CPSC 5031 : Data Structures & Algorithms

Lecture 3: Brute Force

(Levitin, Chapter 3.1-3.4)

Brute Force

A straightforward approach, usually based directly on the problem's statement and definitions of the concepts involved

Examples:

1. Computing a^n ($a > 0$, n a nonnegative integer)

2. Computing $n!$

3. Multiplying two matrices

4. Searching for a key of a given value in a list

Outline

- Sorting
  - Selection sort
  - Bubble sort
- String matching
- Closest-pair problem
- Exhaustive search to combinatorial problems
  - Travelling salesman problem (TSP)
  - Knapsack problem
  - Job assignment problem

Brute Force Sorting Algorithm

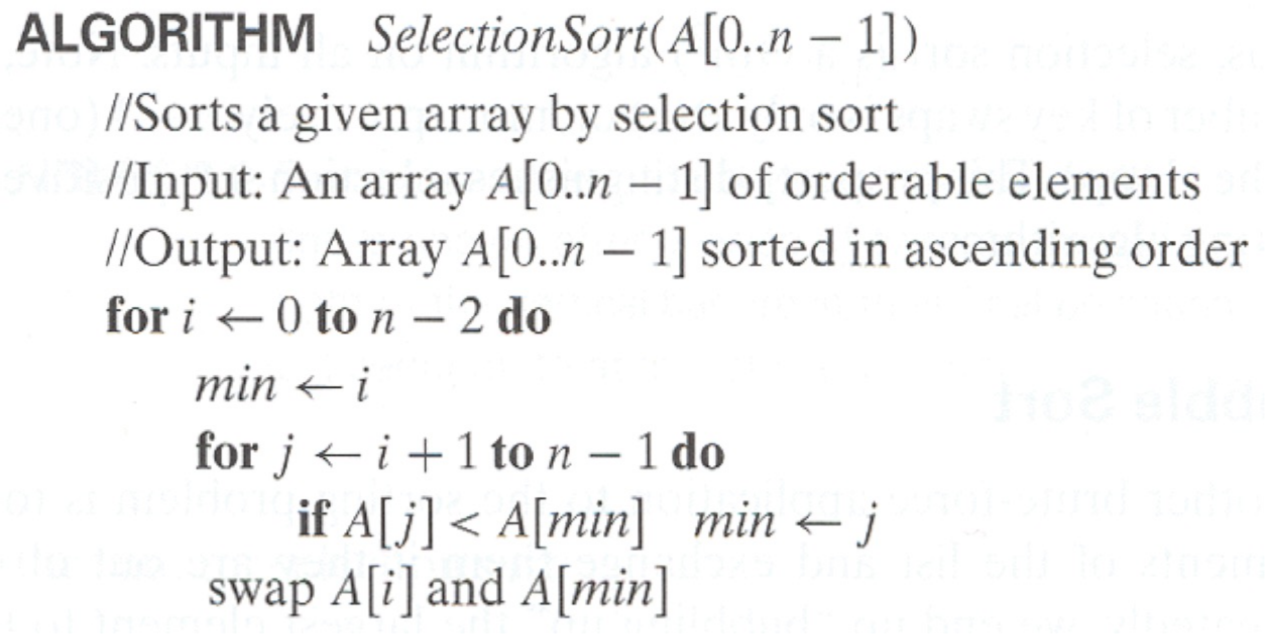
Selection Sort Scan the array to find its smallest element and swap it with the first element. Then, starting with the second element, scan the elements to the right of it to find the smallest among them and swap it with the second elements. Generally, on pass i ($0 \le i \le n$-2), find the smallest element in $A[i..n$-1] and swap it with $A[i]$:

$$\begin{tikzcd}[column sep=small] A[0] \le \ . \ . \ . \ \le A[i-1] \ \mid & A[i], \arrow[rr, leftrightarrow, bend right] & . \ . \ . \ , & A[min], & . \ . \ ., A[n-1] \end{tikzcd}$$

in their final positions

Example: 7 3 2 5

Analysis of Selection Sort



```
ALGORITHM  SelectionSort(A[0..n − 1])
    //Sorts a given array by selection sort
    //Input: An array A[0..n − 1] of orderable elements
    //Output: Array A[0..n − 1] sorted in ascending order
    for i ← 0 to n − 2 do
        min ← i
        for j ← i + 1 to n − 1 do
            if A[j] < A[min]   min ← j
        swap A[i] and A[min]
```

T(n) = ?

Bubble Sort

- Exchange sort, makes *n*-1 scans over the list. On each pass, swap two neighbor data items only if they are out of order. Each pass *i* will bubbling up the *(i+1)*-th largest item to the right position. Pass *i* ($0 \le i \le n$-2) can be represented as follows:

$$A[0], \ . \ . \ . \ , A[j] \overset{\leftrightarrow}{\quad} \mathrm{A}[j+1], \ . \ . \ ., A[n-i-1] \mid \underbrace{A[n-i] \le \ . \ . \ . \ . \le A[n-1]}_{\text{in their final positions}}$$

Bubble Sort

```
Algorithm BubbleSort(A[0..n-1])
    for $i \leftarrow 0$ to n-2 do
        for $j \leftarrow 0$ to $n$-2 do //or n-2-i, why ?
            if $A[j+1] < A[j]$ swap $A[j]$ and $A[j+1]$
```

$T(n) = ?$

Brute Force String Matching

- ***pattern***: a string of *m* characters to search for
- ***text***: a (longer) string of *n* characters to search in
- ***problem***: find a substring in the text that matches the pattern
- Examples:

1. Pattern: `001011`
   Text: `10010101101001100101111010`

2. Pattern: `happy`
   Text: `It is never too late to have a happy childhood.`

Brute Force String Matching

<u>Brute force algorithm</u>

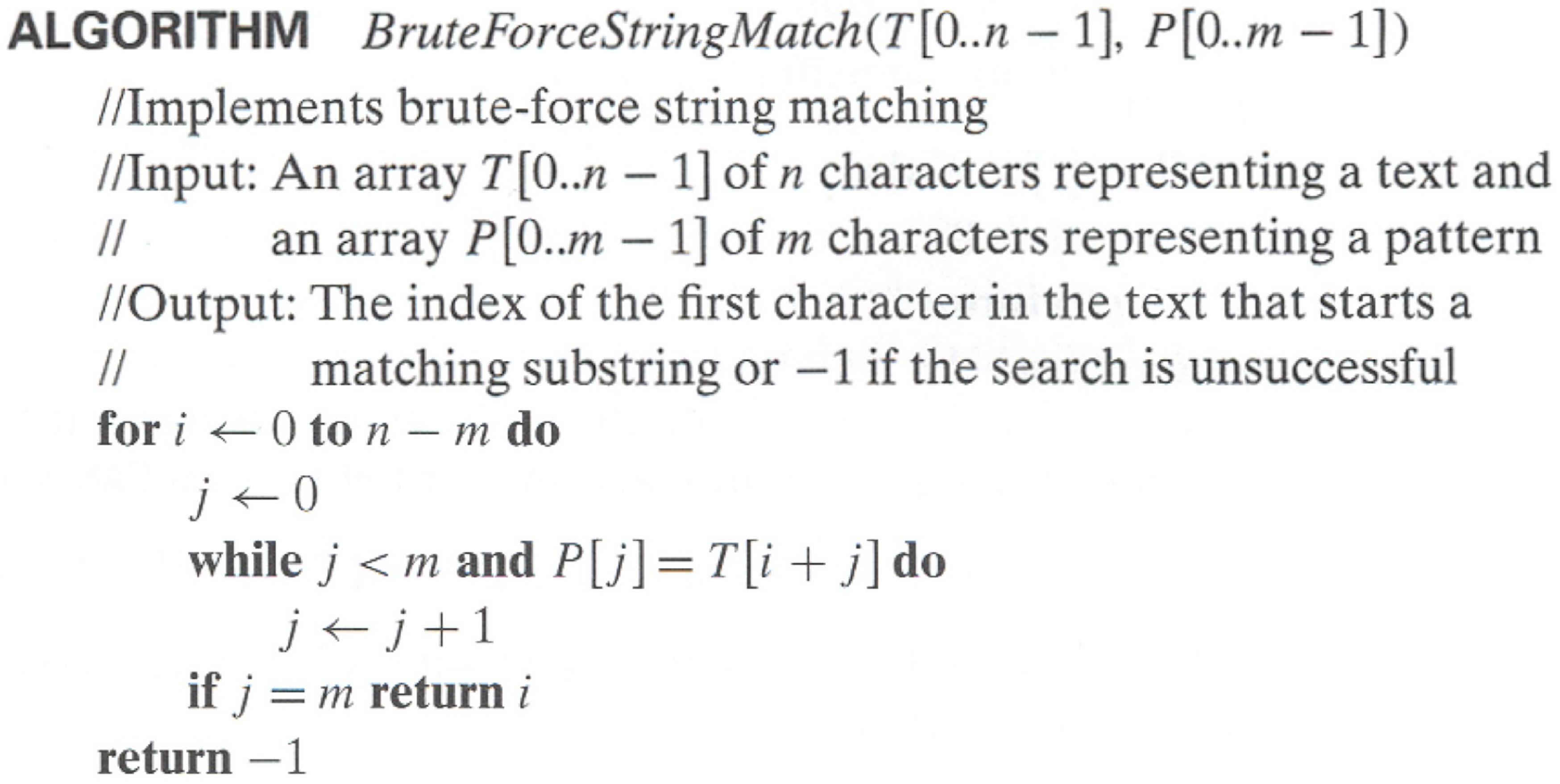
Step 1 Align pattern at beginning of text

Step 2 Moving from left to right, compare each character of pattern to the corresponding character in text until

- all characters are found to match (successful search); or
- a mismatch is detected

Step 3 While pattern is not found and the text is not yet exhausted, realign pattern one position to the right and repeat Step 2

Pseudocode and Efficiency



```
ALGORITHM  BruteForceStringMatch($T[0..n-1]$, $P[0..m-1]$)
    //Implements brute-force string matching
    //Input: An array $T[0..n-1]$ of $n$ characters representing a text and
    //       an array $P[0..m-1]$ of $m$ characters representing a pattern
    //Output: The index of the first character in the text that starts a
    //        matching substring or $-1$ if the search is unsuccessful
    for $i \leftarrow 0$ to $n - m$ do
        $j \leftarrow 0$
        while $j < m$ and $P[j] = T[i + j]$ do
            $j \leftarrow j + 1$
        if $j = m$ return $i$
    return $-1$
```

Efficiency:

Closest-Pair Problem

Find the two closest points in a set of *n* points (in the two-dimensional Cartesian plane).

<u>Brute force algorithm</u>

How?

Take 5 minutes

and try it…

Closest-Pair Problem

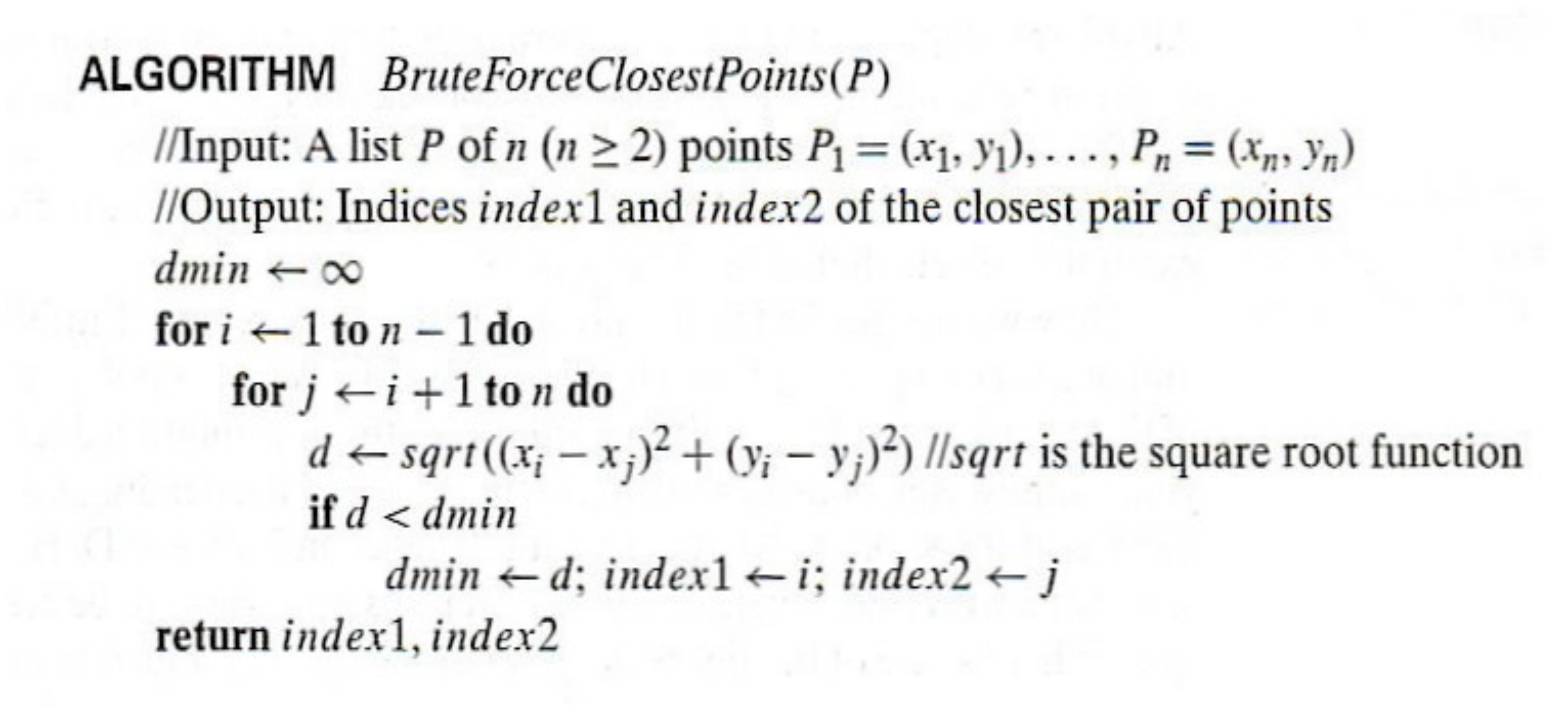
Find the two closest points in a set of *n* points (in the two-dimensional Cartesian plane).

Brute force algorithm

Compute the distance between every pair of distinct points

and return the indexes of the points for which the distance is the smallest.

Closest-Pair Brute Force Algorithm (cont)



```
ALGORITHM  BruteForceClosestPoints(P)
    //Input: A list $P$ of $n$ ($n \geq 2$) points $P_1 = (x_1, y_1), \ldots, P_n = (x_n, y_n)$
    //Output: Indices $index1$ and $index2$ of the closest pair of points
    $dmin \leftarrow \infty$
    for $i \leftarrow 1$ to $n - 1$ do
        for $j \leftarrow i + 1$ to $n$ do
            $d \leftarrow sqrt((x_i - x_j)^2 + (y_i - y_j)^2)$ //sqrt is the square root function
            if $d < dmin$
                $dmin \leftarrow d$; $index1 \leftarrow i$; $index2 \leftarrow j$
    return $index1, index2$
```

Efficiency:

Brute Force Strengths and Weaknesses

- Strengths
  - wide applicability
  - simplicity
  - yields reasonable algorithms for some important problems (e.g., matrix multiplication, sorting, searching, string matching)

- Weaknesses
  - rarely yields efficient algorithms
  - some brute force algorithms are unacceptably slow
  - not as constructive as some other design techniques

Usually results in the most obvious—thus *maintainable*—code.

Exhaustive Search

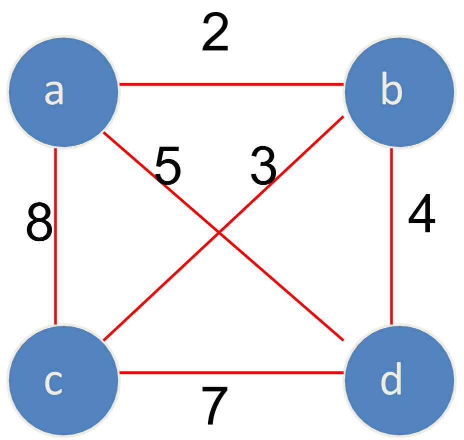
A brute force solution to combinatorial problems. It suggests generating each and every combinatorial object (e.g., permutations, combinations, or subsets of a set) of the problem, selecting those of them that satisfying all the constraints, and then finding a desired object.

Method:

- generate a list of all potential solutions to the problem in a systematic manner
- evaluate potential solutions one by one, disqualifying infeasible ones and, for an optimization problem, keeping track of the best one found so far
- when search ends, announce the solution(s) found

Example 1: Traveling Salesman Problem

- Given *n* cities with known distances between each pair, find the shortest tour that passes through all the cities exactly once before returning to the starting city
- Alternatively: Find shortest *Hamiltonian circuit* in a weighted connected graph
- Example:



TSP by Exhaustive Search

Problem: $v_0, v_1, \ldots, v_{n-1}, v_0$ → permutations of *n* vertices

Tour	Cost	
a→b→c→d→a	2+3+7+5 = 17	optimal
a→b→d→c→a	2+4+7+8 = 21	
a→c→b→d→a	8+3+4+5 = 20	
a→c→d→b→a	8+7+4+2 = 21	
a→d→b→c→a	5+4+3+8 = 20	
a→d→c→b→a	5+7+3+2 = 17	optimal

NP-hard problem!

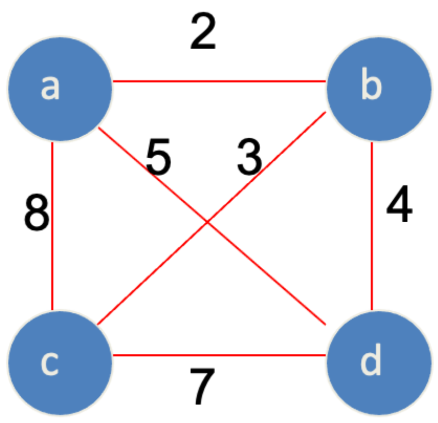
Efficiency:

Common sense check

Start with a. How many paths are left to check?

Say we picked b. How many paths are left now?

…what is the pattern?



Example 2: (0-1) Knapsack Problem

Given n items:

- weights: $w_1 \quad w_2 \ldots \quad w_n$
- values: $v_1 \quad v_2 \ldots \quad v_n$
- a knapsack of capacity W

Find most valuable subset of the items that fit into the knapsack

Example: Knapsack capacity $W=16$

item	weight	value
1	2	\$20
2	5	\$30
3	10	\$50
4	5	\$10

Knapsack Problem by Exhaustive Search

Subset	Total weight	Total value
{1}	2	$20
{2}	5	$30
{3}	10	$50
{4}	5	$10
{1,2}	7	$50
{1,3}	12	$70
{1,4}	7	$30
{2,3}	15	$80
{2,4}	10	$40
{3,4}	15	$60
{1,2,3}	17	not feasible
{1,2,4}	12	$60
{1,3,4}	17	not feasible
{2,3,4}	20	not feasible
{1,2,3,4}	22	not feasible

Efficiency: O(2^n)

Common sense check

You have 4 things.

You need to check all combinations.

Eg. `0110`

How many combinations in 4 bits?

Final Comments on Exhaustive Search

- Exhaustive-search algorithms run in a realistic amount of time <u>only on very small instances</u>

- In some cases, there are much better alternatives!
    - shortest paths
    - minimum spanning tree
    - assignment problem

- In many cases, exhaustive search or its variation is the only known way to get exact solution
    - Approximation may be better in the long run…

Next week…

Decrease and conquer

Chapter 4.1, 4.3-4.4

Divide and conquer

Chapter 5