<u>Quiz W4: Impulse Response, Transfer Functions & Stability</u>

Exercise 2.2, page 99

a) $y(n) = x(n) - 0.75y(n-1)$

Assuming that the y(n) is causal (y(n) = 0 for n < 0) and exciting the system with an impulse signal $x(n) = \delta(n)$, h(n) will be the output of the system:

$$h(0) = x(0) = 1$$
$$h(1) = x(1) - 0.75y(0) = -0.75$$
$$h(2) = x(2) - 0.75y(1) = (-0.75)^2 = 0.56$$
$$h(3) = x(3) - 0.75y(2) = (-0.75)^3 = -0.42$$
$$h(4) = x(4) - 0.75y(3) = (-0.75)^4 = 0.32$$

In general, $h(n) = (-0.75)^n$

b) $y(n) - 0.3y(n-1) - 0.4y(n-2) = x(n) - 2x(n-1)$

Again assuming that (y(n) = 0 for n < 0) and exciting the system with $x(n) = \delta(n)$, h(n) will be the output of the system:

$$h(0) = x(0) - 2x(-1) = 1$$
$$h(1) - 0.3h(0) = x(1) - 2x(0) \rightarrow h(1) = 0.3 - 2 = -1.7$$
$$h(2) - 0.3h(1) - 0.4h(0) = x(2) - 2x(1) \rightarrow h(2) = 0.3\times -1.7 + 0.4\times 1 = -0.11$$
$$h(3) - 0.3h(2) - 0.4h(1) = x(3) - 2x(2) \rightarrow h(3) = 0.3\times -.11 + 0.4\times -1.7 = -0.71$$
$$h(4) - 0.3h(3) - 0.4h(2) = x(3) - 2x(2) \rightarrow h(4) = 0.3\times -0.713 + 0.4\times -0.11 = -0.26$$

c) $y(n) = 2x(n) - 2x(n-1) + 0.5x(n-2)$

Again assuming that (y(n) = 0 for n < 0) and exciting the system with $x(n) = \delta(n)$, h(n) will be the output of the system:

$$h(0) = 2$$
$$h(1) = -2$$
$$h(2) = 0.5$$
$$h(3) = 0$$
$$h(4) = 0$$

Exercise 2.5, page 100

a) Taking z transform of both sides:

$$y(n) = x(n) - 0.75y(n-1) \Rightarrow Y(z) = X(z) - 0.75z^{-1}Y(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1+0.75z^{-1}} = \frac{1+0.0z^{-1}+0.0z^{-2}}{1+0.75z^{-1}+0.0z^{-2}}$$

b) Taking z transform of both sides:

$$y(n) - 0.3y(n-1) - 0.4y(n-2) = x(n) - 2x(n-1)$$

$$Y(z) - 0.3z^{-1}Y(z) - 0.4z^{-2}Y(z) = X(z) - 2z^{-1}X(z)$$

$$H(z) = \frac{1-2z^{-1}}{1-0.3z^{-1}-0.4z^{-2}} = \frac{1-2z^{-1}+0.0z^{-2}}{1-0.3z^{-1}-0.4z^{-2}}$$

c) Taking z transform of both sides:

$$H(z) = 2-2z^{-1}+0.5z^{-2} = \frac{2-2z^{-1}+0.5z^{-2}}{1+0.0z^{-1}+0.0z^{-2}}$$

Exercise 2.6, page 100

a) $$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1+0.75z^{-1}} = \frac{z}{z+0.75}$$

System has one zero, $z_1 = 0$.

System has one pole, $p_1 = -0.75$.

The only pole is inside the unit circle. System is stable.

b) $$H(z) = \frac{1-2z^{-1}}{1-0.3z^{-1}-0.4z^{-2}} = \frac{z^2-2z}{z^2-0.3z-0.4} = \frac{z(z-2)}{(z-0.8)(z+0.5)}$$

System has two zeroes, $z_1 = 0$, $z_2 = 2$

System has two poles, $p_1 = 0.8, p_2 = -0.5$.

All the poles are inside the unit circle. System is stable.

c) $$H(z) = 2-2z^{-1}+0.5z^{-2} = \frac{2z^2-2z+0.5}{z^2} = \frac{2(z-0.5)(z-0.5)}{z^2}$$

System has two repeated zeros, $z_1 = 0.5$, $z_2 = 0.5$.

System has two repeated poles, $p_1 = 0, p_2 = 0$.

All the poles are inside the unit circle. System is stable.