

# Discrete Assignment

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## PROBLEM STATEMENT

The ratio of the A.M and G.M of two positive numbers  $a$  and  $b$  is  $m : n$ . Show that  $a : b = (m + \sqrt{m^2 - n^2}) : (m - \sqrt{m^2 - n^2})$ .

## SOLUTION

Expressing A.M and G.M in terms of  $a$  and  $b$ :

$$\frac{a+b}{2\sqrt{ab}} = \frac{m}{n} \quad (1)$$

Let's assume that  $x = \sqrt{\frac{a}{b}}$ . Then, we have:

$$\frac{a}{b} = x^2 \quad (2)$$

Substituting this into the equation (1):

$$\frac{1+x^2}{2x} = \frac{m}{n} \quad (3)$$

$$\frac{1}{x} + x = \frac{2m}{n} \quad (4)$$

$$x^2 - \frac{2m}{n}x + 1 = 0 \quad (5)$$

$$x = \frac{2m}{n} \pm \frac{2}{n} \sqrt{m^2 - n^2} \quad (6)$$

Since  $x = \sqrt{\frac{a}{b}}$ ,  $x$  must be positive.

$$x = \frac{2}{n}(m + \sqrt{m^2 - n^2}) \quad (7)$$

Referencing the value of  $x$  from equation(2)

$$\frac{a}{b} = \left(\frac{2}{n}\right)^2 (m + \sqrt{m^2 - n^2})^2 \quad (8)$$

Multiplying both the numerator and denominator with  $(m - \sqrt{m^2 - n^2})$ :

$$\frac{a}{b} = \frac{4}{n^2} \frac{(m + \sqrt{m^2 - n^2})^2 (m - \sqrt{m^2 - n^2})}{(m - \sqrt{m^2 - n^2})} \quad (9)$$

$$a : b = (m + \sqrt{m^2 - n^2}) : (m - \sqrt{m^2 - n^2}) \quad (10)$$