Physics Assignment

Karyampudi Meghana Sai
EE23BTECH11031

January 8, 2024

Problem Statement

Write the first five terms of the sequence $a_n = \frac{n(n^2+5)}{4}$.

Solution

The sequence $x(n) = \frac{n(n^2+5)}{4}$ starting from $n = 0$ is:

$$\begin{aligned}
x(0) &= \frac{0(0^2+5)}{4} = \frac{0}{4} = 0 \\
x(1) &= \frac{1(1^2+5)}{4} = \frac{6}{4} = 1.5 \\
x(2) &= \frac{2(2^2+5)}{4} = \frac{18}{4} = 4.5 \\
x(3) &= \frac{3(3^2+5)}{4} = \frac{42}{4} = 10.5 \\
x(4) &= \frac{4(4^2+5)}{4} = \frac{84}{4} = 21 \\
x(5) &= \frac{5(5^2+5)}{4} = \frac{150}{4} = 37.5
\end{aligned}$$

The relation between x(n) and u(n):

$$x(n) = \left(\frac{n^3+5n}{4}\right) \cdot u(n) \tag{1}$$

Finding the z transform of u(n):

$$U(z) = \mathcal{Z}\{u(n)\} = \sum_{n=0}^{\infty} u(n) z^{-n}$$

Given that the unit step function $u(n)$ is:

$$u(n) = \begin{cases} 0 & \text{if } n < 0 \\ 1 & \text{if } n \geq 0 \end{cases}$$

Its Z-transform becomes:

$$\begin{aligned} U(z) &= \sum_{n=0}^{\infty} z^{-n} = 1 + z^{-1} + z^{-2} + z^{-3} + \cdots \\ &= \frac{1}{1 - z^{-1}} \\ U(z) &= \frac{1}{1 - z^{-1}} \end{aligned} \tag{2}$$

ROC for the z transform of u(n):
ROC for U(z):$|z| > 1$

The z transform of $nu(n)$ can be derived as follows:

Given the signal $nu(n)$, its z transform is represented as:

$$\mathcal{Z}\{nu(n)\} = \sum_{n=0}^{\infty} nu(n) z^{-n}$$

The signal $nu(n)$ means the product of n and the unit step function $u(n)$:

$$nu(n) = n \cdot u(n)$$

The z transform of $u(n)$ is $U(z) = \frac{z}{z-1}$. To find the z transform of $nu(n)$, let's derive it step by step:

$$\begin{aligned} \mathcal{Z}\{nu(n)\} &= \sum_{n=0}^{\infty} nu(n) z^{-n} \\ &= \sum_{n=0}^{\infty} n \cdot u(n) z^{-n} \end{aligned}$$

Now, this expression involves the convolution property of the z transform. The z transform of n is derived separately as:

$$\mathcal{Z}\{n\} = \sum_{n=0}^{\infty} n z^{-n-1} = \frac{z^{-1}}{(1 - z^{-1})^2} \tag{3}$$

By convolving the z transform of n with $U(z)$, we get:

$$\mathcal{Z}\{nu(n)\} = U(z) * \mathcal{Z}\{n\}$$

Therefore, after convolution:

$$\mathcal{Z}\{nu(n)\} = U(z) \cdot \mathcal{Z}\{n\} = \frac{1}{1-z^{-1}} \cdot \frac{z^{-1}}{(1-z^{-1})^2}$$

Simplifying the expression:

$$\mathcal{Z}\{nu(n)\} = \frac{z^{-1}}{(1-z^{-1})^3} \tag{4}$$

Given: $x(n) = n^3u(n)$
The z transform 0f $n^3u(n)$ can be derived as follows:

$$\mathcal{Z}\{n^3u(n)\} = \sum_{n=0}^{\infty} n^3u(n)z^{-n} \tag{5}$$

Now, n^3 can be represented as a signal n^3 convolved with the unit step function $u(n)$:

$$n^3u(n) = n^3 * u(n)$$

Using the property that $\mathcal{Z}\{f(n) * g(n)\} = F(z)G(z)$:

$$\mathcal{Z}\{n^3 * u(n)\} = \mathcal{Z}\{n^3\} \cdot \mathcal{Z}\{u(n)\}$$

The z transform of n^3 is found to be:

$$\mathcal{Z}\{n^3\} = \sum_{n=0}^{\infty} n^3z^{-n-1} = \frac{(z^{-1})(1+z^{-1})(1+2z^{-1})}{(1-z^{-1})^4} \tag{6}$$

Therefore, the z transform of $n^3u(n)$ can be obtained by multiplying the z transforms of n^3 and $u(n)$:

$$\begin{aligned}
\mathcal{Z}\{n^3u(n)\} &= \mathcal{Z}\{n^3\} \cdot \mathcal{Z}\{u(n)\} \\
&= \frac{(z^{-1})(1+z^{-1})(1+2z^{-1})}{(1-z^{-1})^4} \frac{1}{1-z^{-1}} \\
\mathcal{Z}\{n^3u(n)\} &= \frac{(z^{-1})(1+z^{-1})(1+2z^{-1})}{(1-z^{-1})^5} \qquad (7)
\end{aligned}$$

Z-Transform of x(n):

$$X(z) = \frac{\mathcal{Z}\{n^3u(n)\}}{4} + \frac{\mathcal{Z}\{5nu(n)\}}{4} \tag{8}$$

$$X(z) = \frac{(z^{-1})(1+z^{-1})(1+2z^{-1})}{4(1-z^{-1})^5} + \frac{5z^{-1}}{4(1-z^{-1})^3} \tag{9}$$