

Discrete Assignment

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PROBLEM STATEMENT

The ratio of the A.M and G.M of two positive numbers a and b is $m : n$. Show that $a : b = (m + \sqrt{m^2 - n^2}) : (m - \sqrt{m^2 - n^2})$.

SOLUTION

Expressing A.M and G.M in terms of a and b :

$$\frac{a+b}{2\sqrt{ab}} = \frac{m}{n} \quad (1)$$

Let's assume that $x = \sqrt{\frac{a}{b}}$. Then, we have:

$$\frac{a}{b} = x^2 \quad (2)$$

Substituting the value of x in equation (1):

$$\frac{1+x^2}{2x} = \frac{m}{n} \quad (3)$$

$$\frac{1}{x} + x = \frac{2m}{n} \quad (4)$$

$$x^2 - \frac{2m}{n}x + 1 = 0 \quad (5)$$

$$\Rightarrow x = \frac{m}{n} \pm \frac{\sqrt{m^2 - n^2}}{n} \quad (6)$$

Since $x = \sqrt{\frac{a}{b}}$, x must be positive.

$$x = \frac{m + \sqrt{m^2 - n^2}}{n} \quad (7)$$

Referencing the value of x from equation(2).

$$\frac{a}{b} = \left(\frac{m + \sqrt{m^2 - n^2}}{n} \right)^2 \quad (8)$$

Multiplying both the numerator and denominator with $(m - \sqrt{m^2 - n^2})$:

$$\frac{a}{b} = \frac{1}{n^2} \frac{(m + \sqrt{m^2 - n^2})^2 (m - \sqrt{m^2 - n^2})}{(m - \sqrt{m^2 - n^2})} \quad (9)$$

$$\Rightarrow a : b = (m + \sqrt{m^2 - n^2}) : (m - \sqrt{m^2 - n^2}) \quad (10)$$