Lecture with Computer Exercises:
Modelling and Simulating Social Systems

Project Report

The Impact of Network Topology on Banking Default Dynamics

Anas Bachiri & Kārlis Mārtiņš Briedis

Zürich
Dec 2018

ETH

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Declaration of originality

The signed declaration of originality is a component of every semester paper, Bachelor's thesis, Master's thesis and any other degree paper undertaken during the course of studies, including the respective electronic versions.

Lecturers may also require a declaration of originality for other written papers compiled for their courses.

I hereby confirm that I am the sole author of the written work here enclosed and that I have compiled it in my own words. Parts excepted are corrections of form and content by the supervisor.

Title of work (in block letters):

THE IMPACT OF NETWORK TOPOLOGY ON BANKING
DEFAULT DYNAMICS

Authored by (in block letters):
For papers written by groups the names of all authors are required.

Name(s):	**First name(s):**
Kārlis Mārtiņš	Briedis
Bachin	Anass

With my signature I confirm that
 - I have committed none of the forms of plagiarism described in the 'Citation etiquette' information sheet.
 - I have documented all methods, data and processes truthfully.
 - I have not manipulated any data.
 - I have mentioned all persons who were significant facilitators of the work.

I am aware that the work may be screened electronically for plagiarism.

Place, date	**Signature(s)**
ZÜRICH, 4/12/2018	

Agreement for free-download

We hereby agree to make our source code for this project freely available for download from the web pages of COSS. Furthermore, we assure that all source code is written by ourselves and is not violating any copyright restrictions.

Anas Bachiri   Karlis Briedis

Contents

1 Abstract

Systemic risk is of crucial importance when dealing with complex systems. In particular, financial networks are an important illustration of such systems. In this case, financial institutions interacts with each others respecting the graph structure of the underlying network.

Recent studies [1] have revealed the relevance of financial network structure in estimating systemic risk. It has also showed that the neglect of the graph topology is seriously underestimating the risk of default of the whole network.

In this project, we address the issue of crisis propagation in a banking network. We construct first a banking system with some parameters, distribute the assets on the banks and finally we simulate shocks in this financial network.

This work investigate the general behaviour of crisis propagation in a financial network, and therefore provides some insights about reducing the systemic risk. For this, we will scan the parameters of the used model and describe the behaviour of the resilience of financial networks.

2 Individual contributions

We collaborated equally on this project, and we had regular meetings were discussed scientific papers we had studied on this topic and further direction of the project. Kārlis Mārtiņš Briedis was responsible for coding and writing subsection "Complex netowrk structure simulation", and Anas Bachiri provided the general approach to tackle the addressed problem and was responsible of writing the flash-talk and the final report and adapted the solution to observe dynamics in networks generated by the stochastic block model.

3   Introduction and Motivations

Systemic risk is the risk of failure of the whole system, or of the majority of its components. In a financial situation, this is equivalent to the risk of collapse of the whole financial network and therefore the collapse of the economy.

One of the driving motivations of this work was the financial crisis 2008. The latter had dramatic consequences on the world's economy, it had affected also the GDP growth rates for several countries.
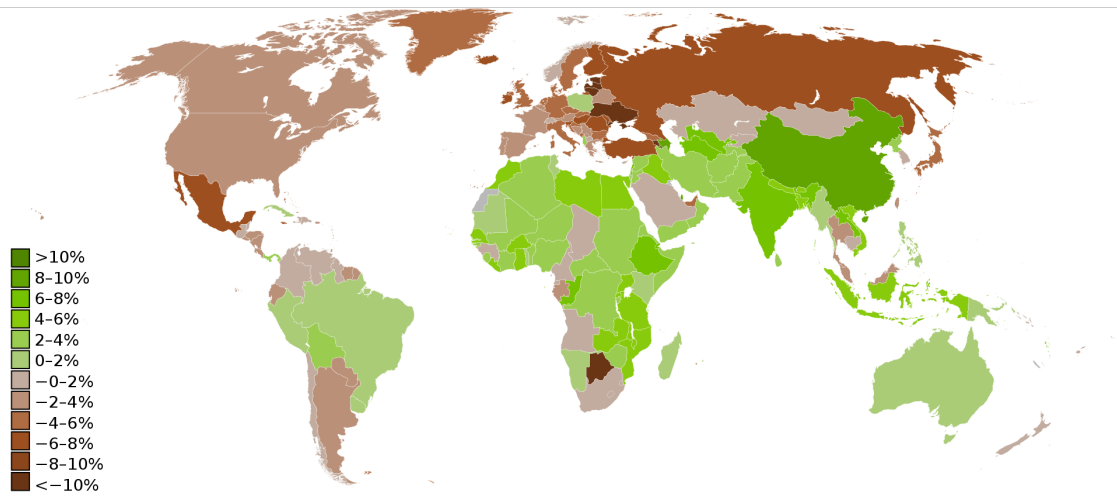

Figure 1: Gross domestic product (GDP) growth rates during 2009[2].

This work was also motivated by our eager to learn how banking networks could be modelled, how contagion in networks is spread. And finally, to give some insights about what can be done in term of regulations to reduce the systemic risk of the banking networks.

There are several mechanisms that can provoke systemic failure in a banking network. The first mechanism which will be focusing on is : financial relations or direct bilateral exposure between banks[3]. In fact, as we shall see later, having a small number of financial partners increases the risk of individual bank failure, "Since there are no partners to absorb the shock, the concerned bank will default".

Other causes of the systemic risk are the informational contagion and feedback effects.

In this work, We will also see the effects of level of capitalization of the banking network on the systemic risk and shock propagation.

4 Theoretical foundations

In this part we will shorty introduce relevant models and their theoretical construction, to establish a basis to the framework used to generate the banking system.

4.1 The $G(n, p)$ model

The $G(n, p)$ or the Erdös-Renyi model refers to a method of constructing random graph that is analytically tractable. The model was introduced by Paul Erdös and Alfred Renyi [4], and it has two parameters n and p.
The first parameter n specifies the number of nodes $|V|$ of the graph $G = (V, E)$, while the second parameter p refers to the rewiring probability, or the probability to link a couple of nodes (v_i, v_j). Since the probability of linking two nodes is uniform, we obtain the following binomial degree distribution : $P(k) = \binom{n-1}{k} p^k (1-p)^{n-1-k}$.

4.2 The Stochastic Block Model (SBM)

In this part, we discuss the generation of graphs that has a "community like" structure. Such a model could be viewed as a generalization of the $G(n, p)$ model.
The SBM requires a set of parameters : n is the number of nodes in the graph, a fixed number of C blocks or communities, a community assignment vector $\vec{z}$ of size n and values in $0, ..., C - 1$. The vector $\vec{z}$ gives to each node a community label. Finally, the last parameter required by SBM is the **stochastic block matrix** $M \in \mathbf{R}^{CxC}$ which gives link probabilities between a pair of nodes (v_i, v_j) with $z_{v_i} = k$ and $z_{v_j} = l$

4.3 Complex network structure simulation

Real-world complex networks are usually characterized by a set of different topological characteristics such as high clustering coefficient, short average path length, have specific communities etc. [5] but with mathematical models like Erdös-Renyi and SBM it is hard to meet such properties. To be able to generate complex networks with arbitrary set of such properties, we deploy evolutionary graph optimization as described in the paper [5]. For this approach specific objective functions are defined and simulated annealing (SA) algorithm is used to find the best matching graph. In each step a mutation is made by choosing multiple local or global graph modifications. It has been shown in [6], that SA can be used successfully for reconstruction of existing banking network, so it is a good choice for banking network generation.

N	Objective function f	Property the function is used for
1	$1 - \|\rho_{optimal} - \rho_{current}\|$	density, clustering coefficient, modularity
2	$1/(\|\rho_{optimal} - \rho_{current}\| + 1)$	average path length, number of communities, number of components

Table 1: Objective functions, where $\rho_{optimal}$ is target value and $\rho_{current}$ is value in the evaluated graph [5]

4.3.1 Objective functions

To generate a random network that approaches some target properties, a set of objective functions must be defined. For this project, we use 6 target functions, which are all calculated by using *igraph* functions:

- **Density** : existing edges over maximum possible edges

- **Clustering** : degree to which nodes tend to cluster together

- **Average path length** : represents the size of the "world"

- **Components** : number of independent components - we want this to always be 1 for regular graph

- **Communities and modularity** : How many distinct clusters are in network and how likely nodes are connected to nodes in the same cluster

For each of these functions an objective function is derived as described in table (1) to scale values in the range [0; 1], where value 1 means that the property is met.

4.3.2 Network mutation

In evolutionary algorithms in each iteration an mutation is applied to the current graph. In the paper [5] 5 types of network modifications are proposed:

1. Connection of random pair of nodes

2. Removal of random edge

3. Rewiring a pair of randomly selected edges

4. Connecting local nodes

5. Local rewiring a pair of random edges

In this project it is randomly selected if global (points 1, 2, 3) or local (2, 4, 5) modifications are applied and they are applied 3 times for each mutation, randomly selecting from the chosen pool.

4.3.3 Simulated annealing algorithm

The simulated annealing algorithm (1) is used to find the near optimal graph for which all objective functions are approaching 1. In [5] it was concluded, that for complex networks the most efficient staring point is empty graph.

Algorithm 1: Simulated annealing algorithm [6]

Let G be the initial graph
Let $\psi(G)$ be the weighted sum of objective functions for graph G
$T \leftarrow T_0$
$t \leftarrow 0$
while $T > T_{min}$ **do**
  $E_{cur} \leftarrow 1 - \psi(G)$
  Mutate G to obtain G'
  $E_{new} \leftarrow 1 - \psi(G')$
  $G \leftarrow G'$ with probability $Pr = [-\frac{1}{T} * max\{0; E_{new} - E_{cur}\}]$
  $T \leftarrow T(t)$
  $t \leftarrow t + 1$
end while

---

5 Description of the Model

We have adopted the framework introduced in [3] to perform this benchmark to study the network resilience to financial crisis. To begin with, we start with the construction of the banking network.

Due to the lack of data on the structure of such banking networks, we are going to adopt a probabilistic methods to generate such networks. There are several methods and models to generate networks, one important example is the Erdös-Renyi graph $G(n, p)$, which generates a network $G = (V, E)$ with n nodes : $|V| = n$, and the links are generated randomly between a pair of nodes with probability p. In our case, the links created are directed and the probability is uniform for both directions.

Considering that every node in fig.(2) is a bank, a directed edge represents the lending relationship between two banks.

Having built the network, we will now distribute the assets in this network. Such a task requires to fill the individual banks balance sheets consistently with the bank level and the aggregate balance sheet identities. We follow the same notation adopted in [3], and capital characters will denote variable at the aggregated level.
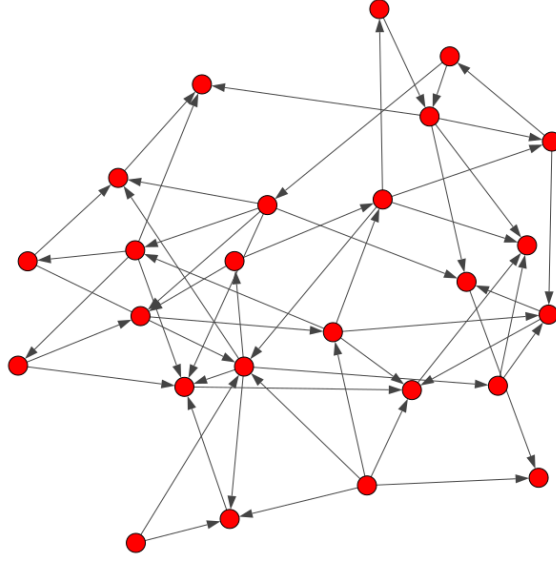
Figure 2: An example of a directed network constructed with the $G(n, p)$ model with $n = 25$ and $p = 0.2$.

Here we define the **parameters** used in the model :

- **A** : Total/aggregated assets of the banking system

- **E** : Total/aggregated external assets of the banking system

- **I** : Total/aggregated internal assets of the banking system

- **Z** : Total number of links $|E|$ in the banking network

- **β** : fraction of external assets $\beta = \frac{E}{A}$

- **θ** : percentage of inter-bank assets $\theta = 1 - \beta$

- **γ** : Net worth as percentage of total assets

Note that each bank is also characterized by a set of parameters : a_i denotes an individual bank assets, e_i denotes external assets borrowing and i_i denotes inter-bank assets. Thus we have : $a_i = e_i + i_i$. Let l_i, c_i, b_i and d_i denotes respectively the individual liabilities, net worth, inter-bank borrowing and finally the customer deposit. We mention also that the aggregated assets satisfies the following rule : $\boldsymbol{A} = \boldsymbol{E} + \boldsymbol{I}$.

In this part, we show how to assign numerical values to individual bank parameters a_i, l_i, c_i, b_i, e_i and d_i respecting the aggregate values of total internal **I** and external assets **E**. We will also how to model the shock spread.

Algorithm 2: Algorithm distributing assets according to the aggregated values	**Algorithm 3:** Algorithm of shocks and shock transmission

$w \leftarrow \frac{I}{Z}$
 for i $\in [1, n]$ **do**
 $\quad i_i \leftarrow w \sum_{j=1}^{n} A_{ij}$
 $\quad b_i \leftarrow w \sum_{j=1}^{n} A_{ji}$
 $\quad \tilde{e}_i \leftarrow max(b_i - i_i, 0)$
 end for
 for i $\in [1, n]$ **do**
 $\quad e_i \leftarrow$
 $\quad \tilde{e}_i + [(E - \sum_{l=1}^{n} \tilde{e}_i)/n]$
 $\quad a_i \leftarrow e_i + i_i$
 $\quad c_i \leftarrow \gamma a_i$
 $\quad d_i \leftarrow a_i - c_i - b_i$
 end for
	Let s_i be the size of initial shock
	 if $s_i > c_i$ **then**
	 $\quad$ the bank i defaults
	 end if
	 if $s_i - c_i - b_i \leq 0$ **then**
	 $\quad$ creditor banks lose
	 $\quad (s_i - c_i)$ distributed
	 $\quad$ equally
	 else
	 $\quad$ Depositor banks lose
	 $\quad s_i - c_i - b_i$
	 end if

Note that for the model, there are only five independent parameters which are scanned. In our case, these parameters are : $(\gamma, \theta, p, N, E)$. The first algorithm (2) shows how to assign individual bank values respecting the aggregated amount of internal and external assets, while the second algorithm (3) is for shock simulation.

6   Implementation

This program was implemented in python, and we used libraries such as : *iGraph* library to generate Erdös-Renyi networks $G(n, p)$, the *Matplot* library to generate plots and figures. And other python libraries were used such as *Numpy* to manipulate vectors and matrices. It is also possible to find our code in a GitHub repository [7]. We have first investigated the behaviour obtained in the paper [3]. For this, we have used the following benchmark values for the model parameters :

$$(\gamma, \theta, p, N, E) = ([0, 0.1], 0.2, 0.2, 25, 100\,000)$$

The numerical value of γ is scanned from 0 to 0.1, while the other parameters are set in fix values.
We start by analyzing the resilience of the banking system and how is it affected by the net worth γ. Then we will study the effects of connectivity on system's resilience, and this is done by changing the probability p of the $G(n, p)$ model. Other questions may be also addressed concerning the network robustness, for example how the latter changes with the percentage of internal assets.

Other parts of the implementation will be performing simulations on different network structures. The most important structures will be "The community like structure in networks". This is analyzed to understand how does crisis propagates in an international financial network scenario. In this case, we can imagine that every "community"/cluster of nodes form the set of banks in some country, where they tend to connect heavily with each others, and have also few other connection with different other clusters "Banks in different countries". We will be studying the same resilience properties in this international network situation.

7 Simulation Results and Discussion

7.1 Percentage net worth and contagion

The first simulation we perform is to count the number of defaults for every value of net worth percentage γ scanned. This is an evaluation method for the network
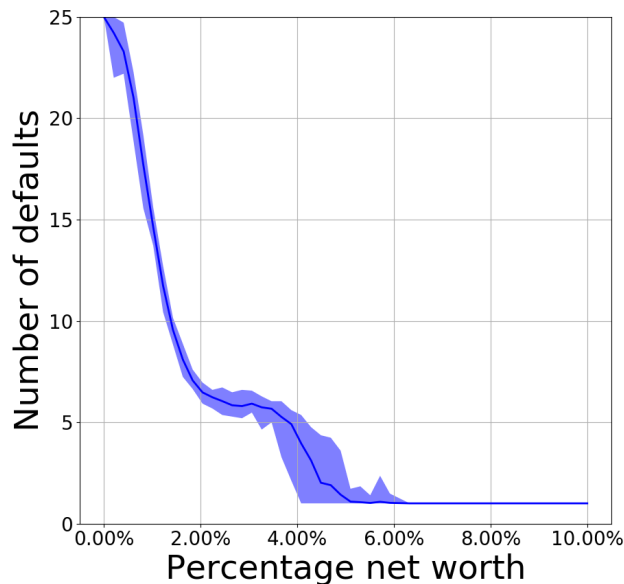

Figure 3: The effect of percentage net worth γ on the number of defaults.

resilience and how it is affected by the parameter γ. The results in figure (3) show that the number of defaults in the banking network decreases with the percentage of net-worth. In fact, as we mentioned previously, the net worth c_i serves as a damping factor to the "financial shock". The biggest the net worth, the more robust the bank

is. Note also the three regimes obtained in the figure (3):

First, there is a fast decreases in the number of defaults for γ in the interval $[0,2]\%$. Second, a constant regime of the number of defaults for $\gamma \in [2,3.5]$, and in this part the number of defaults remains at 5 defaulted banks. Finally, there is another decrease for $\gamma \in [3.5,6]$ and after that the the number of defaults remains the same after $\gamma = 6\%$. The first fast decrease of defaulting banks could be explained by the increase of c_i, this prevent the shock to propagate in the whole network. While the constant regime could be explained by the fact that the shock propagates only in the first neighbours of the initial shocked bank. Note that the defaulting banks stays constant around 6, this value in fact could be obtained if we approximate the neighborhood of a node by the mean degree $\langle k \rangle = np$ for the $G(n,p)$ model. In our case $n = 25$, $p = 0.2$ leads to $\langle k \rangle = 5$ plus the initial shocked bank, which explains the result (3) obtained.

7.2   Connectivity and contagion

Another analysis done concerning the effects of connectivity on network resilience. As we have mentioned before, the connectivity has two different effects. A high connectivity means that the shock can be easily absorbed by the network, since the loss of the shocked bank will be shared by its neighbours. But at the same time, high connectivity means that the shock could propagate easily in the network.

The figure (4) is obtained by scanning the connectivity parameters p of the $G(n,p)$ model, and performing the shocks. This provides us with an idea about how connectivity and net-worth affect the spread of crisis.

As we can see in figure (4), the banks resilience is very sensitive to the parameter γ. For very low values of the net-worth percentage, the number defaulting banks is significantly high. Since there is no enough net-worth to absorb the shock, the latter is easily transmitted in the network. Note also that the graphs in (4) have a similar shape, the contagion spread is remarkably fast at the beginning, after that it remains constant, increase after that to finally decrease. This underlines the fact that there is no "general" rule we can follow, but what we can learn about this is the following : In case of a relatively high value of the net-worth percentage $\gamma \geq 7\%$, a bank should be connected to 20% of the other banks in the network to avoid systemic failure. For the case of very low net-worth percentage, banks should not be very connected to each other.

7.3   Percentage inter-bank assets and contagion

Other aspects of the network resilience were investigated, such as the effects of the percentage inter-bank assets on the number of defaults.
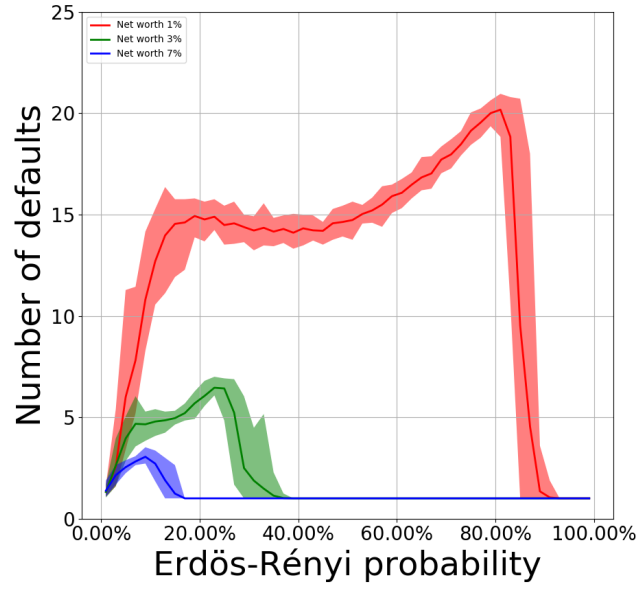
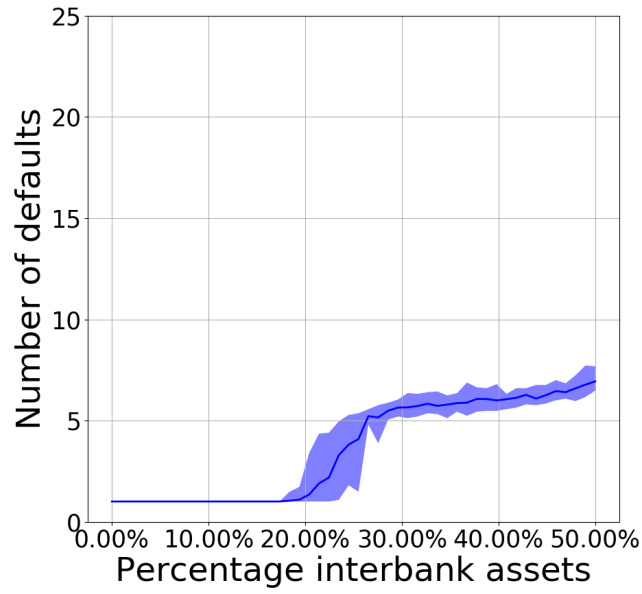Figure 4: The effect of connectivity on the number of defaults for different net-worth values γ.

Figure 5: The effect of percentage inter-bank assets θ on the number of defaults.

10

By maintaining the size of the external assets $\mathbf{E}$ constant, and by increasing the fraction of inter-bank assets. Intuitively, this means an increase in the size of lending in the network. Since we know from the previous sections that the inter-bank borrowing b_i is proportional to $w = \frac{I}{Z}$, and Z being the total number of connections in the network. We mention also that there are two opposite driving forces of the inter-bank assets percentage on the system's resilience. On the one hand, the bigger the size of lending in the network, the easier the transmission of an initial shock. On the other hand, increasing the fraction of inter-bank assets, means an increase in the total net worth, and therefore the capacity of the network to absorb shocks.
Effectively as we can see in figure (5), there are no contagion effects for small values of percentage inter-bank assets, this is due to the absorption of "shocks" by customer deposits d_i. Besides, the loss level is also very so it is easily absorbed by banks net worth.

7.4   Clustering coefficient and network resilience

In this section we are going to study how network resilience is effected by the clustering coefficient of the network.
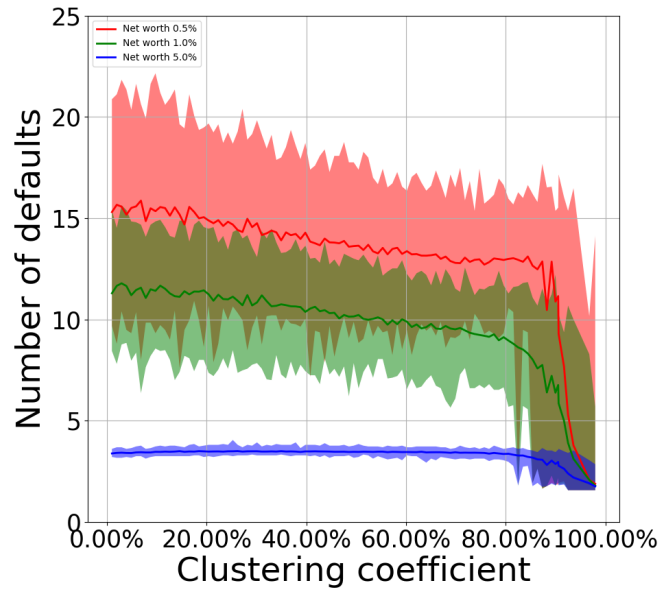

Figure 6: The effect of the clustering coefficient on the number of defaults.

In this part, we use complex network structure simulation discussed above to

generate random graphs respecting an average clustering coefficient.

The results obtained in figure (6) have an extremely noisy character for very low values of the net worth. This property could be explained by the fact that for low values of γ, the number of defaults depends a lot on the graphical topology. In such a regime, network effects becomes extremely important, and this suggests that in the low damping case, it is quiet challenging to estimate the number of defaults without considering the full network topology. Note also that the mean value has a decreasing trend, since the clustering in linked to connectivity. For high values of connectivity, we recover the same behaviour obtained for $G(n, p)$ graphs in figure (4).

7.5   International banking network

After reproducing the results obtained in [3], and obtaining the same results. In this section, we will extend the study to a different class of graphs.

We consider a scenario where the banks belong to different countries. In this case, each country has a specific number of banks, and banks within the same country are highly connected to each others compared to banks of different countries. By this, we model an international banking network, where lots of banks of a country are connected to each other, and only few of them have connections with "foreign" banks.
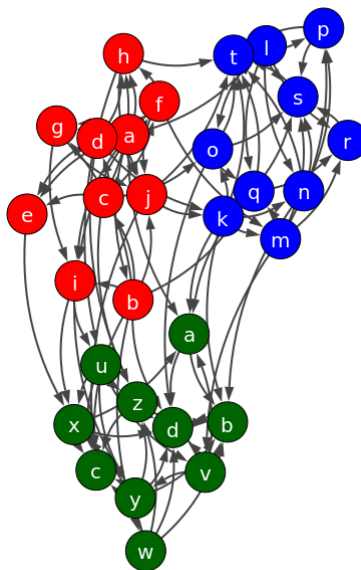

Figure 7: Illustrative example of a 3-communities network of 30 Nodes.

These kind of networks are generated by the **Stochastic Block Model** discussed

before. An illustrative example of such a network is given in figure (7).

The community-like graphs used in the simulations have the following parameters : $n = 75$ nodes, number of communities $C = 3$ and finally we have used two stochastic block matrices M_1 and M_2 :

$$M_1 = \begin{pmatrix} 0.4 & 0.1 & 0.1 \\ 0.1 & 0.4 & 0.1 \\ 0.1 & 0.1 & 0.4 \end{pmatrix} \qquad M_2 = \begin{pmatrix} 0.65 & 0.1 & 0.1 \\ 0.1 & 0.65 & 0.1 \\ 0.1 & 0.1 & 0.65 \end{pmatrix}$$

Note that the matrices $M_{1,2}$ are symmetric and their diagonal terms are higher than the non-diagonal ones. This translates to a case where each country establishes financial relationships with the other countries without any preference. Besides, the diagonal terms refers to the probability to create links within the same "country". The non-diagonal terms are the probabilities to connect to banks in a different country. We have also performed a simulation using the $G(n, p)$ model, using $n = 75$ and
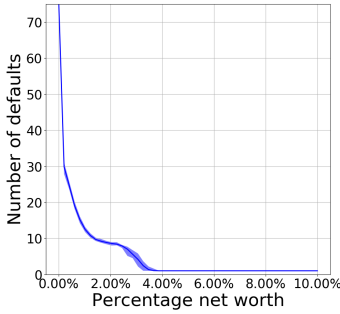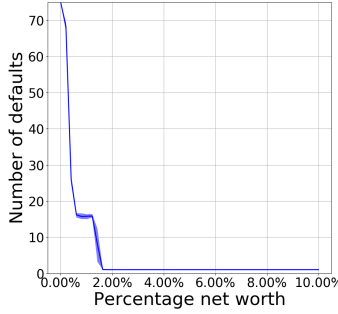

Figure 8: Simulation with SBN with M_1
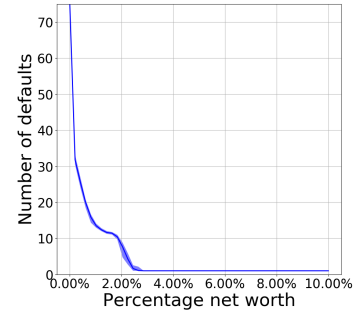
Figure 9: Simulation with $G(n, p)$ model

Figure 10: Simulation with SBN with M_2

$p = 0.2$, and use this simulation as a ground truth to compare the difference between the "National and international" cases.

The results of simulations in figures (8,9,10) shows some interesting behaviours. Note that for the first simulation using the stochastic block matrix M_1, with diagonal probability $p = 0.4$, we observe that the number of default requires a percentage net worth γ of 4% to attain its minimal value. This value is about twice of the value required in a "national" or single country situation : In the figure (9), the number of defaults is minimal already at percentage γ of 2%. We stress also that in the $G(n, p)$ case, the "constant" regime of the function is close to 16 defaulting banks. This number could be explained as before by the mean degree of the $G(n, p)$ model, which is in our case $\langle k \rangle = 15$.

For the last figure (10), we see that the number of defaults reaches the minimum

13

slightly above $\gamma = 2\%$, we also see that the curve is broader than result obtained using M_1. In fact, the simulation obtained using M_2 could be viewed as a middle case between the previous results of $G(n, p)$ model and SBM using M_1.

In the single country situation, the number of defaults decreases very fast and the minimum percentage net worth required in such a network is less than the one required in an international situation. If banks withing the same country are not highly connected -case of M_1-, the defaulting number curve is broadened, and banks need to have higher net worth percentage to dampen shock effects. For higher connected banks -case of M_2-, the curve is less broad, but still a higher percentage net worth is required compared to the single country situation.

8 Summary and Outlook

This work addressed several crucial points on financial networks and crisis. First, it showed the importance of network structures and network effects. We are also able to successfully reproduce the work done in [3], and generalize it to other graph topologies, corresponding to both random and real world networks.

Despite data scarce, we managed to adopt a general investigative approach to understand the effect of network topology on system's resilience. And in the case of banking networks, we have showed that the network robustness is sensitive to the net worth percentage γ. The results suggested also that γ should be at least 5% to reduce systemic risk.

Other important outcomes of this work was to show that there is no general recipe to reduce systemic risk using only connectivity, or if we ignore the net worth value γ. In fact, network connectivity can at the same time foster and damp financial shocks. The analysis of the percentage inter-bank assets showed that for values of θ starting from 20%, could lead to a higher number of defaulting banks. And therefore, banks should have at least 80$ of their assets outside the network.

In the international bank network scenario, we have seen that connecting to banks from different countries could increase the systemic risk. In this situation, the network is more resistant to shocks if banks within the same country are highly connected. This is also equivalent to say that if a bank want to establish international relationships, its percentage net worth γ should be high. This goes in hand with the fact that only "important" banks which have international relationships.

Conserning the effects of average clustering coefficient, the obtained results had high variance, a characteristic property in real world networks. This actually can show the limiting part of using random models can have, real world networks can introduce noise in the observed quantities, and make predictions hard to make. This part also allowed us to check for a second time the sensitivity of the network resilience to the

14

net worth percentage.

Finally, this work was very important to understand how to model banks and financial shocks, there are also several points in which we could extend this work. One of them is to investigate the behaviour of the stochastic block model but for an asymmetric situation. A scenario where countries that are "close to each other" tend to have more connections than other countries, it is also possible to check the behaviour by scanning the entries of the stochastic block matrix by constraining graphs to other real world properties, and finally by considering other important aspects like temporal interactions in the graph.

References

[1] Rahul Kaushik. Amadeo A. Stefano Battiston, Michelangelo Puliga. Debtrank: Too central to fail ? financial networks, the fed and systemic risk. *Scientific Reports*, 2012.

[2] Wikipedia, the free encyclopedia. Financial crisis of 2008-2009, 2013. https://en.wikipedia.org/wiki/Financial_crisis_of_2007%E2%80%932008.

[3] Tanju Y. Amadeo A. Erlend Nier, Jing Yang. Network models and financial stability. *Journal of Economic Dynamics & Control*, 2007.

[4] Paul Erdos & Alfred Renyi. On random graphs i. 1959 (last checked 2018). snap.stanford.edu/class/cs224w-readings/erdos59random.pdf.

[5] Victor V. Kashirin. Evolutionary simulation of complex networks structures with specific topological properties. *Procedia Computer Science*, 29:2401–2411, 12 2014.

[6] Valentina Y. Guleva, Vyacheslav V. Povazhnyuk, Klavdiya O. Bochenina, and Alexander V. Boukhanovsky. Russian interbank network reconstruction via meta-heuristic algorithm. *Procedia Computer Science*, 108:1318 – 1326, 2017. International Conference on Computational Science, ICCS 2017, 12-14 June 2017, Zurich, Switzerland.

[7] Anas Bachiri & Karlis Briedis. Contagion in financial networks github repository, 2018. https://github.com/kmbriedis/msss_project.