

GATE ECE 2023

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Consider a discrete-time signal with period $N = 5$. Let the discrete-time Fourier series (DTFS) representation be $x[n] = \sum_{k=0}^4 a_k e^{\frac{jk2\pi n}{5}}$, where $a_0 = 1$, $a_1 = 3j$, $a_2 = 2j$, $a_3 = -2j$, $a_4 = -3j$. The value of the sum

$$\sum_{n=0}^4 x[n] \sin\left(\frac{4\pi n}{5}\right) \text{ is}$$

Solution:

1) Solving the question for $N=5$:

Parameter	Value	Description
N	5	Time period
$x[n]$	$\sum_{k=0}^4 a_k e^{\frac{jk2\pi n}{5}}$	DTFS representation
a_0	1	DTFS coefficients
a_1	$3j$	
a_2	$2j$	
a_3	$-2j$	
a_4	$-3j$	

TABLE I
INPUT PARAMETERS

$$\sum_{n=0}^4 x(n) \sin\left(\frac{4\pi n}{5}\right) = \sum_{n=0}^4 x[n] \left[\frac{e^{\frac{j4\pi n}{5}} - e^{-\frac{j4\pi n}{5}}}{2j} \right] \quad (1)$$

$$= \frac{1}{2j} \left[\sum_{n=0}^4 x[n] e^{\frac{j2\pi(2)n}{5}} - \sum_{n=0}^4 x[n] e^{-\frac{j2\pi(2)n}{5}} \right] \quad (2)$$

DTFS coefficient is given by,

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-\frac{j2\pi kn}{N}} \quad (3)$$

DFT formula:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-\frac{j2\pi kn}{N}} \quad (4)$$

From (3) and (4),

$$a_k = \frac{X(k)}{N} \quad (5)$$

Given that time period of $x(n)$ is $N=5$ sec.

$$X(k) = \sum_{n=0}^4 x(n) e^{-\frac{j2\pi kn}{5}} \quad (6)$$

Referencing from equation(6), equation(2) can be written as:

$$\sum_{n=0}^4 x(n) \sin\left(\frac{4\pi n}{5}\right) = \frac{1}{2j} [X(-2) - X(2)] \quad (7)$$

From the property of discrete Fourier series.

$$X(k) = X(k + N) \quad (8)$$

So, equation(7) becomes,

$$\sum_{n=0}^4 x(n) \sin\left(\frac{4\pi n}{5}\right) = \frac{1}{2j} [X(3) - X(2)] \quad (9)$$

$$\sum_{n=0}^4 x(n) \sin\left(\frac{4\pi n}{5}\right) = -10 \quad (10)$$

2) Solving the question for N=8:

Parameter	Value	Description
N	8	Time period
$x[n]$	$\sum_{k=0}^7 a_k e^{\frac{jk2\pi n}{8}}$	DTFS representation
a_0	1	DTFS coefficients
a_1	$3j$	
a_2	$2j$	
a_3	$-2j$	
a_4	$-3j$	
a_5	0	
a_6	0	
a_7	0	

TABLE II
INPUT PARAMETERS

$$\sum_{n=0}^7 x(n) \sin\left(\frac{4\pi n}{8}\right) = \sum_{n=0}^7 x[n] \left[\frac{e^{\frac{j4\pi n}{8}} - e^{\frac{-j4\pi n}{8}}}{2j} \right] \quad (11)$$

$$= \frac{1}{2j} \left[\sum_{n=0}^7 x[n] e^{\frac{j2\pi(2)n}{8}} - \sum_{n=0}^7 x[n] e^{\frac{-j2\pi(2)n}{8}} \right] \quad (12)$$

DTFS coefficient is given by,

$$a_K = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}} \quad (13)$$

DFT formula:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}} \quad (14)$$

From (13) and (14),

$$a_k = \frac{X(k)}{N} \quad (15)$$

Given that time period of $x(n)$ is $N=8$ sec.

$$X(k) = \sum_{n=0}^7 x(n) e^{\frac{-j2\pi kn}{8}} \quad (16)$$

Referencing from equation(16), equation(12) can be written as:

$$\sum_{n=0}^7 x(n) \sin\left(\frac{4\pi n}{8}\right) = \frac{1}{2j} [X(-2) - X(2)] \quad (17)$$

From the property of discrete Fourier series.

$$X(k) = X(k + N) \quad (18)$$

So, equation(17) becomes,

$$\sum_{n=0}^7 x(n) \sin\left(\frac{4\pi n}{8}\right) = \frac{1}{2j} [X(6) - X(2)] \quad (19)$$

$$\sum_{n=0}^7 x(n) \sin\left(\frac{4\pi n}{8}\right) = -8 \quad (20)$$