GATE ECE 2023

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Consider a discrete-time signal with period $N = 5$. Let the discrete-time Fourier series (DTFS) representation be $x[n] = \sum_{k=0}^{4} a_k e^{\frac{jk2\pi n}{5}}$, where $a_0 = 1$, $a_1 = 3j$, $a_2 = 2j$, $a_3 = -2j$, $a_4 = -3j$. The value of the sum $\sum_{n=0}^{4} x[n] \sin\left(\frac{4\pi n}{5}\right)$ is

Solution:

Parameter	Value	Description
N	8	Time period
$x[n]$	$\sum_{k=0}^{7} a_k e^{\frac{jk2\pi n}{8}}$	DTFS representation
a_0	1	DTFS coefficients
a_1	$3j$	
a_2	$2j$	
a_3	$-2j$	
a_4	$-3j$	
a_5	0	
a_6	0	
a_7	0	

TABLE I
INPUT PARAMETERS

$$\sum_{n=0}^{7} x(n) \sin\left(\frac{4\pi n}{8}\right) = \sum_{n=0}^{7} x[n] \left[\frac{e^{\frac{j4\pi n}{8}} - e^{\frac{-j4\pi n}{8}}}{2j}\right] \tag{1}$$

$$= \frac{1}{2j} \left[\sum_{n=0}^{7} x[n] e^{\frac{j2\pi(2)n}{8}} - \sum_{n=0}^{7} x[n] e^{\frac{-j2\pi(2)n}{8}}\right] \tag{2}$$

DTFS coefficient is given by,

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}} \tag{3}$$

Given that time period of x(n) is N=8 sec.

$$a_k = \frac{1}{8} \sum_{n=0}^{7} x(n) e^{\frac{-j2\pi kn}{8}} \tag{4}$$

$$\sum_{n=0}^{7} x(n) e^{\frac{-j2\pi kn}{8}} = 8a_k \tag{5}$$

Referencing from equation(5), equation(2) can be written as:

$$\sum_{n=0}^{7} x(n) \sin\left(\frac{4\pi n}{8}\right) = \frac{1}{2j}[8a_{-2} - 8a_2] \tag{6}$$

From the property of discrete Fourier series.

$$a_k = a_{k+N} \tag{7}$$

So, equation(6) becomes,

$$\sum_{n=0}^{7} x(n) \sin\left(\frac{4\pi n}{8}\right) = \frac{1}{2j}[8a_6 - 8a_2] \tag{8}$$

$$\sum_{n=0}^{7} x(n) \sin\left(\frac{4\pi n}{8}\right) = - \tag{9}$$