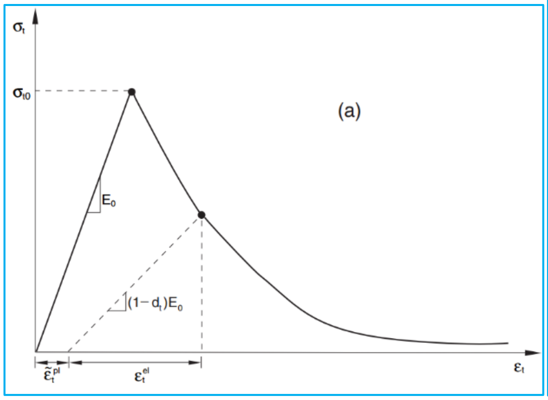
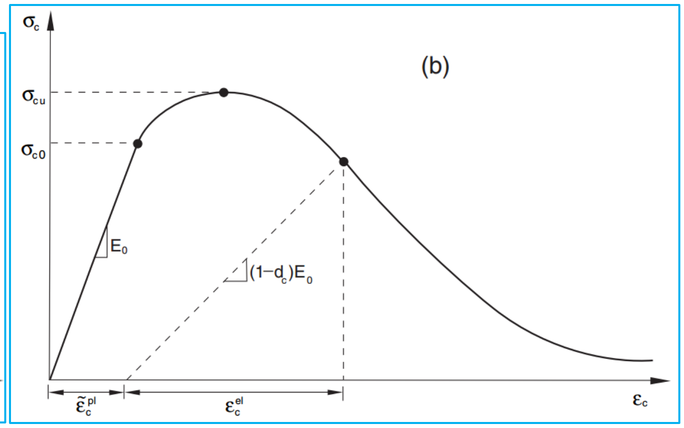




$$
\begin{aligned}
\sigma_t &= (1-d_t)E_0(\varepsilon_t - \tilde{\varepsilon}_t^{pl}), \\
\sigma_c &= (1-d_c)E_0(\varepsilon_c - \tilde{\varepsilon}_c^{pl}).
\end{aligned}
$$

$$
\begin{aligned}
\bar{\sigma}_t &= \frac{\sigma_t}{(1-d_t)} = E_0(\varepsilon_t - \tilde{\varepsilon}_t^{pl}), \\
\bar{\sigma}_c &= \frac{\sigma_c}{(1-d_c)} = E_0(\varepsilon_c - \tilde{\varepsilon}_c^{pl}).
\end{aligned}
$$

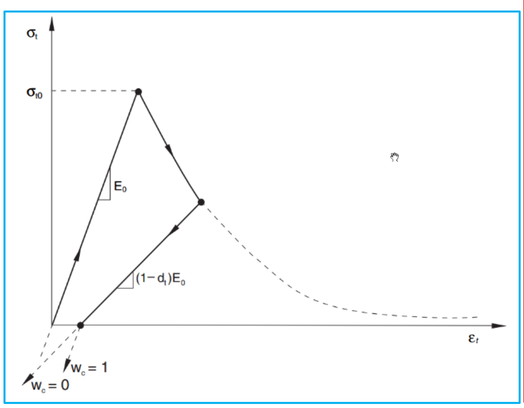
$$
\boldsymbol{\sigma} = (1-d)\mathbf{D}_0^{el} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{pl}) = \mathbf{D}^{el} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{pl}),
$$

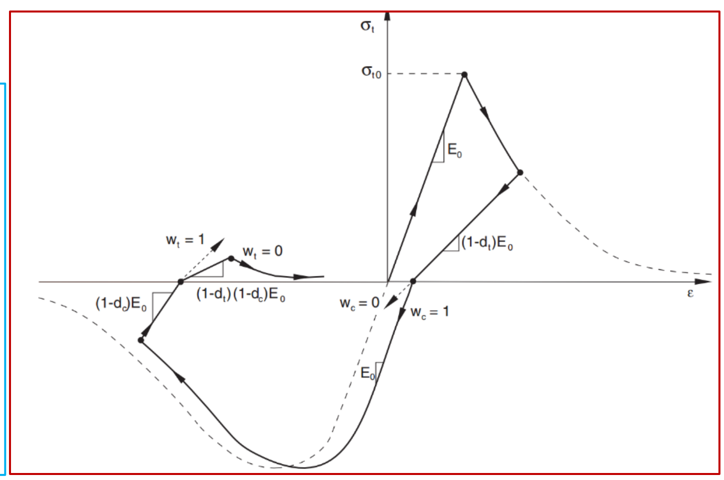
$$
\bar{\boldsymbol{\sigma}} \stackrel{\text{def}}{=} \mathbf{D}_0^{el} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{pl})
$$

$$
\boldsymbol{\sigma} = (1-d)\bar{\boldsymbol{\sigma}}.
$$

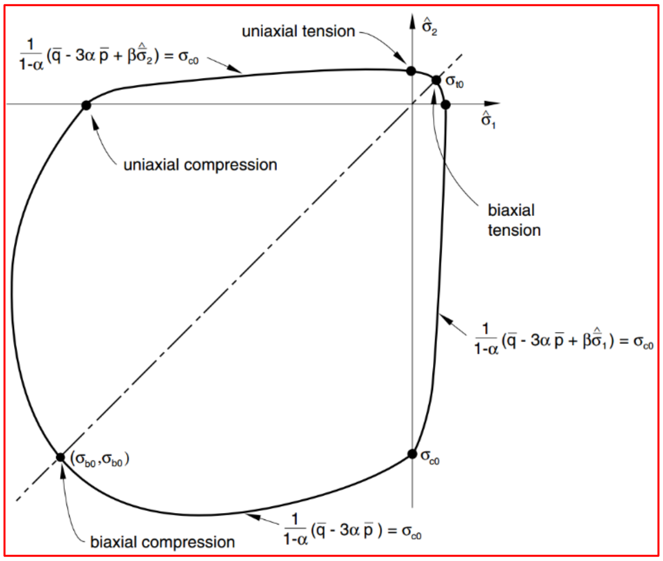
$$
(1-d) = (1-s_t d_c)(1-s_c d_t), \quad 0 \le s_t,\ s_c \le 1,
$$

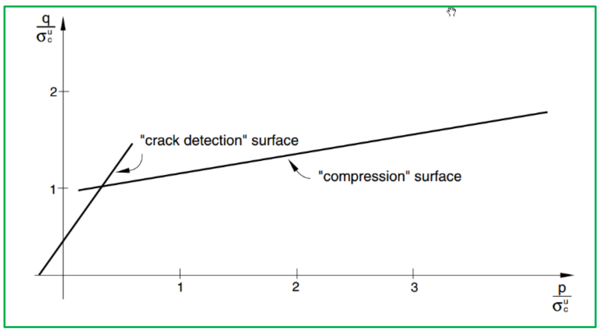
$$
\begin{aligned}
s_t &= 1 - w_t r^*(\bar{\sigma}_{11}); \quad 0 \le w_t \le 1, \\
s_c &= 1 - w_c(1 - r^*(\bar{\sigma}_{11})); \quad 0 \le w_c \le 1,
\end{aligned}
$$





$$F(\bar{\boldsymbol{\sigma}}, \tilde{\boldsymbol{\varepsilon}}^{pl}) = \frac{1}{1-\alpha}\left(\bar{q} - 3\alpha\bar{p} + \beta(\tilde{\boldsymbol{\varepsilon}}^{pl})\langle\hat{\bar{\sigma}}_{\max}\rangle - \gamma\langle -\hat{\bar{\sigma}}_{\max}\rangle\right) - \bar{\sigma}_c(\tilde{\varepsilon}_c^{pl}) \leq 0,$$





$$\bar{p} = -\frac{1}{3}\bar{\boldsymbol{\sigma}} : \mathbf{I}$$

$$\bar{q} = \sqrt{\frac{3}{2}\bar{\mathbf{S}} : \bar{\mathbf{S}}}$$

$$\bar{\mathbf{S}} = \bar{p}\mathbf{I} + \bar{\boldsymbol{\sigma}}$$

$$\beta(\tilde{\boldsymbol{\varepsilon}}^{pl}) = \frac{\bar{\sigma}_c(\tilde{\varepsilon}_c^{pl})}{\bar{\sigma}_t(\tilde{\varepsilon}_t^{pl})}(1-\alpha) - (1+\alpha),$$

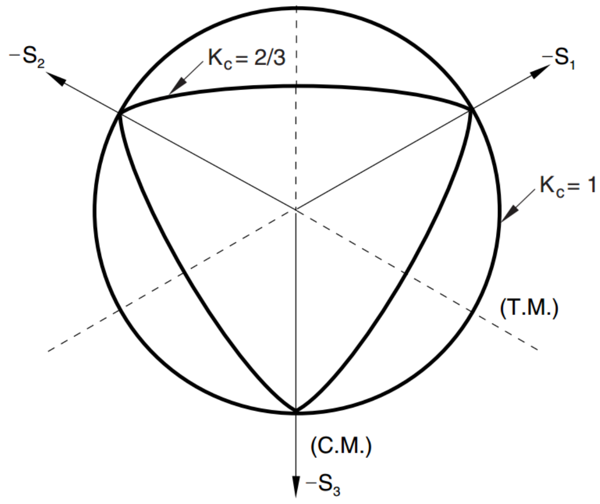
$$\alpha = \frac{\sigma_{b0} - \sigma_{c0}}{2\sigma_{b0} - \sigma_{c0}}$$

$$\left(\frac{2}{3}\gamma + 1\right)\bar{q} - (\gamma + 3\alpha)\bar{p} = (1-\alpha)\bar{\sigma}_c, \quad \text{(TM)}$$

$$\left(\frac{1}{3}\gamma + 1\right)\bar{q} - (\gamma + 3\alpha)\bar{p} = (1-\alpha)\bar{\sigma}_c. \quad \text{(CM)}$$

$$K_c = \frac{\gamma + 3}{2\gamma + 3}$$

$$\gamma = \frac{3(1 - K_c)}{2K_c - 1}$$



$$F(\bar{\boldsymbol{\sigma}}, \tilde{\boldsymbol{\varepsilon}}^{pl}) = \frac{1}{1 - \alpha} \left(\bar{q} - 3\alpha\bar{p} + \beta(\tilde{\boldsymbol{\varepsilon}}^{pl})\langle\hat{\bar{\sigma}}_{\max}\rangle - \gamma\langle-\hat{\bar{\sigma}}_{\max}\rangle \right) - \bar{\sigma}_c(\tilde{\varepsilon}_c^{pl}) \leq 0,$$

If $\hat{\bar{\sigma}}_{\max} > 0$, the yield conditions along the tensile and compressive meridians reduce to

$$\left(\frac{2}{3}\beta + 1\right)\bar{q} - (\beta + 3\alpha)\bar{p} = (1 - \alpha)\bar{\sigma}_c, \quad \text{(TM)}$$

$$\left(\frac{1}{3}\beta + 1\right)\bar{q} - (\beta + 3\alpha)\bar{p} = (1 - \alpha)\bar{\sigma}_c. \quad \text{(CM)}$$

$$K_t = \frac{\beta + 3}{2\beta + 3}$$

$$\dot{\boldsymbol{\varepsilon}}^{pl} = \dot{\lambda}\frac{\partial G(\bar{\boldsymbol{\sigma}})}{\partial \bar{\boldsymbol{\sigma}}}$$

$$G = \sqrt{(\epsilon\sigma_{t0}\tan\psi)^2 + \bar{q}^2} - \bar{p}\tan\psi$$

$$\tilde{\boldsymbol{\varepsilon}}^{pl} = \begin{bmatrix} \tilde{\varepsilon}_t^{pl} \\ \tilde{\varepsilon}_c^{pl} \end{bmatrix}; \quad \dot{\tilde{\boldsymbol{\varepsilon}}}^{pl} = \mathbf{h}(\bar{\boldsymbol{\sigma}}, \tilde{\boldsymbol{\varepsilon}}^{pl}) \cdot \dot{\boldsymbol{\varepsilon}}^{pl},$$

$$\begin{aligned} \bar{\boldsymbol{\sigma}} &= \mathbf{D}_0^{el} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{pl}) \in \{\bar{\boldsymbol{\sigma}} | F(\bar{\boldsymbol{\sigma}}, \tilde{\boldsymbol{\varepsilon}}^{pl}) \leq 0\}, \\ \dot{\tilde{\boldsymbol{\varepsilon}}}^{pl} &= \mathbf{h}(\bar{\boldsymbol{\sigma}}, \tilde{\boldsymbol{\varepsilon}}^{pl}) \cdot \dot{\boldsymbol{\varepsilon}}^{pl}, \\ \dot{\boldsymbol{\varepsilon}}^{pl} &= \dot{\lambda} \frac{\partial G(\bar{\boldsymbol{\sigma}})}{\partial \bar{\boldsymbol{\sigma}}}, \end{aligned}$$

$$\begin{aligned} \dot{\tilde{\varepsilon}}_t^{pl} &= r^* \dot{\varepsilon}_{11}^{pl}, \\ \dot{\tilde{\varepsilon}}_c^{pl} &= -(1 - r^*) \dot{\varepsilon}_{11}^{pl}, \end{aligned}$$

$$\begin{aligned} \dot{\tilde{\varepsilon}}_t^{pl} &\stackrel{\text{def}}{=} r(\hat{\bar{\boldsymbol{\sigma}}}) \hat{\dot{\varepsilon}}_{\max}^{pl}, \\ \dot{\tilde{\varepsilon}}_c^{pl} &\stackrel{\text{def}}{=} -(1 - r(\hat{\bar{\boldsymbol{\sigma}}})) \hat{\dot{\varepsilon}}_{\min}^{pl}, \end{aligned}$$

$\hat{\dot{\varepsilon}}_{\max}^{pl}$ and $\hat{\dot{\varepsilon}}_{\min}^{pl}$ are, respectively, the maximum and minimum eigenvalues of the plastic strain rate

$$r(\hat{\bar{\boldsymbol{\sigma}}}) \stackrel{\text{def}}{=} \frac{\sum_{i=1}^{3} \langle \hat{\bar{\sigma}}_i \rangle}{\sum_{i=1}^{3} |\hat{\bar{\sigma}}_i|}; \quad 0 \leq r(\hat{\bar{\boldsymbol{\sigma}}}) \leq 1$$

$$\dot{\tilde{\boldsymbol{\varepsilon}}}^{pl} = \begin{bmatrix} \dot{\tilde{\varepsilon}}_t^{pl} \\ \dot{\tilde{\varepsilon}}_c^{pl} \end{bmatrix} = \hat{\mathbf{h}}(\hat{\bar{\boldsymbol{\sigma}}}, \tilde{\boldsymbol{\varepsilon}}^{pl}) \cdot \hat{\dot{\boldsymbol{\varepsilon}}}^{pl}$$

$$\hat{\mathbf{h}}(\hat{\bar{\boldsymbol{\sigma}}}, \tilde{\boldsymbol{\varepsilon}}^{pl}) = \begin{bmatrix} r(\hat{\bar{\boldsymbol{\sigma}}}) & 0 & 0 \\ 0 & 0 & -(1 - r(\hat{\bar{\boldsymbol{\sigma}}})) \end{bmatrix}$$

$$\hat{\dot{\boldsymbol{\varepsilon}}}^{pl} = \begin{bmatrix} \hat{\dot{\varepsilon}}_1 \\ \hat{\dot{\varepsilon}}_2 \\ \hat{\dot{\varepsilon}}_3 \end{bmatrix}$$

Kuhn-Tucker conditions: $\dot{\lambda} F = 0; \ \dot{\lambda} \geq 0; \ F \leq 0.$

$$F(\bar{\boldsymbol{\sigma}}, \tilde{\boldsymbol{\varepsilon}}^{pl}) \leq 0.$$

$$\begin{aligned} \sigma_t &= \sigma_t(\tilde{\varepsilon}_t^{pl}, \dot{\tilde{\varepsilon}}_t^{pl}, \theta, f_i), \\ \sigma_c &= \sigma_c(\tilde{\varepsilon}_c^{pl}, \dot{\tilde{\varepsilon}}_c^{pl}, \theta, f_i), \end{aligned}$$

$$\tilde{\varepsilon}_t^{pl} = \int_0^t \dot{\tilde{\varepsilon}}_t^{pl} dt \text{ and } \tilde{\varepsilon}_c^{pl} = \int_0^t \dot{\tilde{\varepsilon}}_c^{pl} dt$$

$$\begin{aligned} \dot{\tilde{\varepsilon}}_t^{pl} &= \dot{\varepsilon}_{11}^{pl}, \ \text{in uniaxial tension and} \\ \dot{\tilde{\varepsilon}}_c^{pl} &= -\dot{\varepsilon}_{11}^{pl}, \ \text{in uniaxial compression} \end{aligned}$$

$$d_t = d_t(\tilde{\varepsilon}_t^{pl}, \theta, f_i), \quad (0 \leq d_t \leq 1),$$
$$d_c = d_c(\tilde{\varepsilon}_c^{pl}, \theta, f_i), \quad (0 \leq d_c \leq 1).$$

$$\dot{\boldsymbol{\varepsilon}}_v^{pl} = \frac{1}{\mu}(\boldsymbol{\varepsilon}^{pl} - \boldsymbol{\varepsilon}_v^{pl})$$

$$\dot{d}_v = \frac{1}{\mu}(d - d_v)$$

$$\boldsymbol{\sigma} = (1 - d_v)\mathbf{D}_0^{el} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_v^{pl})$$

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^p, \qquad \boldsymbol{\sigma} = (1-D)\bar{\boldsymbol{\sigma}}, \qquad \bar{\boldsymbol{\sigma}} = \mathbf{C}_0\boldsymbol{\varepsilon}^e,$$
$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}\mathbf{m}(\bar{\boldsymbol{\sigma}}, \boldsymbol{\kappa}), \qquad \dot{\boldsymbol{\kappa}} = \dot{\lambda}\mathbf{h}(\bar{\boldsymbol{\sigma}}, \boldsymbol{\kappa})$$

$$\dot{\lambda} \geq 0, \quad F(\bar{\boldsymbol{\sigma}}, \boldsymbol{\kappa}) \leq 0 \quad \text{and} \quad \dot{\lambda}F(\bar{\boldsymbol{\sigma}}, \boldsymbol{\kappa}) = 0$$

$\left\{\varepsilon_n, \boldsymbol{\varepsilon}_n^p, \boldsymbol{\kappa}_n\right\}$ is assumed to be known at time t_n.

the effective stress $\bar{\boldsymbol{\sigma}}_n$ and the damage variable D_n

$$\bar{\boldsymbol{\sigma}}_n = \mathbf{C}_0\left(\boldsymbol{\varepsilon}_n - \boldsymbol{\varepsilon}_n^p\right) \quad \text{and} \quad D_n = D\left(\boldsymbol{\kappa}_n\right)$$

$$\boldsymbol{\kappa} \quad = \quad \{\kappa_t; \ \kappa_c\}$$

$$\boldsymbol{\varepsilon}_{n+1} = \boldsymbol{\varepsilon}_n + \Delta\boldsymbol{\varepsilon}$$

$$\bar{\boldsymbol{\sigma}}_{n+1} = \bar{\boldsymbol{\sigma}}_{n+1}^{trial} - \Delta\lambda\mathbf{C}_0\,\mathbf{m} \quad \text{with } \bar{\boldsymbol{\sigma}}_{n+1}^{trial} = \mathbf{C}_0\left(\boldsymbol{\varepsilon}_{n+1} - \boldsymbol{\varepsilon}_n^p\right)$$

$F\left(\bar{\boldsymbol{\sigma}}_{n+1}^{trial}, \boldsymbol{\kappa}_n\right) \leq 0$, then this is an elastic state

$$\bar{\boldsymbol{\sigma}}_{n+1} = \bar{\boldsymbol{\sigma}}_{n+1}^{trial} \text{ and } \boldsymbol{\kappa}_{n+1} = \boldsymbol{\kappa}_n$$

$$\boldsymbol{\kappa}_{n+1} = \boldsymbol{\kappa}_n + \Delta\lambda\mathbf{h}$$

$$F\left(\bar{\boldsymbol{\sigma}}_{n+1}, \boldsymbol{\kappa}_{n+1}\right) = 0$$

$$\mathbf{R}_{\hat{\bar{\boldsymbol{\sigma}}}} = \hat{\bar{\boldsymbol{\sigma}}}_{n+1} + \Delta\lambda\hat{\mathbf{C}}_0\mathbf{m} - \hat{\bar{\boldsymbol{\sigma}}}_{n+1}^{trial}$$
$$\mathbf{R}_{\boldsymbol{\kappa}} = \boldsymbol{\kappa}_{n+1} - \Delta\lambda\,\mathbf{h} - \boldsymbol{\kappa}_n$$
$$\mathbf{R}_{\Delta\lambda} = F\left(\hat{\bar{\boldsymbol{\sigma}}}_{n+1}, \boldsymbol{\kappa}_{n+1}\right)$$

$$\hat{\mathbf{C}}_0 = \begin{bmatrix} \lambda_L + 2G & \lambda_L & \lambda_L \\ \lambda_L & \lambda_L + 2G & \lambda_L \\ \lambda_L & \lambda_L & \lambda_L + 2G \end{bmatrix}$$

$$\mathbf{x} = \left[\hat{\bar{\boldsymbol{\sigma}}}_{n+1} \quad \boldsymbol{\kappa}_{n+1} \quad \Delta\lambda\right]^{\mathrm{T}}$$

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta\mathbf{x} \quad \text{with } \Delta\mathbf{x} = -\mathbf{J}^{-1}\mathbf{R}(\mathbf{x}^{(k)})$$

$\mathbf{J}$ is the Jacobian

$$\mathbf{J} = \begin{bmatrix} \mathbf{I}_{n\hat{\bar{\sigma}}} + \Delta\lambda\hat{\mathbf{C}}_0 \dfrac{\partial \mathbf{m}}{\partial \hat{\bar{\boldsymbol{\sigma}}}}; & \Delta\lambda\hat{\mathbf{C}}_0 \dfrac{\partial \mathbf{m}}{\partial \boldsymbol{\kappa}}; & \hat{\mathbf{C}}_0\mathbf{m} \\ -\Delta\lambda \dfrac{\partial \mathbf{h}}{\partial \hat{\bar{\boldsymbol{\sigma}}}} & \mathbf{I}_{n\kappa} - \Delta\lambda \dfrac{\partial \mathbf{h}}{\partial \boldsymbol{\kappa}} & -\mathbf{h} \\ \left\{\partial F / \partial \hat{\bar{\boldsymbol{\sigma}}}\right\}^{\mathrm{T}} & \{\partial F / \partial \boldsymbol{\kappa}\}^{\mathrm{T}} & 0 \end{bmatrix}$$

$$\mathbf{x}^{(0)} = \begin{bmatrix} \hat{\bar{\boldsymbol{\sigma}}}_{n+1}^{(0)} \\ \boldsymbol{\kappa}_{n+1}^{(0)} \\ \Delta\lambda^{(0)} \end{bmatrix} = \begin{bmatrix} \hat{\bar{\boldsymbol{\sigma}}}_{n+1}^{trial} \\ \boldsymbol{\kappa}_n \\ 0 \end{bmatrix}$$

$$\left\|\mathbf{R}(\mathbf{x}^{(k+1)})\right\| \leq \mathrm{TOL}$$

Sub-stepping strategy

Damage corrector step

$$D_{n+1} = D\left(\boldsymbol{\kappa}_{n+1}\right)$$
$$\boldsymbol{\sigma}_{n+1} = \left(1 - D_{n+1}\right)\bar{\boldsymbol{\sigma}}_{n+1}.$$

$$\frac{\mathrm{d}\boldsymbol{\sigma}_{n+1}}{\mathrm{d}\boldsymbol{\varepsilon}_{n+1}} = -\bar{\boldsymbol{\sigma}}_{n+1}\frac{\mathrm{d}D_{n+1}}{\mathrm{d}\boldsymbol{\kappa}_{n+1}}\frac{\mathrm{d}\boldsymbol{\kappa}_{n+1}}{\mathrm{d}\boldsymbol{\varepsilon}_{n+1}} + \left(1 - D_{n+1}\right)\frac{\mathrm{d}\bar{\boldsymbol{\sigma}}_{n+1}}{\mathrm{d}\boldsymbol{\varepsilon}_{n+1}}.$$

$$\mathbf{J}\begin{pmatrix}\mathrm{d}\hat{\bar{\boldsymbol{\sigma}}}_{n+1}\\ \mathrm{d}\boldsymbol{\kappa}_{n+1}\\ \mathrm{d}\Delta\lambda\end{pmatrix} = \begin{pmatrix}\hat{\mathbf{C}}_0\mathrm{d}\hat{\boldsymbol{\varepsilon}}_{n+1}\\ \mathbf{0}_{n\kappa}\\ 0\end{pmatrix}$$

$$\begin{pmatrix}\mathrm{d}\hat{\bar{\boldsymbol{\sigma}}}_{n+1}\\ \mathrm{d}\boldsymbol{\kappa}_{n+1}\\ \mathrm{d}\Delta\lambda\end{pmatrix} = \begin{bmatrix}\hat{\boldsymbol{\Xi}}_{\hat{\bar{\sigma}}} & \bullet & \bullet\\ \hat{\boldsymbol{\Xi}}_{\kappa} & \bullet & \bullet\\ \bullet & \bullet & \bullet\end{bmatrix}\begin{pmatrix}\hat{\mathbf{C}}_0\mathrm{d}\hat{\boldsymbol{\varepsilon}}_{n+1}\\ \mathbf{0}_{n\kappa,1}\\ 0\end{pmatrix}$$

$$\frac{\mathrm{d}\hat{\bar{\boldsymbol{\sigma}}}_{n+1}}{\mathrm{d}\hat{\boldsymbol{\varepsilon}}_{n+1}} = \hat{\boldsymbol{\Xi}}_{\hat{\bar{\sigma}}}\hat{\mathbf{C}}_0$$

$$\frac{\mathrm{d}\boldsymbol{\kappa}_{n+1}}{\mathrm{d}\hat{\boldsymbol{\varepsilon}}_{n+1}} = \hat{\boldsymbol{\Xi}}_{\kappa}\hat{\mathbf{C}}_0$$

$$\frac{\mathrm{d}\hat{\boldsymbol{\sigma}}_{n+1}}{\mathrm{d}\hat{\boldsymbol{\varepsilon}}_{n+1}} = \left[\left(1 - D_{n+1}\right)\boldsymbol{\Xi}_{\hat{\bar{\sigma}}} - \hat{\bar{\boldsymbol{\sigma}}}_{n+1}\frac{\mathrm{d}D_{n+1}}{\mathrm{d}\boldsymbol{\kappa}_{n+1}}\boldsymbol{\Xi}_{\kappa}\right]\hat{\mathbf{C}}_0$$

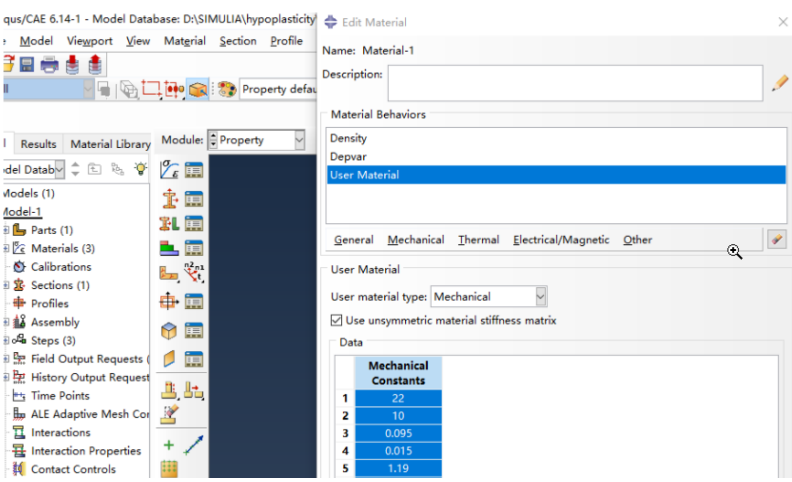
$$\frac{\mathrm{d}\boldsymbol{\sigma}_{n+1}}{\mathrm{d}\boldsymbol{\varepsilon}_{n+1}} = \sum_{A=1}^{3}\sum_{B=1}^{3}\frac{\mathrm{d}\hat{\sigma}_A}{\mathrm{d}\hat{\varepsilon}_B}\mathbf{m}_A^{\mathrm{T}}\mathbf{m}_B + \frac{\left(1 - D_{n+1}\right)}{2}$$
$$\times\sum_{A=1}^{3}\sum_{B\neq A}\left[\left(\frac{\hat{\bar{\sigma}}_B - \hat{\bar{\sigma}}_A}{\hat{\varepsilon}_B - \hat{\varepsilon}_A}\right)\left(\mathbf{m}_{AB}^{\mathrm{T}}\mathbf{m}_{AB} + \mathbf{m}_{AB}^{\mathrm{T}}\mathbf{m}_{BA}\right)\right]$$

$$\mathbf{m}_A = \mathbf{v}_A^{\mathrm{T}}\mathbf{v}_A, \qquad \mathbf{m}_{AB} = \mathbf{v}_A^{\mathrm{T}}\mathbf{v}_B, \quad A \neq B.$$

$\left(\hat{\bar{\sigma}}_B - \hat{\bar{\sigma}}_A\right)/\left(\hat{\varepsilon}_B - \hat{\varepsilon}_A\right)$ by $\partial\left(\hat{\bar{\sigma}}_B - \hat{\bar{\sigma}}_A\right)/\partial\hat{\varepsilon}_B$

```
c       see Lee,  J.,  and G. L. Fenves, "Plastic-Damage Model
c       for Cyclic Loading of Concrete Structures," Journal of
c       Engineering Mechanics, vol. 124, no. 8, pp. 892-900, 1998.
        SUBROUTINE UMAT(sig,statev,DDSDDE,SSE,SPD,SCD,
      1 RPL,DDSDDT,DRPLDE,DRPLDT,
      2 stran,dstran,TIME,DTIME,TEMP,DTEMP,PREDEF,DPRED,CMNAME,
      3 NDI,NSHR,NTENS,NSTATV,PROPS,NPROPS,COORDS,DROT,PNEWDT,
      4 CELENT,DFGRD0,DFGRD1,NOEL,NPT,LAYER,KSPT,JSTEP,KINC)

 INCLUDE 'ABA_PARAM.INC'
 CHARACTER*80 CMNAME
 DIMENSION sig(NTENS),statev(NSTATV),
1 DDSDDE(NTENS,NTENS),
2 DDSDDT(NTENS),DRPLDE(NTENS),
3 stran(NTENS),dstran(NTENS),TIME(2),PREDEF(1),DPRED(1),
4 PROPS(NPROPS),COORDS(3),DROT(3,3),DFGRD0(3,3),DFGRD1(3,3),
5 JSTEP(4),cc(ntens,ntens),ps(3),an(3,3),dd1(3,6),dd2(3,6),
$ sig_pr(3),sde_pr(3),dj2_ds2(3,3),cin(3,3),uni(3),dx(6),
$ x(6),ress(6),pjac(6,6),dm(3),dm_ds(3,3),dh(2),dh_ds(2,3),
$drr_ds(3),pjac2(6,6),dx6(6),dx66(6),dx5(5),px55(5,5),
$px33(3,3),px23(2,3),sig_tr(6),sig_pr1(3),e_tr(3),e_tr1(3),
$df_ds(3),df_dk(2),ress1(6),dx55(5),yx33(3,3),dir63(6,3)
 PARAMETER(zr=0.0D0,on=1.0D0,tw=2.0D0,thr=3.0D0,fr=4.0D0,
%tol=1.0d-6,tol2=10.0d0)
```



```
pi=atan(on)*fr;
Em=PROPS(1);emu=PROPS(2);fco=props(3);fto=props(4);
fb_fc=props(5);ecc=props(6);omega=props(7);pkc=props(8)
rec_c=props(11);rec_t=props(12);omega=omega*pi/180.0;
```

$$F(\bar{\boldsymbol{\sigma}}, \tilde{\varepsilon}^{pl}) = \frac{1}{1-\alpha}\left(\bar{q} - 3\alpha\bar{p} + \beta(\tilde{\varepsilon}^{pl})\langle\hat{\bar{\sigma}}_{\max}\rangle - \gamma\langle-\hat{\bar{\sigma}}_{\max}\rangle\right) - \bar{\sigma}_c(\tilde{\varepsilon}_c^{pl}) \leq 0,$$

$$\alpha = \frac{\sigma_{b0} - \sigma_{c0}}{2\sigma_{b0} - \sigma_{c0}} \qquad \gamma = \frac{3(1-K_c)}{2K_c - 1}$$

```
tan_o=tan(omega);alpha=(fb_fc-on)/(fb_fc*tw-on);
gamma=thr*(on-pkc)/(tw*pkc-on);
```

```
p1=Em/(on+emu)/(on-tw*emu);cc=zr;
```

```
tan_o=tan(omega);alpha=(fb_fc-on)/(fb_fc*tw-on);
gamma=thr*(on-pkc)/(tw*pkc-on);
ept=statev(1);epc=statev(2);dd=statev(3);
p1=Em/(on+emu)/(on-tw*emu);cc=zr;
```

- Hooke's law in terms of the stress and strain vectors:

$$\{\boldsymbol{\varepsilon}\} \overset{def}{=} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \in \mathbb{R}^6 \qquad \{\boldsymbol{\sigma}\} \overset{def}{=} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} \in \mathbb{R}^6$$

$$\begin{cases} \boldsymbol{\sigma} = \mathbb{C} : \boldsymbol{\varepsilon} \\ \mathbb{C} = \lambda \mathbf{1} \otimes \mathbf{1} + 2\mu \mathbb{I} \end{cases} \Rightarrow \begin{cases} \{\boldsymbol{\sigma}\} = \boldsymbol{D} \cdot \{\boldsymbol{\varepsilon}\} \\ \sigma_i = D_{ij}\varepsilon_j \quad i \in \{1, \ldots, 6\} \end{cases}$$

Where $\boldsymbol{D}$ is the matrix of elastic constants:

$$\boldsymbol{D} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & 1 & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix}$$

$\{\varepsilon_n, \boldsymbol{\varepsilon}_n^p, \boldsymbol{\kappa}_n\}$ is assumed to be known at time t_n. $\quad \boldsymbol{\varepsilon}_{n+1} = \boldsymbol{\varepsilon}_n + \Delta\boldsymbol{\varepsilon}$

$\bar{\boldsymbol{\sigma}}_n = \mathbf{C}_0\left(\boldsymbol{\varepsilon}_n - \boldsymbol{\varepsilon}_n^p\right)$ and $D_n = D(\boldsymbol{\kappa}_n)$ $\quad \bar{\boldsymbol{\sigma}}_{n+1} = \bar{\boldsymbol{\sigma}}_{n+1}^{trial} - \Delta\lambda\mathbf{C}_0\,\mathbf{m}$ with $\bar{\boldsymbol{\sigma}}_{n+1}^{trial} = \mathbf{C}_0\left(\boldsymbol{\varepsilon}_{n+1} - \boldsymbol{\varepsilon}_n^p\right)$

```
cc(1:3,1:3)=p1*emu;
cc(1,1)=(on-emu)*p1;cc(2,2)=(on-emu)*p1;
cc(3,3)=(on-emu)*p1;Gm=Em/(on+emu)/tw;
cc(4,4)=Gm;cc(5,5)=Gm;cc(6,6)=Gm;ddsdde=cc*(on-dd);
sig_tr=sig/(on-dd)+matmul(cc,dstran);
```

```
ept=statev(1);epc=statev(2);dd=statev(3);
```

```
sig_tr=zr;sig_tr(1)=-12.5;ept=zr;epc=zr!0.002-fco/Em+0.0001!!!!
write(*,*) 'epc1',epc
call sprind(sig_tr,ps,an,1,ndi,nshr)
call korder(ps,an,dd1,dd2,ndi)
```

```
        pi1=ps(1)+ps(2)+ps(3);sig_pr=ps;sde_pr=sig_pr-pi1/thr
        pj2=(sde_pr(1)**2+sde_pr(2)**2+sde_pr(3)**2)/tw
        call kaxialy(fc_eff,ft_eff,fc,ft,hc,ht,hc_eff,ht_eff,
       $ ddt_dept,ddc_depc,epc,ept,dc,dt,fto,fco,Em,tol,tol2)
        beta=fc_eff/ft_eff*(on-alpha)-(on+alpha)
        ff=(alpha*pi1+sqrt(thr*pj2)+beta*(sig_pr(1)+abs(sig_pr(1)))
       %/tw-gamma*(abs(sig_pr(1))-sig_pr(1))/tw)/(on-alpha)-fc_eff
```

$$\boxed{F(\bar{\boldsymbol{\sigma}}, \tilde{\varepsilon}^{pl}) = \frac{1}{1-\alpha}\left(\bar{q} - 3\alpha\bar{p} + \beta(\tilde{\varepsilon}^{pl})\langle\hat{\bar{\sigma}}_{\max}\rangle - \gamma\langle-\hat{\bar{\sigma}}_{\max}\rangle\right) - \bar{\sigma}_c(\tilde{\varepsilon}_c^{pl}) \leq 0,}$$

```
if ((ff/Em)<(tol*tw)) then
      sig=(on-dd)*sig_tr
else
      dj2_ds2=-on;dj2_ds2(1,1)=tw;dj2_ds2(2,2)=tw;
      dj2_ds2(3,3)=tw;dj2_ds2=dj2_ds2/thr;
      cin=-emu/Em;cin(1,1)=on/Em;cin(2,2)=on/Em;
      cin(3,3)=on/Em;dx=zr;x=zr;x(1:3)=sig_pr;
      x(4)=ept;x(5)=epc;sig_pr1=sig_pr;ept1=ept;epc1=epc
      e_tr=matmul(cin,sig_pr);e_tr1=e_tr;ress=zr;
      ress(6)=ff;res_n=abs(ress(6)/Em/tol2);res_n1=on;
      iter=0;uni=zr;uni(1)=on;
```

the matrix
nstants:

$$D = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & 1 & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix}$$

$$\boxed{\mathbf{x}^{(0)} = \begin{bmatrix} \hat{\boldsymbol{\sigma}}_{n+1}^{(0)} \\ \boldsymbol{\kappa}_{n+1}^{(0)} \\ \Delta\lambda^{(0)} \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{\sigma}}_{n+1}^{trial} \\ \boldsymbol{\kappa}_n \\ 0 \end{bmatrix}}$$

$$\boxed{\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta\mathbf{x} \quad \text{with } \Delta\mathbf{x} = -\mathbf{J}^{-1}\mathbf{R}(\mathbf{x}^{(k)})}$$

$$\boxed{\begin{aligned} \dot{\hat{\varepsilon}}_t^{pl} &\stackrel{\text{def}}{=} r(\hat{\bar{\boldsymbol{\sigma}}})\dot{\hat{\varepsilon}}_{\max}^{pl}, \\ \dot{\hat{\varepsilon}}_c^{pl} &\stackrel{\text{def}}{=} -(1 - r(\hat{\bar{\boldsymbol{\sigma}}}))\dot{\hat{\varepsilon}}_{\min}^{pl}, \end{aligned}}$$

$$\mathbf{J}=\begin{bmatrix} \mathbf{I}_{n\hat{\bar{\sigma}}}+\Delta\lambda\hat{\mathbf{C}}_0\dfrac{\partial\mathbf{m}}{\partial\hat{\bar{\boldsymbol{\sigma}}}}; & \Delta\lambda\hat{\mathbf{C}}_0\dfrac{\partial\mathbf{m}}{\partial\boldsymbol{\kappa}}; & \hat{\mathbf{C}}_0\mathbf{m} \\ -\Delta\lambda\dfrac{\partial\mathbf{h}}{\partial\hat{\bar{\boldsymbol{\sigma}}}} & \mathbf{I}_{n\kappa}-\Delta\lambda\dfrac{\partial\mathbf{h}}{\partial\boldsymbol{\kappa}} & -\mathbf{h} \\ \left\{\partial F/\partial\hat{\bar{\boldsymbol{\sigma}}}\right\}^{\mathrm{T}} & \{\partial F/\partial\boldsymbol{\kappa}\}^{\mathrm{T}} & \mathbf{0} \end{bmatrix}$$

$$\boxed{\dot{\boldsymbol{\varepsilon}}^{pl}=\dot{\lambda}\frac{\partial G(\bar{\boldsymbol{\sigma}})}{\partial\bar{\boldsymbol{\sigma}}}}$$

$$\boxed{G=\sqrt{(\epsilon\sigma_{t0}\tan\psi)^2+\bar{q}^2}-\bar{p}\tan\psi}$$

```
do while(res_n1>=tol/tol2/tw)
        par=tol/tol2



        iter=iter+1;pjac=zr;
        sqrt1=sqrt((ecc*fto*tan_o)**2+thr*pj2)
        dm=tan_o/thr+thr/tw*sde_pr/sqrt1
        do iii=1,3
              dm_ds(:,iii)=thr/tw*dj2_ds2(:,iii)/sqrt1
%-thr*thr/fr*sde_pr*sde_pr(iii)/sqrt1**3
        end do
        pi1_n=abs(sig_pr(1))+abs(sig_pr(2))+
%abs(sig_pr(3))
        rr=(pi1+pi1_n)/pi1_n/tw
        dh(1)=rr*dm(1);dh(2)=(rr-on)*dm(3)
        par=tw*pi1_n**2;
        do iii=1,3
              if (sig_pr(iii)<zr) then
                    drr_ds(iii)=(pi1_n+pi1)/par
              else
                    drr_ds(iii)=(pi1_n-pi1)/par
              end if
        end do
        dh_ds(1,:)=drr_ds*dm(1)+rr*dm_ds(1,:)
        dh_ds(2,:)=drr_ds*dm(3)+(rr-on)*dm_ds(3,:)
```

$$\boxed{r(\hat{\bar{\boldsymbol{\sigma}}})\stackrel{\text{def}}{=}\frac{\sum_{i=1}^{3}\langle\hat{\bar{\sigma}}_i\rangle}{\sum_{i=1}^{3}|\hat{\bar{\sigma}}_i|};\quad 0\le r(\hat{\bar{\boldsymbol{\sigma}}})\le 1}$$

$$\boxed{\dot{\tilde{\boldsymbol{\varepsilon}}}^{pl}=\begin{bmatrix}\dot{\tilde{\varepsilon}}_t^{pl}\\ \dot{\tilde{\varepsilon}}_c^{pl}\end{bmatrix}=\hat{\mathbf{h}}(\hat{\bar{\boldsymbol{\sigma}}},\tilde{\boldsymbol{\varepsilon}}^{pl})\cdot\dot{\hat{\boldsymbol{\varepsilon}}}^{pl}}$$

$$\boxed{\hat{\mathbf{h}}(\hat{\bar{\boldsymbol{\sigma}}},\tilde{\boldsymbol{\varepsilon}}^{pl})=\begin{bmatrix} r(\hat{\bar{\boldsymbol{\sigma}}}) & 0 & 0\\ 0 & 0 & -(1-r(\hat{\bar{\boldsymbol{\sigma}}}))\end{bmatrix}}$$

$$\mathbf{J} = \begin{bmatrix} \mathbf{I}_{n\hat{\hat{\sigma}}} + \Delta\lambda \hat{\mathbf{C}}_0 \dfrac{\partial \mathbf{m}}{\partial \hat{\hat{\boldsymbol{\sigma}}}}; & \Delta\lambda \hat{\mathbf{C}}_0 \dfrac{\partial \mathbf{m}}{\partial \boldsymbol{\kappa}}; & \hat{\mathbf{C}}_0 \mathbf{m} \\ -\Delta\lambda \dfrac{\partial \mathbf{h}}{\partial \hat{\hat{\boldsymbol{\sigma}}}} & \mathbf{I}_{n\kappa} - \Delta\lambda \dfrac{\partial \mathbf{h}}{\partial \boldsymbol{\kappa}} & -\mathbf{h} \\ \left\{\partial F / \partial \hat{\hat{\boldsymbol{\sigma}}}\right\}^{\mathrm{T}} & \{\partial F / \partial \boldsymbol{\kappa}\}^{\mathrm{T}} & 0 \end{bmatrix}$$

$$F(\bar{\boldsymbol{\sigma}}, \tilde{\varepsilon}^{pl}) = \frac{1}{1-\alpha} \left(\bar{q} - 3\alpha\bar{p} + \beta(\tilde{\varepsilon}^{pl})\langle\hat{\bar{\sigma}}_{\max}\rangle - \gamma\langle -\hat{\bar{\sigma}}_{\max}\rangle\right) - \bar{\sigma}_c(\tilde{\varepsilon}_c^{pl}) \leq 0,$$

```
pjac(1:3,1:3)=cin+x(6)*dm_ds;pjac(1:3,6)=dm
pjac(4:5,1:3)=-x(6)*dh_ds;pjac(4:5,6)=-dh;
pjac(4,4)=on;pjac(5,5)=on;
if (sig_pr(1)>zr) then
df_ds=(alpha+thr/tw*sde_pr/sqrt(thr*pj2)
%+beta*uni)/(on-alpha)
df_dk(1)=-ht_eff*fc_eff/ft_eff**2*sig_pr(1)
df_dk(2)=hc_eff/ft_eff*sig_pr(1)-hc_eff
else
df_ds=(alpha+thr/tw*sde_pr/sqrt(thr*pj2)
%+gamma*uni)/(on-alpha)
df_dk(1)=zr;df_dk(2)=-hc_eff
end if
pjac(6,1:3)=df_ds;pjac(6,4:5)=df_dk

pjac2=pjac;ress1=ress;
call gauss_2(pjac2,-on*ress1,dx,6)
pjac2=pjac
```

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta\mathbf{x} \quad \text{with } \Delta\mathbf{x} = -\mathbf{J}^{-1}\mathbf{R}(\mathbf{x}^{(k)})$$

```
x=x+dx;sig_pr=x(1:3);
ress(1:3)=matmul(cin,x(1:3))-e_tr1+x(6)*dm
if(x(4)<ept1+tol/tol2**5) x(4)=ept1
if(x(5)<epc1+tol/tol2**5) x(5)=epc1
ress(4)=x(4)-ept1-x(6)*dh(1);ept=x(4)
ress(5)=x(5)-epc1-x(6)*dh(2);epc=x(5)
```

$$\begin{aligned} \mathbf{R}_{\hat{\hat{\sigma}}} &= \hat{\hat{\boldsymbol{\sigma}}}_{n+1} + \Delta\lambda \hat{\mathbf{C}}_0 \mathbf{m} - \hat{\hat{\boldsymbol{\sigma}}}_{n+1}^{trial} \\ \mathbf{R}_{\kappa} &= \boldsymbol{\kappa}_{n+1} - \Delta\lambda\, \mathbf{h} - \boldsymbol{\kappa}_n \\ \mathbf{R}_{\Delta\lambda} &= F\left(\hat{\hat{\boldsymbol{\sigma}}}_{n+1}, \boldsymbol{\kappa}_{n+1}\right) \end{aligned}$$

$$\boxed{F(\bar{\boldsymbol{\sigma}}, \tilde{\boldsymbol{\varepsilon}}^{pl}) = \frac{1}{1-\alpha}\left(\bar{q} - 3\alpha\bar{p} + \beta(\tilde{\boldsymbol{\varepsilon}}^{pl})\langle\hat{\bar{\sigma}}_{\max}\rangle - \gamma\langle-\hat{\bar{\sigma}}_{\max}\rangle\right) - \bar{\sigma}_c(\tilde{\varepsilon}_c^{pl}) \leq 0,}$$

```
                pi1=sig_pr(1)+sig_pr(2)+sig_pr(3)
                sde_pr=sig_pr-pi1/thr
                pj2=(sde_pr(1)**2+sde_pr(2)**2
 %+sde_pr(3)**2)/tw
                call kaxialy(fc_eff,ft_eff,fc,ft,hc,ht,hc_eff,
 $ ht_eff,ddt_dept,ddc_depc,epc,ept,dc,dt,fto,fco,Em,tol,tol2)
                beta=fc_eff/ft_eff*(on-alpha)-(on+alpha);
                ff=(alpha*pi1+sqrt(thr*pj2)+beta*(sig_pr(1)
 %+abs(sig_pr(1)))/tw-gamma*(abs(sig_pr(1))-sig_pr(1))/tw)
 $/(on-alpha)-fc_eff
                ress(6)=ff;res_n=sqrt(ress(1)**2+ress(2)**2+
 $ress(3)**2+ress(4)**2+ress(5)**2+(ress(6)/Em/tol2)**2)
                res_n1=sqrt(ress(1)**2+ress(2)**2+ress(3)**2)+on

      if (iter==200) then
            write(*,*) 'trials alot'
            !exit
            call xit
      end if
end do
```

$$\boxed{\left\|\mathbf{R}(\mathbf{x}^{(k+1)})\right\| \leq \mathrm{TOL}}$$

$$\mathbf{J}\begin{pmatrix} \mathrm{d}\hat{\hat{\boldsymbol{\sigma}}}_{n+1} \\ \mathrm{d}\boldsymbol{\kappa}_{n+1} \\ \mathrm{d}\Delta\lambda \end{pmatrix} = \begin{pmatrix} \hat{\mathbf{C}}_0 \mathrm{d}\hat{\boldsymbol{\varepsilon}}_{n+1} \\ \mathbf{0}_{n\kappa} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \mathrm{d}\hat{\hat{\boldsymbol{\sigma}}}_{n+1} \\ \mathrm{d}\boldsymbol{\kappa}_{n+1} \\ \mathrm{d}\Delta\lambda \end{pmatrix} = \begin{bmatrix} \hat{\boldsymbol{\Xi}}_{\hat{\hat{\sigma}}} & \bullet & \bullet \\ \hat{\boldsymbol{\Xi}}_{\kappa} & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{bmatrix} \begin{pmatrix} \hat{\mathbf{C}}_0 \mathrm{d}\hat{\boldsymbol{\varepsilon}}_{n+1} \\ \mathbf{0}_{n\kappa,1} \\ 0 \end{pmatrix}$$

$$\mathbf{J} = \begin{bmatrix} \mathbf{I}_{n\hat{\hat{\sigma}}} + \Delta\lambda \hat{\mathbf{C}}_0 \dfrac{\partial \mathbf{m}}{\partial \hat{\hat{\boldsymbol{\sigma}}}}; & \Delta\lambda \hat{\mathbf{C}}_0 \dfrac{\partial \mathbf{m}}{\partial \boldsymbol{\kappa}}; & \hat{\mathbf{C}}_0 \mathbf{m} \\ -\Delta\lambda \dfrac{\partial \mathbf{h}}{\partial \hat{\hat{\boldsymbol{\sigma}}}} & \mathbf{I}_{n\kappa} - \Delta\lambda \dfrac{\partial \mathbf{h}}{\partial \boldsymbol{\kappa}} & -\mathbf{h} \\ \left\{\partial F / \partial \hat{\hat{\boldsymbol{\sigma}}}\right\}^{\mathrm{T}} & \{\partial F / \partial \boldsymbol{\kappa}\}^{\mathrm{T}} & 0 \end{bmatrix}$$

```
          pjac2=pjac;


          call kinverse(pjac2(1:5,1:5),px55,5)
       pjac2=pjac;
       call kinverse(pjac2,pjac2,6)
       write(*,*) 'px55',px55
       dx5=matmul(px55,pjac(1:5,6))
       dx55=matmul(transpose(px55),pjac(6,1:5))
       par=dx5(1)*pjac(6,1)+dx5(2)*pjac(6,2)+
$dx5(3)*pjac(6,3)+dx5(4)*pjac(6,4)+dx5(5)*pjac(6,5)
       do ii=1,3
         px33(ii,:)=px55(ii,1:3)-dx5(ii)*dx55(1:3)/par
       end do
do ii=1,2
  px23(ii,:)=px55(ii+3,1:3)-dx5(ii+3)*dx55(1:3)/par
end do
dd=on-(on-dc)*(on-dt)
ddc=ddc_depc*(on-dt);ddt=ddt_dept*(on-dc)
do i=1,3
do j=1,3
 yx33(i,j)=sig_pr(i)*(px23(1,j)*ddt+px23(2,j)*ddc)
end do
end do

sig_tr=matmul(transpose(dd2),sig_pr)
sig=(on-dd)*sig_tr;
px33=(on-dd)*px33-yx33
```

$$\frac{\mathrm{d}\hat{\hat{\boldsymbol{\sigma}}}_{n+1}}{\mathrm{d}\hat{\boldsymbol{\varepsilon}}_{n+1}} = \hat{\boldsymbol{\Xi}}_{\hat{\hat{\sigma}}} \hat{\mathbf{C}}_0$$

$$\frac{\mathrm{d}\boldsymbol{\kappa}_{n+1}}{\mathrm{d}\hat{\boldsymbol{\varepsilon}}_{n+1}} = \hat{\boldsymbol{\Xi}}_{\kappa} \hat{\mathbf{C}}_0$$

$$(1-d) = (1 - s_t d_c)(1 - s_c d_t), \quad 0 \le s_t,\ s_c \le 1,$$

$$s_t = 1 - w_t r^*(\bar{\sigma}_{11}); \quad 0 \le w_t \le 1,$$
$$s_c = 1 - w_c(1 - r^*(\bar{\sigma}_{11})); \quad 0 \le w_c \le 1,$$

$$D_{n+1} = D\left(\boldsymbol{\kappa}_{n+1}\right)$$
$$\boldsymbol{\sigma}_{n+1} = (1 - D_{n+1})\, \bar{\boldsymbol{\sigma}}_{n+1}.$$

$$\frac{\mathrm{d}\boldsymbol{\sigma}_{n+1}}{\mathrm{d}\boldsymbol{\varepsilon}_{n+1}} = -\bar{\boldsymbol{\sigma}}_{n+1} \frac{\mathrm{d}D_{n+1}}{\mathrm{d}\boldsymbol{\kappa}_{n+1}} \frac{\mathrm{d}\boldsymbol{\kappa}_{n+1}}{\mathrm{d}\boldsymbol{\varepsilon}_{n+1}} + (1 - D_{n+1}) \frac{\mathrm{d}\bar{\boldsymbol{\sigma}}_{n+1}}{\mathrm{d}\boldsymbol{\varepsilon}_{n+1}}$$

$$\frac{\mathrm{d}\hat{\boldsymbol{\sigma}}_{n+1}}{\mathrm{d}\hat{\boldsymbol{\varepsilon}}_{n+1}} = \left[(1 - D_{n+1})\, \boldsymbol{\Xi}_{\hat{\hat{\sigma}}} - \hat{\hat{\boldsymbol{\sigma}}}_{n+1} \frac{\mathrm{d}D_{n+1}}{\mathrm{d}\boldsymbol{\kappa}_{n+1}} \boldsymbol{\Xi}_{\kappa}\right] \hat{\mathbf{C}}_0$$

```
      call kinverse(px33,px33,3)
      e_tr=matmul(px33,sig_pr)
      !!!!!!!!!!!!!!!!!!!
      do i=1,2
        do j=i+1,3
          iii=iii+1
          dir63(1,iii)=2.0*an(i,1)*an(j,1)
          dir63(2,iii)=2.0*an(i,2)*an(j,2)
          dir63(3,iii)=2.0*an(i,3)*an(j,3)
          dir63(4,iii)=an(i,1)*an(j,2)+an(i,2)*an(j,1)
          dir63(5,iii)=an(i,1)*an(j,3)+an(i,3)*an(j,1)
          dir63(6,iii)=an(i,3)*an(j,2)+an(i,2)*an(j,3)
          if (abs(e_tr(i)-e_tr(j))<tol) then
            rati=cc(i,i)-cc(i,j)
          else
            rati=(sig_pr(i)-sig_pr(j))/(e_tr(i)-e_tr(j))
          end if
          do ii=1,6
           do jj=1,6
              ddsdde(ii,jj)=ddsdde(ii,jj)
$+0.5*(on-dd)*rati*dir63(ii,iii)*dir63(jj,iii)
           end do
          end do
        end do
      end do
    end if
    write(*,*) 'ddsdde',ddsdde
    statev(1)=ept;statev(2)=epc;statev(3)=dd;
    call xit
    return
```

$$\mathbf{m}_A = \mathbf{v}_A^{\mathrm{T}}\mathbf{v}_A, \qquad \mathbf{m}_{AB} = \mathbf{v}_A^{\mathrm{T}}\mathbf{v}_B, \quad A \neq B.$$

$$\boxed{\begin{aligned}\frac{\mathrm{d}\boldsymbol{\sigma}_{n+1}}{\mathrm{d}\boldsymbol{\varepsilon}_{n+1}} &= \sum_{A=1}^{3}\sum_{B=1}^{3}\frac{\mathrm{d}\hat{\sigma}_A}{\mathrm{d}\hat{\varepsilon}_B}\mathbf{m}_A^{\mathrm{T}}\mathbf{m}_B + \frac{(1-D_{n+1})}{2}\\ &\times \sum_{A=1}^{3}\sum_{B\neq A}\left[\left(\frac{\hat{\bar{\sigma}}_B - \hat{\bar{\sigma}}_A}{\hat{\varepsilon}_B - \hat{\varepsilon}_A}\right)(\mathbf{m}_{AB}^{\mathrm{T}}\mathbf{m}_{AB} + \mathbf{m}_{AB}^{\mathrm{T}}\mathbf{m}_{BA})\right]\end{aligned}}$$

$\left(\hat{\bar{\sigma}}_B - \hat{\bar{\sigma}}_A\right) / \left(\hat{\varepsilon}_B - \hat{\varepsilon}_A\right)$ by $\partial\left(\hat{\bar{\sigma}}_B - \hat{\bar{\sigma}}_A\right)/\partial\hat{\varepsilon}_B$

```
subroutine korder(ps,an,dd1,dd2,ndi)
      real*8 ps(3),an(3,3),dd1(3,6),dd2(3,6),temp11(3),temp1
      integer ndi
      do ii=1,ndi
            if (ps(ii)>ps(1)) then
                  temp1=ps(1);ps(1)=ps(ii);ps(ii)=temp1
                  temp11=an(1,:);an(1,:)=an(ii,:);
                  an(ii,:)=temp11
            end if
            if (ps(ii)<ps(ndi)) then
                  temp1=ps(ndi);ps(ndi)=ps(ii);ps(ii)=temp1
                  temp11=an(ndi,:);an(ndi,:)=an(ii,:);
                  an(ii,:)=temp11
            end if
      end do
      do ii=1,ndi
            dd1(ii,1)=an(ii,1)*an(ii,1);
            dd1(ii,2)=an(ii,2)*an(ii,2);
            dd1(ii,3)=an(ii,3)*an(ii,3);
            dd1(ii,4)=2.*an(ii,1)*an(ii,2);
            dd1(ii,5)=2.*an(ii,1)*an(ii,3);
            dd1(ii,6)=2.*an(ii,2)*an(ii,3);
      end do
      dd2=dd1;dd2(:,4:6)=0.5*dd1(:,4:6)
      !ds_pr=dd1*ds     ds=transpose(dd2)*ds_pr
 return
 end
```

```
 subroutine kaxialy(fc_eff,ft_eff,fc,ft,hc,ht,hc_eff,ht_eff,
$ ddt_dept,ddc_depc,epc,ept,dc,dt,fto,fco,Em,tol,tol2)
      real*8 fc_eff,ft_eff,fc,ft,hc,ht,hc_eff,ht_eff,
$ ddt_dept,ddc_depc,epc,ept,dc,dt,fto,fco,Em,faci,tol,tol2
      pbb=0.002-fco/Em;pcc=0.01;faci=100.0
      if (abs(epc)<tol/tol2**5) then
            fc_eff=0.4*fco;fc=fc_eff
            hc_eff=Em*faci;hc=Em*faci;
            ddc_depc=0.0;dc=0.0;
      elseif (epc<pbb) then
            paa=epc/pbb;fc_eff=0.4+(0.6-0.01)*paa**3+
$(0.01-0.6)*3.0*paa**2+(0.6*3.0-0.01*2.0)*paa;
            fc_eff=fc_eff*fco;fc=fc_eff;
            hc_eff=(0.6-0.01)*3.*paa**2+(0.01-0.6)*6.0*paa
$+(0.6*3.0-0.01*2.0);hc_eff=hc_eff*fco/pbb;hc=hc_eff;
            ddc_depc=0.0;dc=0.0
      elseif (epc>=pbb.and.epc<pbb*5.0) then
            paa=epc/pbb;fc_eff=(1.0+0.01*(paa-1.0))*fco;
            fc=(1.0-0.95/4.0*(paa-1))*fco;
            hc_eff=0.01*fco/pbb;hc=-0.95/4.0*fco/pbb;
            ddc_depc=-hc/fc_eff;
            dc=1.0-fc/fc_eff;
            !write(*,*) 'hc',hc,dc,ddc_depc,pbb,paa,fc

       else
             paa=epc/pbb;fc_eff=(1.0+0.01*(paa-1.0))*fco;
             fc=0.05*fco;
             hc_eff=0.01*fco/pbb;hc=0.0;
             ddc_depc=0.0;dc=1.0-fc/fc_eff;
       end if
       if (abs(ept)<tol/tol2**5) then
             ft_eff=fto;ft=ft_eff;
             ht_eff=Em*faci;ht=Em*faci;
             ddt_dept=0.0;dt=0.0;
       elseif (ept<pcc*0.9) then
             ft_eff=fto+0.01*fco/pbb*ept;
             ft=fto*(pcc-ept)/pcc;
             ht_eff=0.01*fco/pbb;ht=-1.0*fto/pcc;
             ddt_dept=-ht/fto;dt=1.0-ft/ft_eff;
       else
             ft_eff=fto+0.01*fco/pbb*ept;
             ft=fto*0.1;
             ht_eff=0.01*fco/pbb;ht=0.0;
             ddt_dept=0.0;dt=1.0-ft/ft_eff;
       end if
```

```
subroutine kinverse(a,c,n)
       !============================================================
       ! kinverse matrix
       ! Method: Based on Doolittle LU factorization for Ax=b
       ! Alex G. December 2009
       !-----------------------------------------------------------
       ! input ...
       ! a(n,n) - array of coefficients for matrix A
       ! n      - dimension
       ! output ...
       ! c(n,n) - kinverse matrix of A
       ! comments ...
       ! the original matrix a(n,n) will be destroyed
       ! during the calculation

subroutine gauss_2(a,b,x,n)
      !============================================================
           ! Solutions to a system of linear equations A*x=b
           ! Method: Gauss elimination (with scaling and pivoting)
           ! Alex G. (November 2009)
           !-----------------------------------------------------
```