David L. Ferguson Computational Physics Memorial Workshop
Brookhaven National Laboratory
Exam 01

Please email Dave Biersach (dbiersach@bnl.gov) a single Mathematica notebook (.nb) file containing all of your work. Clearly label all answer (output) cells in the notebook with the corresponding question # to which they pertain. You are free to work in pairs or groups, but only submit one notebook per team. Ensure the full name and email address of each team member appears at the start of the notebook. Feel free to consult with professors and Internet reference sites as needed.

1. Find all the roots of the equation $z^7 - 5z^4 + 3z^2 + 6z - 4 = 0$

2. Plot the function in problem #1 from $z = -2 - 2i$ to $z = 2 + 2i$

3. Express in polar form the complex number $4 - 3i$

4. Evaluate
$$\lim_{x\to\infty} \frac{\sqrt{9x^2 + 2}}{(3 - 4x)}$$

5. Evaluate
$$\lim_{x\to 0} \left(\frac{1}{x} \ln\left(\frac{x+2}{2}\right)\right)$$

6. Plot the function
$$y = \frac{2x^2}{(9 - x^2)}$$
from $x = 1$ to $x = 5$

7. Prove the limit exists, or does not exist, at $x = 3$ for the function in problem #6

8. Create a single plot showing the functions $x^2, x^2 \sin\frac{1}{x}, and\ -x^2$ from $x = -0.1$ to $x = 0.1$

9. Evaluate $\lim_{x\to 0} \left(x^2 \sin\frac{1}{x}\right)$ and explain how the plot in problem #8 proves that this limit exists

10. Plot the function $y = \left(x - \sqrt{\sin x + 10x + x^2}\right)^2$ from $x = 0$ to $x = 1{,}000{,}000$

11. What is the limit of the function in problem #10 as x goes to infinity?

12. Compare and contrast the numerical roots of these three equations:
$$f(x) = 0.99x^3 - 21x^2 + 120x - 100$$
$$g(x) = 1.00x^3 - 21x^2 + 120x - 100$$
$$h(x) = 1.01x^3 - 21x^2 + 120x - 100$$

13. Plot the three functions in problem #12 from $x = -1$ to $x = 12$ on the same graph

14. Solve the system of equations $4x^2 + y^2 = 4$ and $x - y = 3$ over the complex domain

15. Use the Mathematica function **ContourPlot[]** to graph in the real domain the two functions in problem #14 from $x = -3$ to $x = 3$ and $y = -3$ to $y = 3$. How does this plot explain the necessity for complex solutions to problem #14?

16. Find the <u>period</u> of this polynomial: $p(x) = 15\sin^2(12x) + 12\sin^2(15x)$

17. Plot the polynomial in problem #16 from $x = 0$ to $x = 2\pi/3$

18. Plot the two sinusoids $y = \cos(t)$ and $y = \cos\left(t\sqrt{2}\right)$ from $t = 0$ to $t = 50$ and then plot their superposition over the same domain. Explain how these three plots demonstrate that the superposition of two periodic waveforms can be aperiodic

19. Given $f(x) = \ln(1 - 5x^2 + x^3)$ find $f'(x)$

20. Given $f(x) = (3x)(5x^2)$ find $f'(x)$

21. Given
$$f(x) = \frac{(1 + e^{-2x})}{(x + \tan(12x))}$$
express $f'(x)$ fully simplified

22. The company Oculus has realized it can sell 400 VR headsets per month at a price of $300 per headset. For each $5.00 drop in price, they can sell 10 more VR headsets. What would be the optimal selling price so they can maximize the number of headsets sold? What would be their gross monthly sales revenue at that price point?

23. Evaluate $\int \csc(x)\cot(x)\,dx$ in the real domain

24. Evaluate $\int 2\sin(x)\cos(x)\,dx$ in the real domain

25. Given $f(x) = e^{\sin x}$ and $g(x) = \sin(\sin x)$ plot both functions on the same graph from $x = 0$ to $x = 2$ and calculate the numerical area between both curves across that same domain

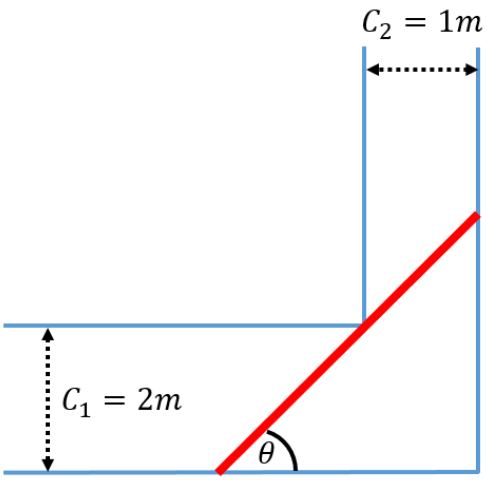
26. Using the Mathematica functions **Plot3D[]** and **Disk[]**, graph the function $z = \left|\sqrt{x}\right|$ where <u>both</u> x and y coordinates are <u>elements of</u> the unit circle. Notice there is no term containing y in this equation for z

27. Find the numeric volume under the surface given in problem #26

28. Evaluate
$$\int_0^1 \int_0^{2-2x} (x^2 + y^2 + 1)\,dy\,dx$$

29. The Archimedes Spiral follows the equations $x = t\cos t$ and $y = t\sin t$. Find the total arclength along this spiral from $t = 0$ to $t = 8\pi$

30. What is the maximum length ladder that can fit around the corner depicted below? Note: the ladder must maintain a *constant* length **L** throughput its entire rotation between $0^\circ \leq \theta \leq 90^\circ$



31. Fully simplify the factors of the polynomial $x^3y^2 - 3x^2y - 2xy^3 + 6y^2$

32. Demonstrate Benford's Law by using the Mathematica functions **RandomReal[]**, **RealDigits[]**, and **Histogram[]** to first create a list of 100 random real numbers between 0 and 100 inclusive. Then make a histogram of the first digit frequency of the numbers in that list. Then take the reciprocal of each number in that list, and again make a histogram of the first digit frequency of those reciprocals. Though the original list was all random numbers, which first digit occurs most often in the reciprocals of those random numbers?

33. Given the function

$$f(n) = \frac{\left(1 + \sqrt{4n^2 - 4n + 5}\right)}{2}$$

generate a table of the continued fractions of this function as n goes from 0 to 9 inclusive. What is the relationship between each n and the repeated sequence in the continued fraction for each value of $f(n)$?

34. Which two successive primes, both less than 300, mark the beginning and end of the largest prime desert in that region? Hint: Refer to the Mathematica functions **Prime[]**, **PrimePi[]**, **Differences[]**, **Position[]**, and **Ordering[]**

35. Solve the following system of equations:

$$
\begin{aligned}
a + b - 2c + d + 3e - f &= 4 \\
2a - b + c + 2d + e - 3f &= 20 \\
a + 3b - 3c - d + 2e + f &= -15 \\
5a + 2b - c - d + 2e + f &= -3 \\
-3a - b + 2c + 3d + e + 3f &= 16 \\
4a + 3b + c - 6d - 3e - 2f &= -27
\end{aligned}
$$