Discrete Assignment

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Problem Statement

The ratio of the A.M and G.M of two positive numbers a and b is $m : n$. Show that $a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$.

Solution

Expressing A.M and G.M in terms of a and b:

$$\frac{a+b}{2\sqrt{ab}} = \frac{m}{n} \tag{1}$$

Let's assume that $x = \sqrt{\frac{a}{b}}$. Then, we have:

$$\frac{a}{b} = x^2 \tag{2}$$

Substituting this into the equation (1):

$$\frac{1+x^2}{2x} = \frac{m}{n} \tag{3}$$

$$\frac{1}{x} + x = \frac{2m}{n} \tag{4}$$

$$x^2 - \frac{2m}{n}x + 1 = 0 \tag{5}$$

$$x = \frac{2m}{n} \pm \frac{2}{n}\sqrt{m^2 - n^2} \tag{6}$$

Since $x = \sqrt{\frac{a}{b}}$, x must be positive.

$$x = \frac{2}{n}\left(m + \sqrt{m^2 - n^2}\right) \tag{7}$$

Referencing the value of x from equation(2)

$$\frac{a}{b} = \left(\frac{2}{n}\right)^2 \left(m + \sqrt{m^2 - n^2}\right)^2 \tag{8}$$

Multiplying both the numerator and denominator with $\left(m - \sqrt{m^2 - n^2}\right)$:

$$\frac{a}{b} = \frac{4}{n^2} \frac{\left(m + \sqrt{m^2 - n^2}\right)^2 \left(m - \sqrt{m^2 - n^2}\right)}{\left(m - \sqrt{m^2 - n^2}\right)} \tag{9}$$

$$a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right) \tag{10}$$