Maths Assignment

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Problem Statement

Write the first five terms of the sequence $a_n = \frac{n(n^2+5)}{4}$.

Solution

The relation between x(n) and u(n):

$$x(n) = \left(\frac{(n+1)^3 + 5(n+1)}{4}\right) u(n) \tag{1}$$

Given that the unit step function $u(n)$ is:

$$u(n) = \begin{cases} 0 & \text{if } n < 0 \\ 1 & \text{if } n \geq 0 \end{cases} \tag{2}$$

Its Z-transform becomes:

$$U(z) = \sum_{n=0}^{\infty} z^{-n} = 1 + z^{-1} + z^{-2} + z^{-3} + \cdots \tag{3}$$

$$U(z) = \frac{1}{1 - z^{-1}} \tag{4}$$

ROC for the z transform of u(n):
ROC of U(z):$|z| > 1$
The Z-transform of $nu(n)$ is given by:

$$\mathcal{Z}\{nu(n)\} = -\frac{1}{z^{-1}} \frac{d}{dz}[U(z)] \tag{5}$$

$$\mathcal{Z}\{nu(n)\} = \frac{z^{-1}}{(1 - z^{-1})^2} \tag{6}$$

ROC is $|z| > 1$
The Z-transform of $n^2u(n)$ is given by:

$$\mathcal{Z}\{n^2u(n)\} = \frac{1}{z^{-1}}\frac{d}{dz}[U(z)] + \frac{1}{z^{-2}}\frac{d^2}{dz^2}[U(z)] \tag{7}$$

$$\mathcal{Z}\{n^2u(n)\} = \frac{(z^{-1})(1+z^{-1})}{(1-z^{-1})^3} \tag{8}$$

ROC is $|z| > 1$
The Z-transform of $n^3u(n)$ is given by:

$$\mathcal{Z}\{n^3u(n)\} = -\frac{1}{z^{-1}}\frac{d}{dz}[U(z)] - \frac{3}{z^{-2}}\frac{d^2}{dz^2}[U(z)] - \frac{1}{z^{-3}}\frac{d^3}{dz^3}[U(z)] \tag{9}$$

$$\mathcal{Z}\{n^3u(n)\} = \frac{(z^{-1})(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4} \tag{10}$$

ROC is $|z| > 1$
Now Z-transform of x(n) is given by:

$$\mathcal{X}\{z\} = \frac{\mathcal{Z}\{n^3u(n)\}}{4} + \frac{3\mathcal{Z}\{n^2u(n)\}}{4} + 2\mathcal{Z}\{nu(n)\} + \frac{3\mathcal{Z}\{u(n)\}}{2} \tag{11}$$

$$\mathcal{X}\{z\} = \frac{(z^{-1})(1+4z^{-1}+z^{-2})}{4(1-z^{-1})^4} + \frac{3(z^{-1})(1+z^{-1})}{4(1-z^{-1})^3} + \frac{2z^{-1}}{(1-z^{-1})^2} + \frac{3}{2(1-z^{-1})} \tag{12}$$

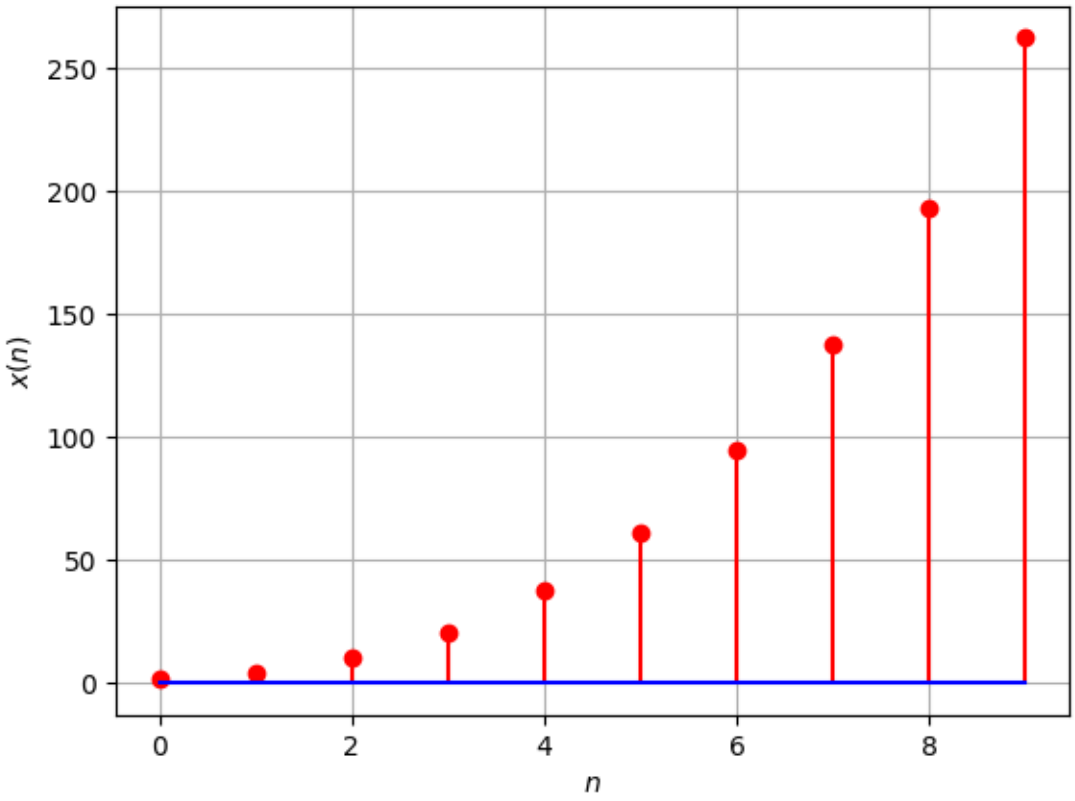

Figure 1: Plot of equation(1)