Maths Assignment

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Problem Statement

The ratio of the A.M and G.M of two positive numbers a and b is $m : n$. Show that $a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$.

Solution

Expressing A.M and G.M in terms of a and b:

$$\frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$$

From the given expression, let's express a and b in terms of m and n using the quadratic equation.

Let's assume that $x = \sqrt{\frac{a}{b}}$. Then, we have:

$$\frac{a}{b} = x^2$$

Substituting this into the equation for A.M and G.M:

$$\frac{1+x^2}{2x} = \frac{m}{n}$$

$$\frac{1}{x} + x = \frac{2m}{n}$$

$$x^2 - \frac{2m}{n}x + 1 = 0$$

This is a quadratic equation in terms of x, and the roots of this equation are $x = \frac{2m}{n} \pm \sqrt{\left(\frac{2m}{n}\right)^2 - 4}$. Simplifying the expression inside the square root gives us:

$$x = \frac{2m}{n} \pm \sqrt{\frac{4m^2}{n^2} - 4}$$

$$x = \frac{2m}{n} \pm \sqrt{\frac{4m^2 - 4n^2}{n^2}}$$
$$x = \frac{2m}{n} \pm \frac{2}{n}\sqrt{m^2 - n^2}$$

Since $x = \sqrt{\frac{a}{b}}$, x must be positive, so we take the positive sign before the square root. Therefore:

$$x = \frac{2m}{n} + \frac{2}{n}\sqrt{m^2 - n^2}$$
$$x = \frac{2}{n}(m + \sqrt{m^2 - n^2})$$

Now, since $x = \sqrt{\frac{a}{b}}$, we have:

$$\frac{a}{b} = \left(\frac{2}{n}\right)^2 (m + \sqrt{m^2 - n^2})^2$$

Now, let's express $a : b$ using this result:

$$\frac{a}{b} = \frac{4}{n^2}(m + \sqrt{m^2 - n^2})^2$$

Multiplying both the numerator and denominator with $(m - \sqrt{m^2 - n^2})$:

$$\frac{a}{b} = \frac{4}{n^2}\frac{(m + \sqrt{m^2 - n^2})^2(m - \sqrt{m^2 - n^2})}{(m - \sqrt{m^2 - n^2})}$$

$$a : b = 2\left(m + \sqrt{m^2 - n^2}\right) : 2\left(m - \sqrt{m^2 - n^2}\right)$$

Finally, simplify by dividing both sides by 2:

$$a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$$

Thus, we've arrived at the required result: $a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$.