

Gate 2023_nm_33

Karyampudi Meghana Sai
EE23BTECH11031

For a regular sinusoidal wave propagating in deep water having wave height of 3.5 m and wave period of 9 s, the wave steepness is ____ (round off to three decimal places). Gate 2023 NM 33

Solution:

Symbol	Value	Description
H	3.5m	wave height
T	9s	wave period
S	?	wave steepness
λ		wave length
η		surface elevation of water

TABLE I
INPUT PARAMETERS

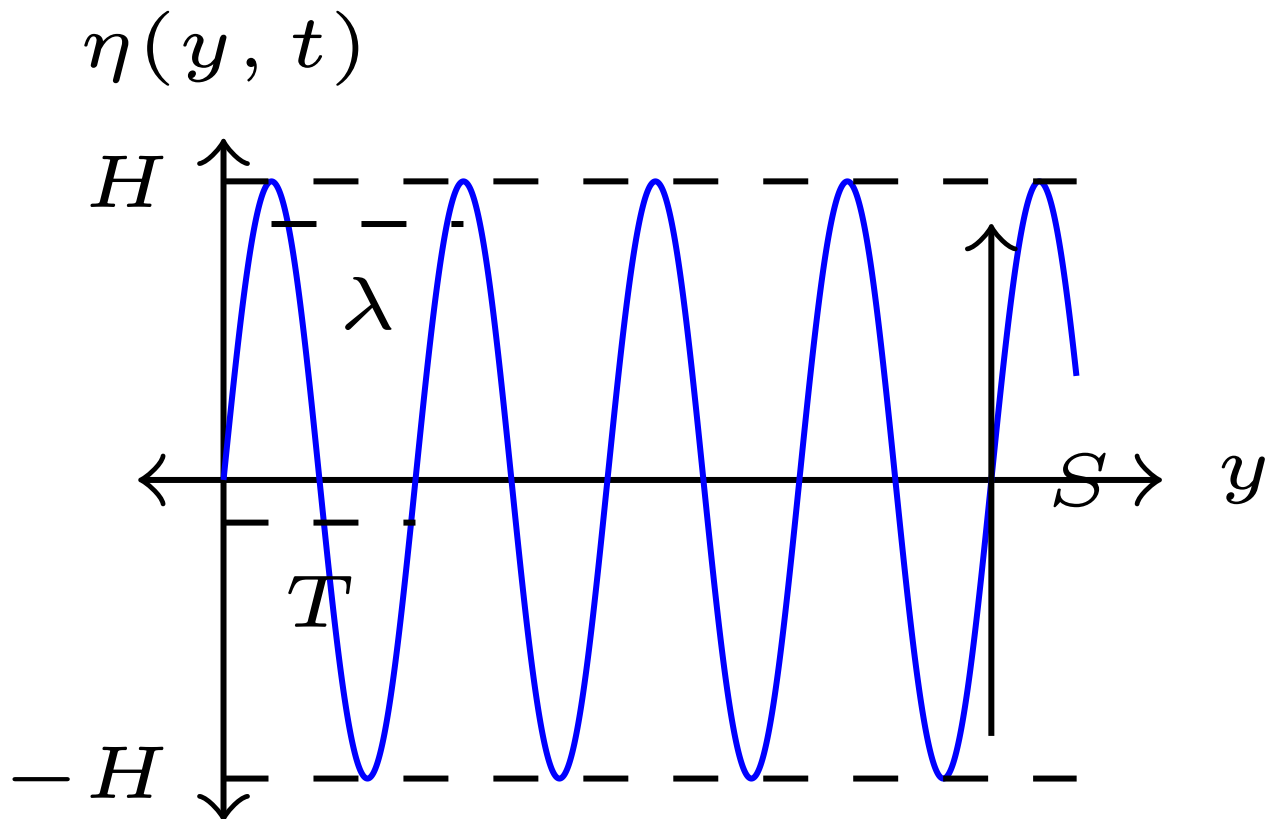


Fig. 1. Sinusoidal wave

$$S = \frac{H}{\lambda} \quad (1)$$

Deriving the formula for wavelength of deep water wave:

Let's start with the linearized shallow water wave equation.

$$\frac{\partial^2 \eta}{\partial t^2} = g \frac{\partial \eta}{\partial y} \quad (2)$$

$$\eta = A \sin(ky - \omega t) \quad (3)$$

$$\frac{\partial^2 \eta}{\partial t^2} = -(\omega)^2 A \sin(ky - \omega t) \quad (4)$$

For deep water waves:

$$\frac{\partial \eta}{\partial y} \approx -k\eta \quad (5)$$

Using the equation (2).

$$\frac{\partial^2 \eta}{\partial t^2} = -gk\eta \quad (6)$$

$$= -gkA \sin(ky - \omega t) \quad (7)$$

where, k is wave number.

Comparing equations (4) and (7),

$$\omega^2 = gk \quad (8)$$

$$\omega = \frac{2\pi}{T} \quad (9)$$

$$k = \frac{2\pi}{\lambda} \quad (10)$$

$$\lambda = \frac{g \cdot T^2}{2\pi} \quad (11)$$

Substituting numericals,

$$\lambda = \frac{g \cdot T^2}{2\pi} \quad (12)$$

$$= \frac{9.81 (9)^2}{2\pi} \quad (13)$$

$$= 126.53m \quad (14)$$

Using the equation (1).

$$S = \frac{3.5}{126.53} \quad (15)$$

$$= 0.028 \quad (16)$$

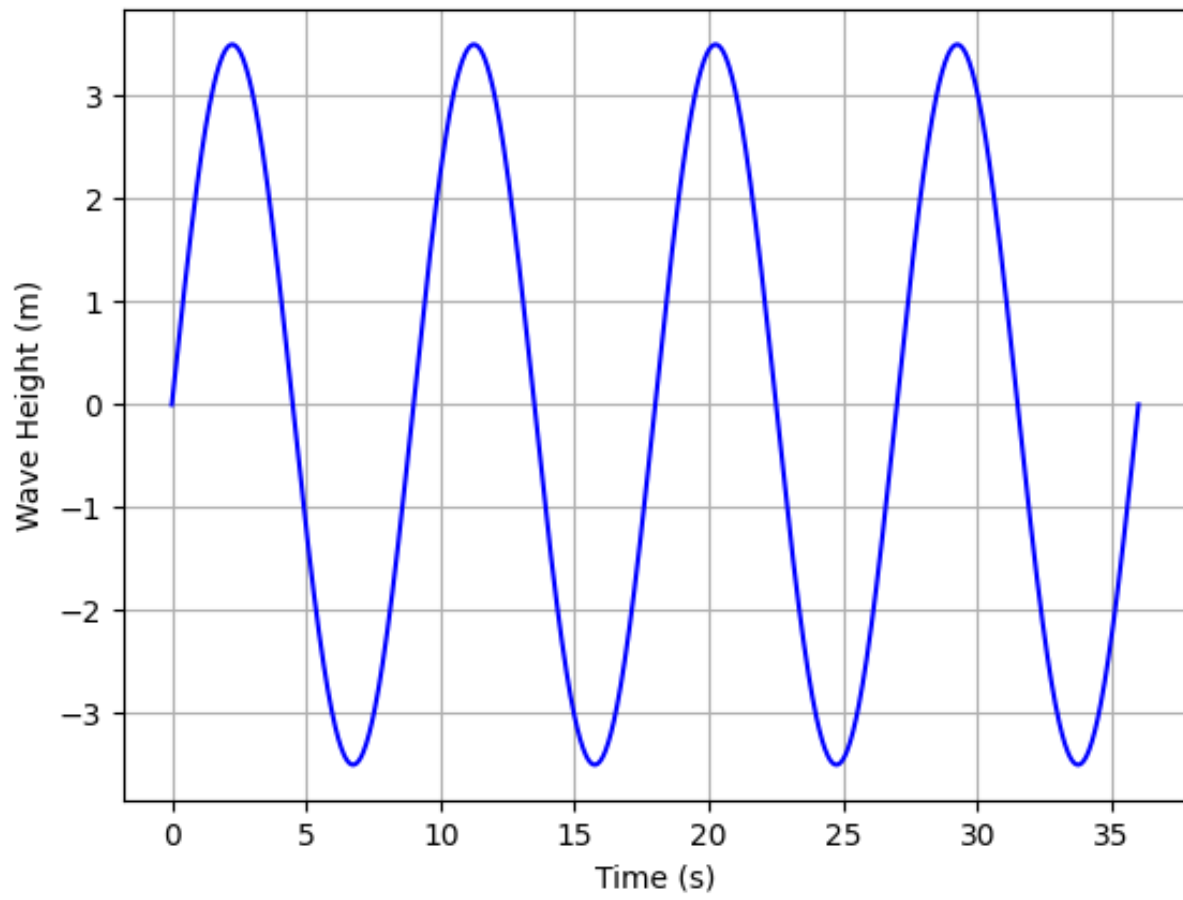


Fig. 2. Sinusoidal wave