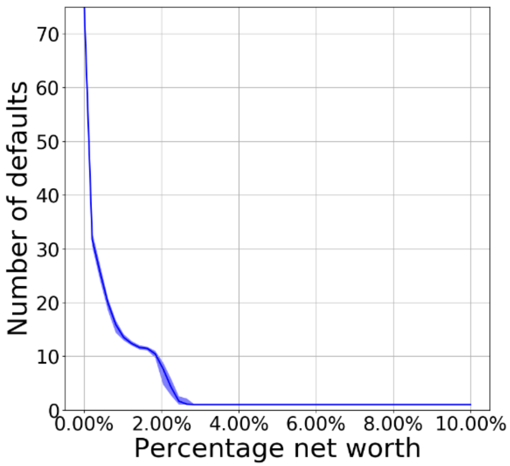
Results obtained from SBM



(a)
$SBM\ with\ M_1$



(b)
$G(n = 75, p = 0.2)$



(c)
$SBM\ with\ M_2$

$$M_1 = \begin{pmatrix} 0.4 & 0.1 & 0.1 \\ 0.1 & 0.4 & 0.1 \\ 0.1 & 0.1 & 0.4 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 0.65 & 0.1 & 0.1 \\ 0.1 & 0.65 & 0.1 \\ 0.1 & 0.1 & 0.65 \end{pmatrix}$$

Summary

- Implemented crisis simulation and network generation models
- Models applied to observe the impact of different topological parameters on banking default dynamics

- Propose regulations for the minimum percentage net worth
- Full graph topology matters in real world banking networks
- International banking networks require higher percentage net worth

Improvements

- Use our framework to generate networks and observe the impact of variating other properties
- Use different distributions of assets:
  - Observe the impact of distribution
  - Evaluate existing network to get the distribution
- Introduce the temporal characteristic of networks

Thank you for your attention!