Physics Assignment

Karyampudi Meghana Sai
EE23BTECH11031

January 7, 2024

Problem Statement

Write the first five terms of the sequence $a_n = \frac{n(n^2+5)}{4}$.

Solution

The sequence $x(n) = \frac{n(n^2+5)}{4}$ starting from $n = 0$ is:

$$
\begin{aligned}
x(0) &= \frac{0(0^2+5)}{4} = \frac{0}{4} = 0 \\
x(1) &= \frac{1(1^2+5)}{4} = \frac{6}{4} = 1.5 \\
x(2) &= \frac{2(2^2+5)}{4} = \frac{18}{4} = 4.5 \\
x(3) &= \frac{3(3^2+5)}{4} = \frac{42}{4} = 10.5 \\
x(4) &= \frac{4(4^2+5)}{4} = \frac{84}{4} = 21 \\
x(5) &= \frac{5(5^2+5)}{4} = \frac{150}{4} = 37.5
\end{aligned}
$$

The relation between x(n) and u(n):

$$x(n) = \frac{n^3+5n}{4} \cdot u(n) \tag{1}$$

The unit step function $u(n)$ is defined as:

$$u(n) = \begin{cases} 0 & \text{if } n < 0 \\ 1 & \text{if } n \geq 0 \end{cases}$$

The Z-transform of $u(n)$, denoted as $U(z)$, is calculated as follows:

$$U(z) = \mathcal{Z}\{u(n)\} = \sum_{n=0}^{\infty} u(n) z^{-n} \tag{2}$$

$$U(z) = \sum_{n=0}^{\infty} z^{-n} = 1 + z^{-1} + z^{-2} + z^{-3} + \dots$$

$$U(z) = \frac{1}{1 - z^{-1}} = \frac{z}{z - 1} \tag{3}$$

The Z-transform of $u(n)$ is $U(z) = \frac{1}{1-z^{-1}}$. To find $X(z)$ from $U(z)$:

$$Z\{n^k\} = \frac{z}{(z-1)^{k+1}} \tag{4}$$

$$\begin{aligned}
X(z) &= \mathcal{Z}\left\{\frac{n^3 + 5n}{4} \cdot u(n)\right\} && (5)\\
&= \frac{1}{4} \cdot \mathcal{Z}\{n^3 + 5n\} \cdot U(z)\\
&= \frac{1}{4} \cdot \left(\frac{z}{(z-1)^4} + \frac{5z}{(z-1)^2}\right) \cdot U(z)\\
&= \frac{1}{4} \cdot \left(\frac{z}{(z-1)^4} + \frac{5z}{(z-1)^2}\right) \cdot \frac{1}{1 - z^{-1}}\\
&= \frac{1}{4} \cdot \left(\frac{z}{(z-1)^4} + \frac{5z}{(z-1)^2}\right) \cdot \frac{z}{z - 1}\\
X(z) &= \frac{1}{4} \cdot \frac{z^2}{(z-1)^5} + \frac{5z^2}{4(z-1)^3} && (6)
\end{aligned}$$

This expression represents $X(z)$ in terms of $U(z)$, relating the z-domain representation of $x(n)$ to that of $u(n)$.