Maths Assignment

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Problem Statement

Write the first five terms of the sequence $a_n = \frac{n(n^2+5)}{4}$.

Solution

The relation between x(n) and u(n):

$$x(n) = \left(\frac{(n+1)^3 + 5(n+1)}{4}\right)u(n) \tag{1}$$

Z-transform of $n^k u(k)$ in terms of the k-th derivative of U(z):

$$n^k u(n) \stackrel{\text{ZT}}{\longleftrightarrow} (-1)^k z^k \frac{d^k}{dz^k} U(z)$$
 (2)

$$\mathcal{Z}\{nu(n)\} = \frac{z^{-1}}{(1-z^{-1})^2} \text{ [ROC: } |z| > 1]$$
(3)

$$\mathcal{Z}\{n^2 u(n)\} = \frac{(z^{-1})(1+z^{-1})}{(1-z^{-1})^3} [ROC: |z| > 1]$$
(4)

$$\mathcal{Z}\{n^3 u(n)\} = \frac{(z^{-1})(1 + 4z^{-1} + z^{-2})}{(1 - z^{-1})^4} \text{ [ROC: } |z| > 1]$$
 (5)

Referencing the equations (3), (4), and (5).

$$\mathcal{X}\{z\} = \frac{(z^{-1})(1+4z^{-1}+z^{-2})}{4(1-z^{-1})^4} + \frac{3(z^{-1})(1+z^{-1})}{4(1-z^{-1})^3} + \frac{2z^{-1}}{(1-z^{-1})^2} + \frac{3}{2(1-z^{-1})}$$

$$\mathcal{X}\{z\} = \frac{3}{2(1-z^{-1})^3} + \frac{3z^{-2}}{2(1-z^{-1})^4} \tag{7}$$

