GATE ECE 2023

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Consider a discrete-time signal with period $N = 5$. Let the discrete-time Fourier series (DTFS) representation be $x[n] = \sum_{k=0}^{4} a_k e^{\frac{jk2\pi n}{5}}$, where $a_0 = 1$, $a_1 = 3j$, $a_2 = 2j$, $a_3 = -2j$, $a_4 = -3j$. The value of the sum $\sum_{n=0}^{4} x[n] \sin\left(\frac{4\pi n}{5}\right)$ is

Solution:

1) Solving the question for N=5:

Parameter	Value	Description
N	5	Time period
$x[n]$	$\sum_{k=0}^{4} a_k e^{\frac{jk2\pi n}{5}}$	DTFS representation
a_0	1	DTFS coefficients
a_1	$3j$	
a_2	$2j$	
a_3	$-2j$	
a_4	$-3j$	

TABLE I
Input Parameters

$$\sum_{n=0}^{4} x(n) \sin\left(\frac{4\pi n}{5}\right) = \sum_{n=0}^{4} x(n) \left[\frac{e^{\frac{j4\pi n}{5}} - e^{\frac{-j4\pi n}{5}}}{2j}\right] \tag{1}$$

$$= \frac{1}{2j}\left[\sum_{n=0}^{4} x(n) e^{\frac{j2\pi(2)n}{5}} - \sum_{n=0}^{4} x(n) e^{\frac{-j2\pi(2)n}{5}}\right] \tag{2}$$

DTFS coefficient is given by,

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}} \tag{3}$$

DFT formula:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}} \tag{4}$$

From (3) and (4),

$$a_k = \frac{X(k)}{N} \tag{5}$$

Given that time period of x(n) is N=5 sec.

$$X(k) = \sum_{n=0}^{4} x(n) e^{\frac{-j2\pi kn}{5}} \tag{6}$$

Referencing from equation(6), equation(2) can be written as:

$$\sum_{n=0}^{4} x(n)\sin\left(\frac{4\pi n}{5}\right) = \frac{1}{2j}\left[X(-2) - X(2)\right] \tag{7}$$

From the property of discrete Fourier series.

$$X(k) = X(k+N) \tag{8}$$

So, equation(7) becomes,

$$\sum_{n=0}^{4} x(n)\sin\left(\frac{4\pi n}{5}\right) = \frac{1}{2j}\left[X(3) - X(2)\right] \tag{9}$$

$$\sum_{n=0}^{4} x(n)\sin\left(\frac{4\pi n}{5}\right) = -10 \tag{10}$$

2) Solving the question for N=8:

Parameter	**Value**	**Description**
N	8	Time period
$x[n]$	$\sum_{k=0}^{7} a_k e^{\frac{jk2\pi n}{8}}$	DTFS representation
a_0	1	DTFS coefficients
a_1	$3j$	
a_2	$2j$	
a_3	$-2j$	
a_4	$-3j$	
a_5	0	
a_6	0	
a_7	0	

TABLE II
INPUT PARAMETERS

$$\sum_{n=0}^{7} x(n)\sin\left(\frac{4\pi n}{8}\right) = \sum_{n=0}^{7} x(n)\left[\frac{e^{\frac{j4\pi n}{8}} - e^{\frac{-j4\pi n}{8}}}{2j}\right] \tag{11}$$

$$= \frac{1}{2j}\left[\sum_{n=0}^{7} x(n)e^{\frac{j2\pi(2)n}{8}} - \sum_{n=0}^{7} x(n)e^{\frac{-j2\pi(2)n}{8}}\right] \tag{12}$$

DTFS coefficient is given by,

$$a_K = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{\frac{-j2\pi kn}{N}} \tag{13}$$

DFT formula:

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{\frac{-j2\pi kn}{N}} \tag{14}$$

From (13) and (14),

$$a_k = \frac{X(k)}{N} \tag{15}$$

Given that time period of x(n) is N=8 sec.

$$X(k) = \sum_{n=0}^{7} x(n) e^{\frac{-j2\pi kn}{8}} \tag{16}$$

Referencing from equation(16), equation(12) can be written as:

$$\sum_{n=0}^{7} x(n) \sin\left(\frac{4\pi n}{8}\right) = \frac{1}{2j} \left[X(-2) - X(2)\right] \tag{17}$$

From the property of discrete Fourier series.

$$X(k) = X(k+N) \tag{18}$$

So, equation(17) becomes,

$$\sum_{n=0}^{7} x(n) \sin\left(\frac{4\pi n}{8}\right) = \frac{1}{2j} \left[X(6) - X(2)\right] \tag{19}$$

$$\sum_{n=0}^{7} x(n) \sin\left(\frac{4\pi n}{8}\right) = -8 \tag{20}$$