

# Physics Assignment

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## Problem Statement

Write the first five terms of the sequence  $a_n = \frac{n(n^2+5)}{4}$ .

## Solution

The sequence  $x(n) = \frac{n(n^2+5)}{4}$  starting from  $n = 0$  is:

$$\begin{aligned}x(0) &= \frac{1(1^2 + 5)}{4} = \frac{6}{4} = 1.5 \\x(1) &= \frac{2(2^2 + 5)}{4} = \frac{18}{4} = 4.5 \\x(2) &= \frac{3(3^2 + 5)}{4} = \frac{42}{4} = 10.5 \\x(3) &= \frac{4(4^2 + 5)}{4} = \frac{84}{4} = 21 \\x(4) &= \frac{5(5^2 + 5)}{4} = \frac{150}{4} = 37.5\end{aligned}$$

The relation between  $x(n)$  and  $u(n)$ :

$$x(n) = \left( \frac{n^3 + 5n}{4} \right) \cdot u(n) \quad (1)$$

Finding the z transform of  $u(n)$ :

$$U(z) = \mathcal{Z}\{u(n)\} = \sum_{n=0}^{\infty} u(n)z^{-n} \quad (2)$$

Given that the unit step function  $u(n)$  is:

$$u(n) = \begin{cases} 0 & \text{if } n < 0 \\ 1 & \text{if } n \geq 0 \end{cases} \quad (3)$$

Its Z-transform becomes:

$$\begin{aligned}
 U(z) &= \sum_{n=0}^{\infty} z^{-n} = 1 + z^{-1} + z^{-2} + z^{-3} + \dots \\
 &= \frac{1}{1 - z^{-1}} \\
 U(z) &= \frac{1}{1 - z^{-1}} \tag{4}
 \end{aligned}$$

ROC for the z transform of  $u(n)$ :

ROC for  $U(z): |z| > 1$

The  $z$  transform of  $nu(n)$  can be derived as follows:

Given the signal  $nu(n)$ , its  $z$  transform is represented as:

$$\mathcal{Z}\{nu(n)\} = \sum_{n=0}^{\infty} nu(n)z^{-n} \tag{5}$$

The signal  $nu(n)$  means the product of  $n$  and the unit step function  $u(n)$ :

$$nu(n) = n \cdot u(n) \tag{6}$$

The  $z$  transform of  $u(n)$  is  $U(z) = \frac{z}{z-1}$ . To find the  $z$  transform of  $nu(n)$ , let's derive it step by step:

$$\begin{aligned}
 \mathcal{Z}\{nu(n)\} &= \sum_{n=0}^{\infty} nu(n)z^{-n} \\
 \mathcal{Z}\{nu(n)\} &= \sum_{n=0}^{\infty} n \cdot u(n)z^{-n} \tag{7}
 \end{aligned}$$

Now, this expression involves the convolution property of the  $z$  transform. The  $z$  transform of  $n$  is derived separately as:

$$\mathcal{Z}\{n\} = \sum_{n=0}^{\infty} nz^{-n-1} = \frac{z^{-1}}{(1 - z^{-1})^2} \tag{8}$$

By convolving the  $z$  transform of  $n$  with  $U(z)$ , we get:

$$\mathcal{Z}\{nu(n)\} = U(z) * \mathcal{Z}\{n\} \tag{9}$$

Therefore, after convolution:

$$\mathcal{Z}\{nu(n)\} = U(z) \cdot \mathcal{Z}\{n\} = \frac{1}{1-z^{-1}} \cdot \frac{z^{-1}}{(1-z^{-1})^2} \quad (10)$$

Simplifying the expression:

$$\mathcal{Z}\{nu(n)\} = \frac{z^{-1}}{(1-z^{-1})^3} \quad (11)$$

Given:  $x(n) = n^3 u(n)$

The  $z$  transform of  $n^3 u(n)$  can be derived as follows:

$$\mathcal{Z}\{n^3 u(n)\} = \sum_{n=0}^{\infty} n^3 u(n) z^{-n} \quad (12)$$

Now,  $n^3$  can be represented as a signal  $n^3$  convolved with the unit step function  $u(n)$ :

$$n^3 u(n) = n^3 * u(n)$$

Using the property that  $\mathcal{Z}\{f(n) * g(n)\} = F(z)G(z)$ :

$$\mathcal{Z}\{n^3 * u(n)\} = \mathcal{Z}\{n^3\} \cdot \mathcal{Z}\{u(n)\} \quad (13)$$

The  $z$  transform of  $n^3$  is found to be:

$$\mathcal{Z}\{n^3\} = \sum_{n=0}^{\infty} n^3 z^{-n-1} = \frac{(z^{-1})(1+z^{-1})(1+2z^{-1})}{(1-z^{-1})^4} \quad (14)$$

Therefore, the  $z$  transform of  $n^3 u(n)$  can be obtained by multiplying the  $z$  transforms of  $n^3$  and  $u(n)$ :

$$\begin{aligned} \mathcal{Z}\{n^3 u(n)\} &= \mathcal{Z}\{n^3\} \cdot \mathcal{Z}\{u(n)\} \\ &= \frac{(z^{-1})(1+z^{-1})(1+2z^{-1})}{(1-z^{-1})^4} \cdot \frac{1}{1-z^{-1}} \\ \mathcal{Z}\{n^3 u(n)\} &= \frac{(z^{-1})(1+z^{-1})(1+2z^{-1})}{(1-z^{-1})^5} \end{aligned} \quad (15)$$

Z-Transform of  $x(n)$ :

$$X(z) = \frac{\mathcal{Z}\{n^3 u(n)\}}{4} + \frac{\mathcal{Z}\{5nu(n)\}}{4} \quad (16)$$

$$X(z) = \frac{(z^{-1})(1+z^{-1})(1+2z^{-1})}{4(1-z^{-1})^5} + \frac{5z^{-1}}{4(1-z^{-1})^3} \quad (17)$$

Here's the plot of  $x(n)$ :

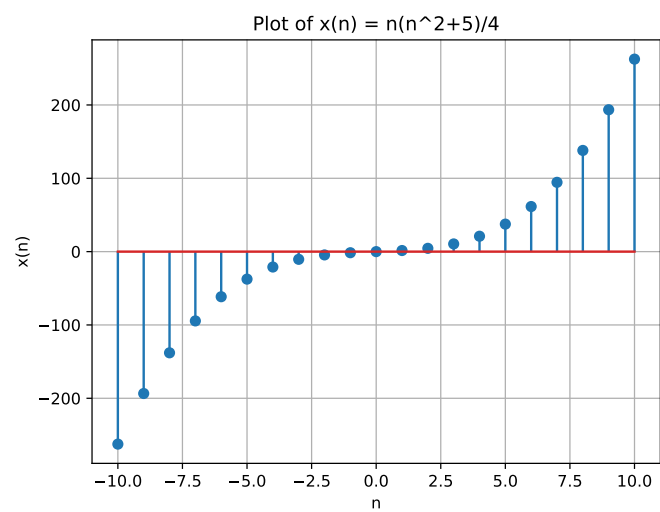


Figure 1: Plot of the function  $x(n) = \frac{n(n^2+5)}{4}$