Maths Assignment

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Problem Statement

Write the first five terms of the sequence $a_n = \frac{n(n^2+5)}{4}$.

Solution

The sequence x(n):

$$x(n) = \frac{(n+1)((n+1)^2+5)}{4} \tag{1}$$

$$x(n) = \frac{(n+1)^3+5(n+1)}{4} \tag{2}$$

$$x(n) = \frac{n^3+3n^2+8n+6}{4} \tag{3}$$

The relation between x(n) and u(n):

$$x(n) = \left(\frac{(n+1)^3+5(n+1)}{4}\right) \cdot u(n) \tag{4}$$

Finding the z transform of u(n):

$$U(z) = \mathcal{Z}\{u(n)\} = \sum_{n=0}^{\infty} u(n) z^{-n} \tag{5}$$

Given that the unit step function $u(n)$ is:

$$u(n) = \begin{cases} 0 & \text{if } n < 0 \\ 1 & \text{if } n \geq 0 \end{cases} \tag{6}$$

Its Z-transform becomes:

$$U(z) = \sum_{n=0}^{\infty} z^{-n} = 1 + z^{-1} + z^{-2} + z^{-3} + \cdots \tag{7}$$

$$U(z) = \frac{1}{1 - z^{-1}} \tag{8}$$

ROC for the z transform of u(n):
ROC for U(z):$|z| > 1$

The z transform of $nu(n)$ can be derived as follows:

Given the signal $nu(n)$, its z transform is represented as:

$$\mathcal{Z}\{nu(n)\} = \sum_{n=0}^{\infty} nu(n) z^{-n} \tag{9}$$

The signal $nu(n)$ means the product of n and the unit step function $u(n)$:

$$nu(n) = n \cdot u(n) \tag{10}$$

The z transform of $u(n)$ is $U(z) = \frac{z}{z-1}$. To find the z transform of $nu(n)$, let's derive it step by step:

$$\mathcal{Z}\{nu(n)\} = \sum_{n=0}^{\infty} nu(n) z^{-n} \tag{11}$$

$$\mathcal{Z}\{nu(n)\} = \sum_{n=0}^{\infty} n \cdot u(n) z^{-n} \tag{12}$$

Now, this expression involves the convolution property of the z transform. The z transform of n is derived separately as:

$$\mathcal{Z}\{n\} = \sum_{n=0}^{\infty} n z^{-n-1} = \frac{z^{-1}}{(1 - z^{-1})^2} \tag{13}$$

By convolving the z transform of n with $U(z)$, we get:

$$\mathcal{Z}\{nu(n)\} = U(z) * \mathcal{Z}\{n\} \tag{14}$$

Therefore, after convolution:

$$\mathcal{Z}\{nu(n)\} = U(z) \cdot \mathcal{Z}\{n\} = \frac{1}{1 - z^{-1}} \cdot \frac{z^{-1}}{(1 - z^{-1})^2} \tag{15}$$

Simplifying the expression:

$$\mathcal{Z}\{nu(n)\} = \frac{z^{-1}}{(1-z^{-1})^3} \tag{16}$$

The z transform 0f $n^2u(n)$ can be derived as follows:

$$\mathcal{Z}\{n^2u(n)\} = \sum_{n=0}^{\infty} n^2 u(n) z^{-n} \tag{17}$$

Now, n^2 can be represented as a signal n^2 convolved with the unit step function $u(n)$:

$$n^2u(n) = n^2 * u(n) \tag{18}$$

Using the property that $\mathcal{Z}\{f(n) * g(n)\} = F(z)G(z)$:

$$\mathcal{Z}\{n^2 * u(n)\} = \mathcal{Z}\{n^2\} \cdot \mathcal{Z}\{u(n)\} \tag{19}$$

The z transform of n^2 is found to be:

$$\mathcal{Z}\{n^2\} = \sum_{n=0}^{\infty} n^2 z^{-n-1} = \frac{(z^{-1})(1+z^{-1})}{(1-z^{-1})^3} \tag{20}$$

Therefore, the z transform of $n^2u(n)$ can be obtained by multiplying the z transforms of n^2 and $u(n)$:

$$\mathcal{Z}\{n^2u(n)\} = \mathcal{Z}\{n^2\} \cdot \mathcal{Z}\{u(n)\} \tag{21}$$

$$\mathcal{Z}\{n^2u(n)\} = \frac{(z^{-1})(1+z^{-1})}{(1-z^{-1})^3} \frac{1}{1-z^{-1}} \tag{22}$$

$$\mathcal{Z}\{n^2u(n)\} = \frac{(z^{-1})(1+z^{-1})}{(1-z^{-1})^4} \tag{23}$$

The z transform 0f $n^3u(n)$ can be derived as follows:

$$\mathcal{Z}\{n^3u(n)\} = \sum_{n=0}^{\infty} n^3 u(n) z^{-n} \tag{24}$$

Now, n^3 can be represented as a signal n^3 convolved with the unit step function $u(n)$:

$$n^3u(n) = n^3 * u(n) \tag{25}$$

Using the property that $\mathcal{Z}\{f(n) * g(n)\} = F(z)G(z)$:

$$\mathcal{Z}\{n^3 * u(n)\} = \mathcal{Z}\{n^3\} \cdot \mathcal{Z}\{u(n)\} \tag{26}$$

The z transform of n^3 is found to be:

$$\mathcal{Z}\{n^3\} = \sum_{n=0}^{\infty} n^3 z^{-n-1} = \frac{(z^{-1})(1+z^{-1})(1+2z^{-1})}{(1-z^{-1})^4} \tag{27}$$

Therefore, the z transform of $n^3u(n)$ can be obtained by multiplying the z transforms of n^3 and $u(n)$:

$$\mathcal{Z}\{n^3u(n)\} = \mathcal{Z}\{n^3\} \cdot \mathcal{Z}\{u(n)\} \tag{28}$$

$$\mathcal{Z}\{n^3u(n)\} = \frac{(z^{-1})(1+z^{-1})(1+2z^{-1})}{(1-z^{-1})^4} \frac{1}{1-z^{-1}} \tag{29}$$

$$\mathcal{Z}\{n^3u(n)\} = \frac{(z^{-1})(1+z^{-1})(1+2z^{-1})}{(1-z^{-1})^5} \tag{30}$$

Z-Transform of x(n):

$$X(z) = \frac{\mathcal{Z}\{n^3u(n)\}}{4} + \frac{\mathcal{Z}\{3n^2u(n)\}}{4} + \frac{\mathcal{Z}\{8nu(n)\}}{4} + \frac{\mathcal{Z}\{6u(n)\}}{4} \tag{31}$$

$$X(z) = \frac{(z^{-1})(1+z^{-1})(1+2z^{-1})}{4(1-z^{-1})^5} + \frac{3(z^{-1})(1+z^{-1})}{4(1-z^{-1})^4} + \frac{8z^{-1}}{4(1-z^{-1})^3} + \frac{6}{4(1-z^{-1})} \tag{32}$$

Here's the plot of x(n):

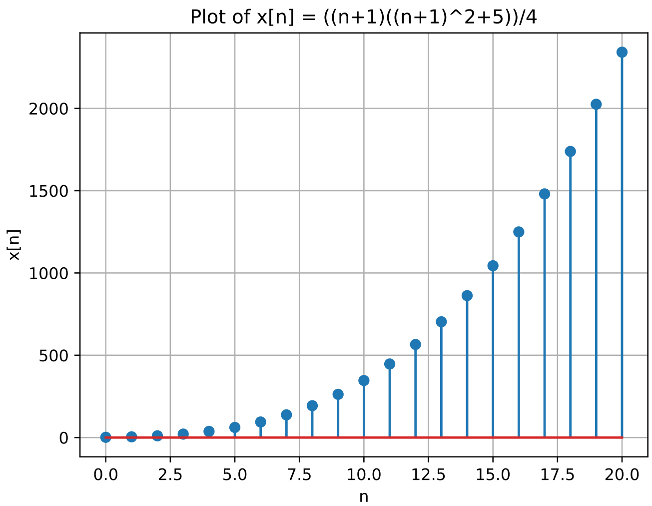

Figure 1: Plot of the function $x(n) = \frac{(n+1)((n+1)^2+5)}{4}$