Discrete Assignment

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Problem Statement

The ratio of the A.M and G.M of two positive numbers a and b is $m : n$. Show that $a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$.

Solution

Expressing A.M and G.M in terms of a and b:

$$\frac{a+b}{2\sqrt{ab}} = \frac{m}{n} \tag{1}$$

Let's assume that $x = \sqrt{\frac{a}{b}}$. Then, we have:

$$\frac{a}{b} = x^2 \tag{2}$$

Substituting the value of x in equation (1):

$$\frac{1+x^2}{2x} = \frac{m}{n} \tag{3}$$

$$\frac{1}{x} + x = \frac{2m}{n} \tag{4}$$

$$x^2 - \frac{2m}{n}x + 1 = 0 \tag{5}$$

$$\implies x = \frac{m}{n} \pm \frac{\sqrt{m^2 - n^2}}{n} \tag{6}$$

Since $x = \sqrt{\frac{a}{b}}$, x must be positive.

$$x = \frac{m + \sqrt{m^2 - n^2}}{n} \tag{7}$$

Referencing the value of x from equation(2).

$$\frac{a}{b} = \left(\frac{m + \sqrt{m^2 - n^2}}{n}\right)^2 \tag{8}$$

Multiplying both the numerator and denominator with $\left(m - \sqrt{m^2 - n^2}\right)$:

$$\frac{a}{b} = \frac{1}{n^2}\frac{\left(m + \sqrt{m^2 - n^2}\right)^2\left(m - \sqrt{m^2 - n^2}\right)}{\left(m - \sqrt{m^2 - n^2}\right)} \tag{9}$$

$$\implies a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right) \tag{10}$$

nth term of the AP :

$$y(n) = \ [a + n\,(b - a)]\,u(n) \tag{11}$$

$$n^k u(n) \xrightarrow{Z} (-1)^k z^k \frac{d^k}{dz^k} U(z) \tag{12}$$

$$u(n) \xrightarrow{Z} \frac{1}{(1 - z^{-1})} \quad |z| > |1| \tag{13}$$

$$nu(n) \xrightarrow{Z} \frac{z^{-1}}{(1 - z^{-1})^2} \quad |z| > |1| \tag{14}$$

Referencing the equations from (13),(14).

$$y(n) \xrightarrow{Z} \frac{a}{(1 - z^{-1})} + \frac{(b - a)\,z^{-1}}{(1 - z^{-1})^2} \quad |z| > |1| \tag{15}$$

nth term of the GP :

$$y(n) = a\left(\frac{b}{a}\right)^n u(n) \tag{16}$$

$$r^k u(n) \xrightarrow{Z} \frac{1}{(1 - rz^{-1})} \quad |z| > |r| \tag{17}$$

Referencing the equation from (17).

$$y(n) \xrightarrow{Z} \frac{a^2 z^{-1}}{(a - bz^{-1})} \quad |z| > \left|\frac{b}{a}\right| \tag{18}$$