Gate 2023_nm_33

Karyampudi Meghana Sai
EE23BTECH11031

For a regular sinusoidal wave propagating in deep water having wave height of 3.5 m and wave period of 9 s, the wave steepness is _____ (round off to three decimal places). Gate 2023 NM 33

Solution:

Symbol	Value	Description
H	$3.5m$	wave height
T	$9s$	wave period
S	?	wave steepness
λ		wave length
η		surface elevation of water

TABLE I
INPUT PARAMETERS

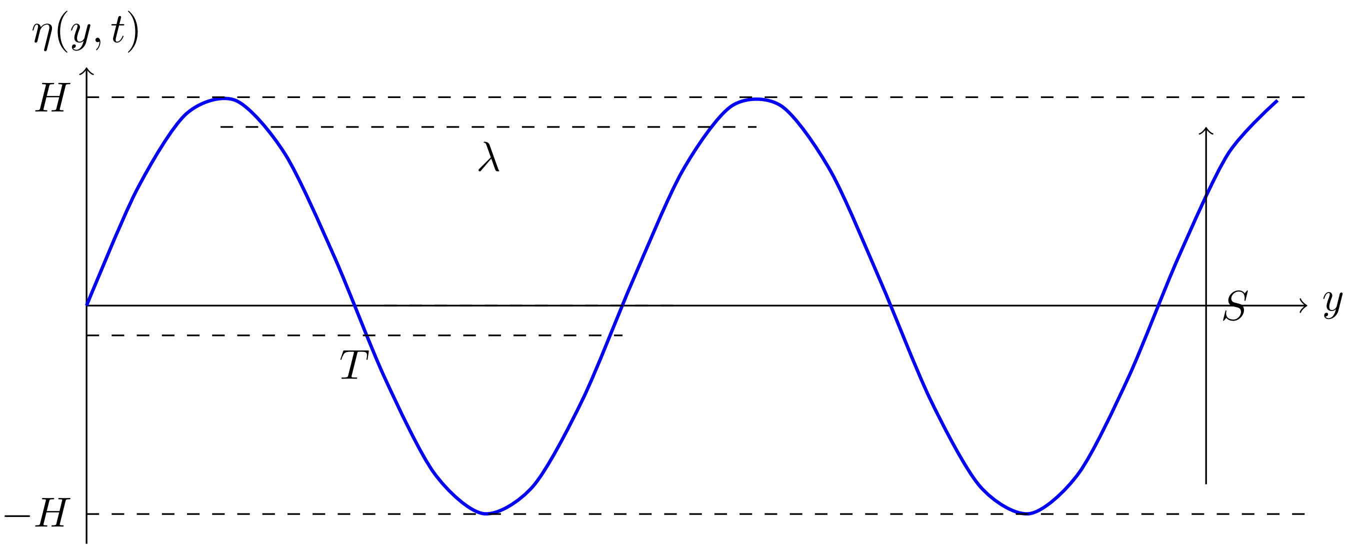

Fig. 1. Sinusoidal wave

$$S = \frac{H}{\lambda} \tag{1}$$

Deriving the formula for wavelength of deep water wave:

Let's start with the linearized shallow water wave equation:

$$\frac{\partial^2 \eta}{\partial t^2} = g\frac{\partial \eta}{\partial y} \tag{2}$$

$$\eta = A\sin(ky - \omega t) \tag{3}$$

$$\frac{\partial^2 \eta}{\partial t^2} = -(\omega)^2 A\sin(ky - \omega t) \tag{4}$$

For deep water waves:

$$\frac{\partial \eta}{\partial y} \approx -k\eta \tag{5}$$

Referencing the equation (2).

$$\frac{\partial^2 \eta}{\partial t^2} = -gk\eta \tag{6}$$

$$= -gkA\sin(ky - \omega t) \tag{7}$$

where, k is wave number.

Comparing equations (4) and (7),

$$\omega^2 = gk \tag{8}$$

$$\omega = \frac{2\pi}{T} \tag{9}$$

$$k = \frac{2\pi}{\lambda} \tag{10}$$

$$\lambda = \frac{g \cdot T^2}{2\pi} \tag{11}$$

Substituting numericals,

$$\lambda = \frac{g \cdot T^2}{2\pi} \tag{12}$$

$$= \frac{9.81\,(9)^2}{2\pi} \tag{13}$$

$$= 126.53m \tag{14}$$

Referecning the equation (1).

$$S = \frac{3.5}{126.53} \tag{15}$$

$$= 0.028 \tag{16}$$

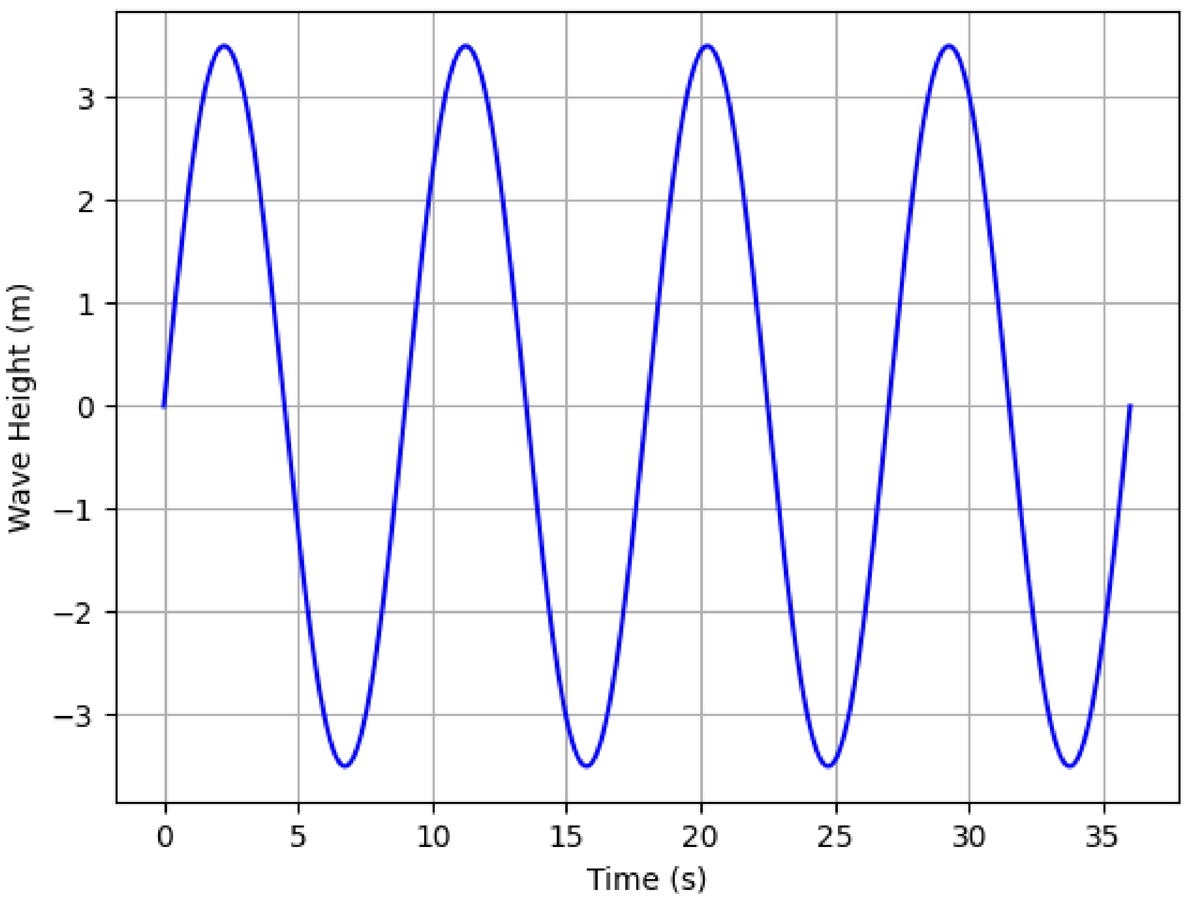

Fig. 2. Sinusoidal wave