

## Important Summation Formulas

1.  $\sum_{i=l}^u 1 = \underbrace{1 + 1 + \cdots + 1}_{u-l+1 \text{ times}} = u - l + 1$  ( $l, u$  are integer limits,  $l \leq u$ );  $\sum_{i=1}^n 1 = n$
2.  $\sum_{i=1}^n i = 1 + 2 + \cdots + n = \frac{n(n+1)}{2} \approx \frac{1}{2}n^2$
3.  $\sum_{i=1}^n i^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} \approx \frac{1}{3}n^3$
4.  $\sum_{i=1}^n i^k = 1^k + 2^k + \cdots + n^k \approx \frac{1}{k+1}n^{k+1}$
5.  $\sum_{i=0}^n a^i = 1 + a + \cdots + a^n = \frac{a^{n+1} - 1}{a - 1}$  ( $a \neq 1$ );  $\sum_{i=0}^n 2^i = 2^{n+1} - 1$
6.  $\sum_{i=1}^n i2^i = 1 \cdot 2 + 2 \cdot 2^2 + \cdots + n2^n = (n-1)2^{n+1} + 2$
7.  $\sum_{i=1}^n \frac{1}{i} = 1 + \frac{1}{2} + \cdots + \frac{1}{n} \approx \ln n + \gamma$ , where  $\gamma \approx 0.5772 \dots$  (Euler's constant)
8.  $\sum_{i=1}^n \lg i \approx n \lg n$

## Sum Manipulation Rules

1.  $\sum_{i=l}^u ca_i = c \sum_{i=l}^u a_i$
2.  $\sum_{i=l}^u (a_i \pm b_i) = \sum_{i=l}^u a_i \pm \sum_{i=l}^u b_i$
3.  $\sum_{i=l}^u a_i = \sum_{i=l}^m a_i + \sum_{i=m+1}^u a_i$ , where  $l \leq m < u$
4.  $\sum_{i=l}^u (a_i - a_{i-1}) = a_u - a_{l-1}$