

Maths Assignment

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Problem Statement

Write the first five terms of the sequence $a_n = \frac{n(n^2+5)}{4}$.

Solution

The relation between $x(n)$ and $u(n)$:

$$x(n) = \left(\frac{(n+1)^3 + 5(n+1)}{4} \right) u(n) \quad (1)$$

Z-transform of $n^k u(k)$ in terms of k th derivative of $U(z)$:

$$n^k u(n) \xleftrightarrow{\text{ZT}} (-1)^k z^k \frac{d^k}{dz^k} U(z) \quad (2)$$

$$\mathcal{Z}\{nu(n)\} = \frac{z^{-1}}{(1-z^{-1})^2} \quad [\text{ROC: } |z| > 1] \quad (3)$$

$$\mathcal{Z}\{n^2 u(n)\} = \frac{(z^{-1})(1+z^{-1})}{(1-z^{-1})^3} \quad [\text{ROC: } |z| > 1] \quad (4)$$

$$\mathcal{Z}\{n^3 u(n)\} = \frac{(z^{-1})(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4} \quad [\text{ROC: } |z| > 1] \quad (5)$$

Now Z-transform of $x(n)$ is given by:

$$\mathcal{X}\{z\} = \frac{\mathcal{Z}\{n^3 u(n)\}}{4} + \frac{3\mathcal{Z}\{n^2 u(n)\}}{4} + 2\mathcal{Z}\{nu(n)\} + \frac{3\mathcal{Z}\{u(n)\}}{2} \quad (6)$$

Referencing the equations (3), (4), and (5).

$$\mathcal{X}\{z\} = \frac{(z^{-1})(1+4z^{-1}+z^{-2})}{4(1-z^{-1})^4} + \frac{3(z^{-1})(1+z^{-1})}{4(1-z^{-1})^3} + \frac{2z^{-1}}{(1-z^{-1})^2} + \frac{3}{2(1-z^{-1})} \quad [\text{ROC: } |z| > 1] \quad (7)$$

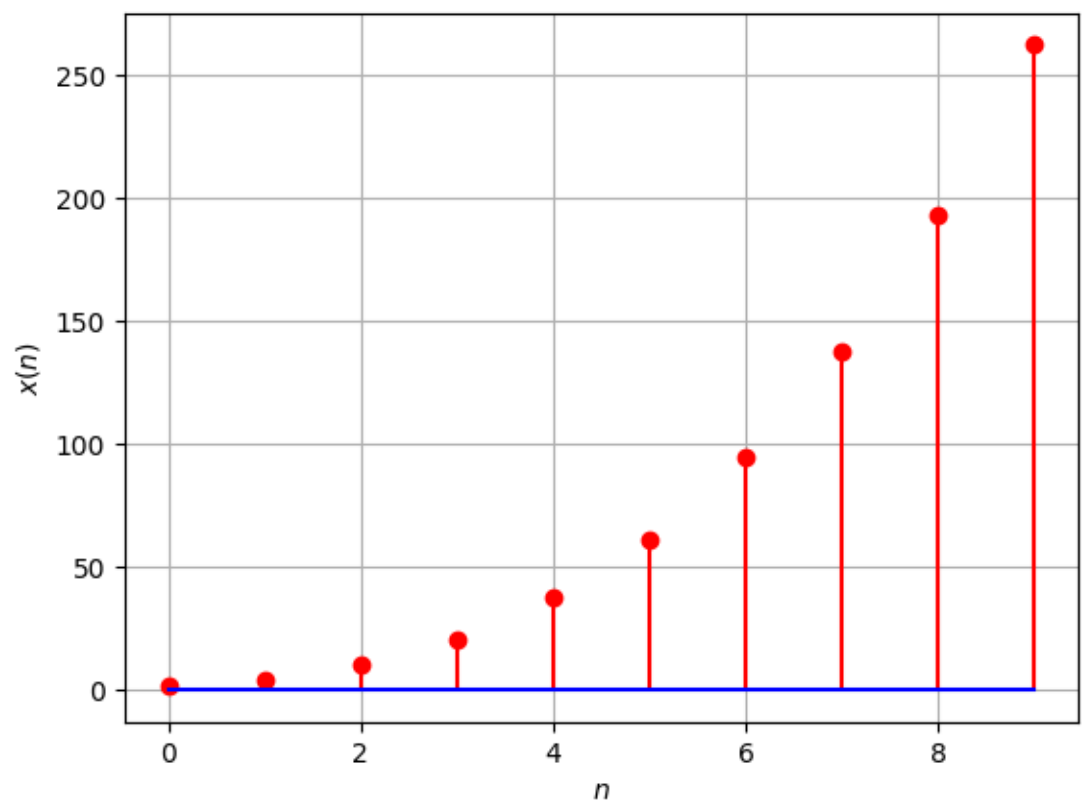


Figure 1: Plot of equation (1)