Kevin Gong kg2445 HW02 Stat W4240 Section 2

Homework 2

Problem #1

Part a)

```
> colMeans(set1,na.rm=TRUE)
x1 x2 x3 x4 x5
6.049104 -8.277221 4.665532 7.914270 62.138753
[1] -0.1277116 20.8162864
[15] 29.6357898 11.0316832
                                    -8.8984358 25.5999204
-2.5453007 8.6124471
3.7478608 18.9230712
                                                                 -9.7472153
33.8364419
                                                                               64.0626702
24.9647839
                                                                                              22.0392371
34.8385372
                                                                                                                           31.7598224
                                                                                                             23.3914888
34.1951748
                                                                                                                                                                       6.5478369
21.4051827
                                                                                                                                         -13.8680290
                                                                                                                                                        43.8318898
                                                                                                                                                                                     14.1665143
                                                                                                                           25.8869897
                                                                                                                                          -0.4545730
                                                                                                                                                                                      3.2291136
[29] 21.1031545
                                                                                                                                         16.7027735 -34.4147885
                      6.5535668
                                                                  -9.2447158
                                                                                 6.3811655
                                                                                              16.8358750
                                                                                                              7.9628124
                                                                                                                           16.6264489
                                                                                                                                                                       0.4138282
                                                                                                                                                                                     12.6572899
                                                                                                                                                                                                   35.4589880
      17.3456417
-5.4779341
27.1881673
                                                                                                                                                                                                   -3.1939531
-4.2948679
                     17 2383651
                                     0 5124620
                                                  -24.7073649
                                                                 17 1498949
                                                                               52 3665782
                                                                                               9.6993053
                                                                                                              0.3079195
                                                                                                                           15.6758568
                                                                                                                                        -13 3093667
                                                                                                                                                         8 2062088
                                                                                                                                                                       34 8247664
                                                                                                                                                                                     12 1909900
                                                                               4.4008921
18.1517706
                                                                                                                           27.9510161
                     10.7689107
                                                  19.5034554
                                                                  8.9492321
7.6480638
                                                                                                             14.7207124
                                                                                                                                                                                     -6.7256757
                                    36.2253846
                                                                                                                                         -14.3617846
                                    19.1932996
                                                  23.5607379
                                                                                              16.9872267
                                                                                                             -46.6660940
                                                                                                                             7.2223867
                                                                                                                                          28.8378401
                                                                                                                                                         6.5043155
                                                                                                                                                                       26.5206768
                                                                                                                                                                                     -2.4442159
                                                                                                                                                                                                   15.3802055
                                    20.1409435 63.2646829
                                                                   9.1977728
                                                                               29.2026018
                                                                                               1.2105932 21.2145724
                                                                                                                           -8.4896595
                                                                                                                                         19.0639963
                                                                                                                                                        20.9767512
                                                                                                                                                                        3.5962333
                                                                                                                                                                                     22.3461063
                                                                                                                                                                                                    0.7145014
```

This matrix has 5 columns and 100 rows, and we can see that the column and row means vary in both magnitude and sign. The second column has a negative mean, while the fifth column's mean is significantly larger than those of the other columns. Among the rows, there are more positive row means than negative ones.

Part b)

```
set1_emp
                      x2
           x1
                                 x3
                                            x4
                                                        x5
                          0.9903675
    1.0000000 -0.9328319
                                     0.9229496
                                                0.8391148
x1
x2 -0.9328319
               1.0000000 -0.9641852 -0.7226496 -0.9787417
x3 0.9903675 -0.9641852
                          1.0000000
                                     0.8733612
                                                0.8921875
   0.9229496 -0.7226496
x4
                          0.8733612
                                     1.0000000
                                                0.5657457
   0.8391148 -0.9787417
                          0.8921875
                                                1.0000000
x5
                                     0.5657457
```

The diagonal values of the covariance matrix are all 1, while the off-diagonal values are symmetric around the main diagonal. Thus, we know that the covariance matrix is symmetric and standardized.

Part c)

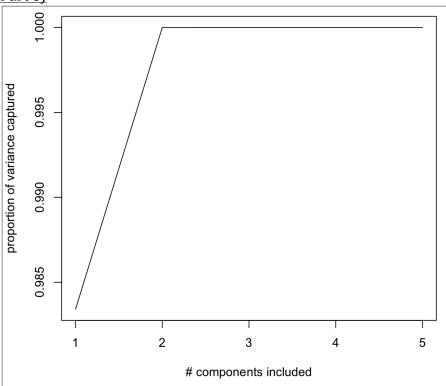
```
> eigen(set1_emp)
$values
[1] 4.486912e+00 5.087101e-01 4.368873e-03 8.952428e-06 5.481364e-07
$vectors
           [,1]
                                              [,4]
                       [,2]
                                   [,3]
[1,] 0.4673470 -0.19431684 -0.42408896
                                         0.7368413
                                                    0.14505575
[2,] -0.4596583 -0.31879336  0.25264423
                                         0.2026734
                                                    0.76300680
[3,] 0.4711378 -0.04048796 0.85611177
                                        0.1967856 -0.06883262
[4,] 0.4064079 -0.71331462 -0.12223569 -0.5420731
                                                    0.13126328
[5,] 0.4279751 0.59173874 -0.09192674 -0.2888262
                                                    0.61221849
```

We know that the left eignenvectors are transposes of the right eigenvectors of the transposed covariance matrix. In this case, our left and right eigenvectors are the same since our covariance matrix is symmetric.

Part d)

```
> set1_pca$scores
                                               Comp.4
       Comp.1
                    Comp.2
                                 Comp.3
                                                             Comp.5
                                         6.118564e-06
x1 0.9171061 -0.058803201 -2.038744e-03
                                                       1.221961e-10
x2 -3.2422362 0.007687044 9.159998e-05
                                         4.785668e-07
                                                       9.545031e-12
x3 0.9365474 0.018887709 3.482194e-03 1.813234e-06 3.599721e-11
x4 0.6355725 -0.314161290 -5.228455e-04 -4.897621e-06 -9.769873e-11
x5 0.7530102 0.346389738 -1.012205e-03 -3.512744e-06 -7.003975e-11
> set1_pca$loadings
Loadings:
   Comp.1 Comp.2 Comp.3 Comp.4 Comp.5
x1 0.461 -0.209 -0.424 0.735 0.154
x2 -0.469 -0.304 0.253
                        0.194
                               0.765
x3 0.469
                 0.856 0.198
x4 0.384 -0.726 -0.121 -0.544
                               0.125
                        -0.296
x5 0.446 0.578
               Comp.1 Comp.2 Comp.3 Comp.4 Comp.5
SS loadings
                  1.0
                        1.0
                               1.0
                                      1.0
                                             1.0
Proportion Var
                  0.2
                        0.2
                               0.2
                                      0.2
                                             0.2
                                             1.0
Cumulative Var
                        0.4
                               0.6
                                      0.8
                  0.2
```

Part e)



We should include 2 components, as additional components beyond the first two contribute very insignificantly toward the proportion of variance captured.

Part f)

Part g)

Coordinates of the projection in the original space:

```
> Y
          x1
                      x2
                                x3
                                           x4
                                                       x5
  1.6021865
             0.04605328
                         1.0635890 -0.5996748
                                               -4.0227381
2 -1.2877658 -4.14192024 -1.1392189 1.7976776 -18.8652256
3 -3.2729844 6.43439645 -2.5496028 -7.6101484 -29.9975327
  0.9713501 0.74796859 0.4469463 -3.5978859
                                                0.6046244
5 0.8458003 -2.76899602 1.2421640 -0.2266484
                                               21.2489825
```

Euclidean distance:

```
> sqrt(sum((Y-set1_centered))^2)
[1] 46.39402
```

Part h)

```
> as.vector(errors)
[1] 1.838789e-01 6.410107e-01 4.796689e-02 9.753955e-04 9.249128e-09
```

The errors are all positive, though very small, for the 5 new points. The direction of the error for the 5 new points is likely perpendicular to the direction of the projections in the original space since this generates the smallest possible error.

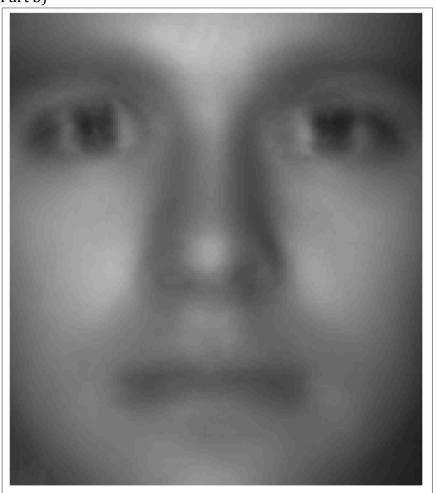
Problem #2

Part a)

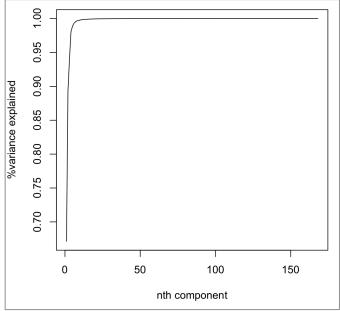
```
> dim(faces_matrix)
[1] 152 32256
```

This matrix is 152 rows by 32,256 columns, where each row represents a different picture.

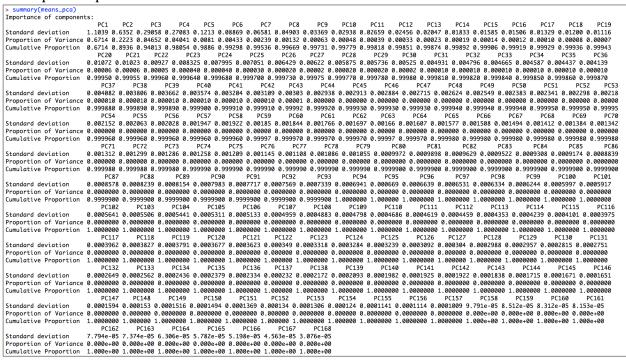
Part b)



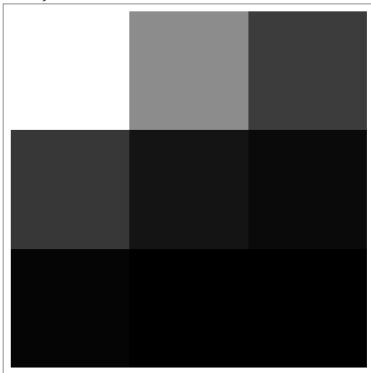
Part c)
Number of components vs. proportion of variance explained:



Principal components:



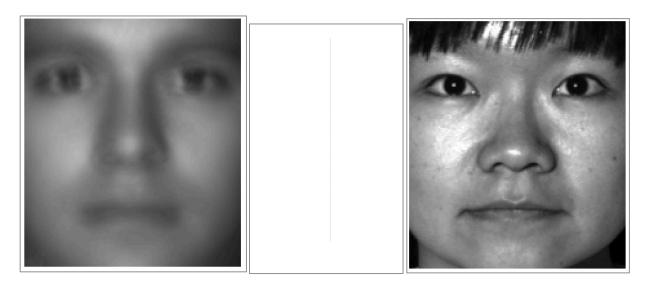
Part d)



The eigenfaces take on the standard ranges of pixmap class objects of 0 to 1, where 1 represents white and 0 represents black. These eigenfaces represent eigenvectors of our covariance matrix of faces, or more generally the principal components of a distribution of

faces. We generated our principal components in part c, and these form a basis for the images of all of the faces.

Part e)



Unfortunately, we failed to generate the 5x5 plot, as our code only produced an image of a vertical grey line (see middle image above). However, we understand that by adding eigenfaces in the following manner...

mean_face + score[1]*loading[1] + ... + score[m]*loading[m]

...we slowly transforms our mean face into the face of Subject 5. It would likely take us a fairly substantial number of eigenvectors (at least about $80 \sim 100$) for us to be able to recognize the person.

Part f)

```
> dim(new_matrix)
[1] 148 32256
```

Removing subject 1 leaves us with a new matrix of faces of dimension 148 by 32256. We also successfully run prcomp() to obtain new principal components (see below).

<pre>> summary(new_pca)</pre>																			
Importance of componen	ts:																		
	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11	PC12	PC13	PC14	PC15	PC16	PC17	PC18	PC19
Standard deviation	15.5266	7.2094	6.15662	5.99079	5.57408	5.31375	4.94628	4.23696	3.65965	3.44747	3.25434	3.17436	3.02384	2.91959	2.78488	2.64503	2.58397	2.50496	2.35343
Proportion of Variance	0.3678	0.0793	0.05783	0.05476	0.04741	0.04308	0.03733	0.02739	0.02044	0.01813	0.01616	0.01537	0.01395	0.01301	0.01183	0.01067	0.01019	0.00957	0.00845
Cumulative Proportion	0.3678	0.4471	0.50497	0.55973	0.60714	0.65022	0.68755	0.71494	0.73538	0.75351	0.76967	0.78504	0.79900	0.81200	0.82383	0.83451	0.84470	0.85427	0.86272
	PC20	PC21	PC22	PC23	PC24	PC25	PC2	6 PC2	7 PC28	3 PC29	PC36	PC31	PC32	PC33	PC34	PC35	PC36	PC37	PC38
Standard deviation	2.23564	2.17815	2.11883	2.08600	1.96834	1.93196	1.8600	6 1.8358	3 1.81298	3 1.76646	1.70521	1.64866	1.6394	1.58417	1.49621	1.46367	1.43663	1.38629	1.30114
Proportion of Variance	0.00763	0.00724	0.00685	0.00664	0.00591	0.00569	0.0052	8 0.0051	4 0.00502	0.00476	0.00444	0.00415	0.0041	0.00383	0.00342	0.00327	0.00315	0.00293	0.00258
Cumulative Proportion	0.87035	0.87759	0.88444	0.89108	0.89699	0.90268	0.9079	6 0.9131	0.91812	0.92288	0.92732	0.93146	0.9356	0.93939	0.94281	0.94608	0.94923	0.95216	0.95474
·	PC39	PC40	PC41	PC42	PC43	PC44	PC45	PC46	PC47	PC48	PC49	PC50	PC51	PC52	PC53	PC54	PC55	PC56	PC57
Standard deviation	1.26920	1.19783	1.1732	1.05725	1.03599	0.96921	0.92505	0.90463	0.90216	0.87494	0.86756	0.83367	0.82290	0.79682	0.77706	0.76083	0.73611	0.73041	0.72038
Proportion of Variance	0.00246	0.00219	0.0021	0.00171	0.00164	0.00143	0.00131	0.00125	0.00124	0.00117	0.00115	0.00106	0.00103	0.00097	0.00092	0.00088	0.00083	0.00081	0.00079
Cumulative Proportion	0.95720	0.95939	0.9615	0.96320	0.96483	0.96627	0.96757	0.96882	0.97006	0.97123	0.97238	0.97344	0.97447	0.97544	0.97636	0.97725	0.97807	0.97889	0.97968
	PC58	PC59	PC60	PC61	PC62	PC63	PC6	4 PC6	5 PC66	PC67	PC68	PC69	PC70	PC71	PC72	PC73	PC74	PC75	PC76
Standard deviation	0.71667	0.69545	0.68247	0.66793	0.65500	0.65219	0.6392	7 0.6178	0.60503	0.58738	0.5732	0.56603	0.53497	0.53026	0.52260	0.5092	0.49680	0.49485	0.49008
Proportion of Variance	0.00078	0.00074	0.00071	0.00068	0.00065	0.00065	0.0006	2 0.0005	8 0.00056	0.00053	0.0005	0.00049	0.00044	0.00043	0.00042	0.0004	0.00038	0.00037	0.00037
Cumulative Proportion	0.98046	0.98120	0.98191	0.98259	0.98325	0.98390	0.9845	2 0.9851	0.98566	0.98619	0.9867	0.98718	0.98761	0.98804	0.98846	0.9889	0.98923	0.98961	0.98997
	PC77	PC78	PC79	PC80	PC81	PC82	PC83	PC84	PC85	PC86	PC87	PC88	PC89	PC90	PC91	PC92	PC93	PC94	PC95
Standard deviation	0.48523	0.47223	0.46992	0.46132	0.45510	0.44976	0.4454	0.43751	0.42826	0.41397	0.41002	0.40595	0.39723	0.39520	0.38811	0.38240	0.37561	0.37085	0.3649
Proportion of Variance	0.00036	0.00034	0.00034	0.00032	0.00032	0.00031	L 0.0003	0.00029	0.00028	0.00026	0.00026	0.00025	0.00024	0.00024	0.00023	0.00022	0.00022	0.00021	0.0002
Cumulative Proportion	0.99033	0.99067	0.99101	0.99133	0.99165	0.99196	0.9923	0.99255	0.99283	0.99309	0.99335	0.99360	0.99384	0.99408	0.99431	0.99453	0.99475	0.99496	0.9952
	PC96	PC97	PC98	PC99	PC100	PC101	PC102		PC104	PC105	PC106	PC107	PC108	PC109	PC110	PC111	PC112		
Standard deviation	0.3602	0.35378	0.35102	0.34909	0.34080	0.33564	0.33374	0.32756		0.31477	0.31069	0.30445		0.29722	0.29136		0.28450	0.27622	0.27216
Proportion of Variance	0.0002	0.00019	0.00019	0.00019	0.00018	0.00017	0.00017	0.00016	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00013	0.00012	0.00012	0.00011
Cumulative Proportion	0.9954	0.99555	0.99574	0.99592	0.99610	0.99627	0.99644	0.99661	0.99677	0.99692	0.99706	0.99721	0.99734	0.99748	0.99761	0.99773	0.99786	0.99797	0.99809
			PC117	PC118	PC119	PC120	PC121	PC122	PC123	PC124	PC125	PC126	PC127	PC128	PC129	PC130	PC131	PC132	PC133
Standard deviation	0.26435	0.2588	0.2528 0	.23946 @	.23648 @	.23472 @	0.23028					0.20087 0				0.18712	0.18534	0.18175	0.17965
Proportion of Variance	0.00011	0.0001	0.0001 0	.00009 0	.00009 0	.00008 0	0.00008	0.00008	0.00007	0.00007 0	.00007	0.00006 0	.00006	0.00006	0.00006	0.00005	0.00005	0.00005	0.00005
Cumulative Proportion	0.99819	0.9983	0.9984 0	.99848 @	.99857 @	.99865 @	9.99873	0.99881	0.99888	9.99895	.99902	0.99908 0	.99914 (0.99920	9.99925	99931	0.99936	0.99941	0.99946
	PC134																		
Standard deviation	0.17872	0.17647	0.17424	0.16913	0.16850							0.14053			L 1.234e	-14			
Proportion of Variance																			
Cumulative Proportion																			
						2.333.12													

Unfortunately, we were unable to generate a reconstructed image, though we understand that we are using the eigenfaces of the remaining images to reconstruct subject 1's image. We know that the reconstructed image will look somewhat like the original image since we are eliminating data on the other subjects' faces from the mean face which thus allows us to approximate the image of subject 1.

Code

```
###################################
# Kevin Gong
# STAT W4240
# Homework 2 , Problem 1
# 2/19/14
################################
#################
# Setup
#################
# make sure R is in the proper working directory
# note that this will be a different path for every machine
setwd("~/Dropbox/SIPA/Data Mining/HW2")
# first include the relevant libraries
# note that a loading error might mean that you have to
# install the package into your R distribution. From the
# command line, type install.packages("pixmap")
library(pdist)
#################
# Problem 1a
#################
set1 = read.csv(file="hw02_q1_p1.csv")
head(set1)
colMeans(set1,na.rm=TRUE)
rowMeans(set1,na.rm=TRUE)
#################
# Problem 1b
#################
set1_centered = scale(set1, center=TRUE)
set1_emp = cov(set1_centered)
set1_emp
##################
# Problem 1c
#################
eigen(set1_emp)
#################
# Problem 1d
#################
set1_pca = princomp(set1_emp)
set1_pca$scores
set1_pca$loadings
#################
# Problem 1e ??
#################
```

```
set1_pca$sdev^2/sum(set1_pca$sdev^2)
cumsum(set1_pca$sdev^2/sum(set1_pca$sdev^2))
plot(cumsum(set1_pca$sdev^2/sum(set1_pca$sdev^2)),type='l',xlab="# components included",
ylab="proportion of variance captured")
##################
# Problem 1f
##################
set2 = read.csv(file="hw02_q1_p2.csv")
set2_centered = scale(set2, center=TRUE)
set2_pca = princomp(set2_centered)
set2_pca$scores
##################
# Problem 1g
#################
W = set1_pca$loadings[,1:2]
X = set2\_centered[1:5,1:5]
Y = X*t(W)
#euclidean distances
sqrt(sum((Y-set1_centered))^2)
pdist(Y,set1_centered)
dist(Y,set1_centered)
################
# Problem 1h
##################
errors = set2_pca$sdev-set1_pca$sdev
as.vector(errors)
```

```
# make sure R is in the proper working directory
# note that this will be a different path for every machine
setwd("~/Dropbox/SIPA/Data Mining/HW2")
# first include the relevant libraries
# note that a loading error might mean that you have to
# install the package into your R distribution. From the
# command line, type install.packages("pixmap")
library(pixmap)
#################
# Problem 2a
#################
# the list of pictures
dir_list_1 = dir(path="CroppedYale/",all.files=FALSE)
view_list = c( 'P00A+000E+00' , 'P00A+005E+10' , 'P00A+005E-10','P00A+010E+00')
# preallocate an empty list
this_face_row = vector()
# initialize an empty matrix of faces data
faces_matrix = vector()
pic_list = c(
01,02,03,04,05,06,07,08,09,10,11,12,13,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33
34,35,36,37,38,39)
#generating a matrix of photos of the 4 desired views
for (i in 1:38) {
placeholder_matrix <- NULL</pre>
for (j in 1:4) {
filename= sprintf("CroppedYale/%s/%s_%s.pgm",dir_list_1[i],dir_list_1[i],view_list[j])
print(filename)
face=read.pnm(file=filename)
pic_data <-getChannels(face)</pre>
pic_data_vec <- as.vector(pic_data)</pre>
placeholder_matrix=rbind(placeholder_matrix,pic_data_vec)
faces_matrix=rbind(faces_matrix,placeholder_matrix)
dim(faces_matrix)
#################
# Problem 2b
################
#computing the mean of the faces
faces.matrix_v2=as.matrix(faces_matrix)
means_col=colMeans(faces.matrix_v2)
means_mat=as.matrix(means_col)
#reshape this matrix into 192x168
means_mat_ref=matrix(means_mat,192,168)
#double check dimensions to make sure it worked
dim(means_mat_ref)
```

```
#generate image of mean face
means_face=pixmapGrey(means_mat_ref)
plot(means_face)
#save image of mean face
filename = 'hw02_02b.png'
dev.copy(device=png, file=filename, height=600, width=800)
dev.off()
##################
# Problem 2c
#################
means_pca = prcomp(means_mat_ref)
summary(means_pca)
plot(summary(means_pca)$importance[3,], type="l", ylab="%variance explained", xlab="nth
component")
means_pca$x
#################
# Problem 2d
#################
eigenfaces_matrix = matrix(summary(means_pca)$sdev[1:9], 3,3, byrow=TRUE)
eigenfaces_matrix
eigenfaces = pixmapGrey(eigenfaces_matrix)
plot(eigenfaces)
#################
# Problem 2e
#################
summary(means_pca)$loadings
means_pca
means_prin = princomp(means_mat_ref)
means_prin$loadings[1,]
means_prin$scores[,1]
dim(means_pca)
yaleB05 P00A+010E+00.pgm
face_05 = read.pnm(file = "CroppedYale/yaleB05/yaleB05_P00A+010E+00.pgm")
plot(face_05)
final_matrix=NULL
p=1
for(i in 1:5){
face_row=NULL
for(j in 1:5){
if(i==1 && j==1){
     z=as.vector(means_mat_ref)
     #z=rbind(z,z)
     }
else {
     means_mat=means_mat_ref+((as.matrix(means_pca$x[,p]))%*%t(as.matrix(means_pca$x[p,])))
     mean_face_t=t(means_mat)
     p=p+1
```

```
face_vec=as.vector(mean_face_t)
      z=cbind(z,face_vec)
      }
final_matrix=rbind(final,z)
faces = pixmapGrey(final_matrix)
plot(faces)
as.matrix(means_pca$x[,1])
t(as.matrix(means_pca[1,]))
loadings(means_pca)
head(means_pca)
dim(means_pca$x)
##################
# Problem 2e
##################
new_matrix = faces_matrix[5:152,]
dim(new_matrix)
new_pca = prcomp(new_matrix)
summary(new_pca)
final_matrix2=NULL
p=1
for(i in 1:5){
face_row=NULL
for(j in 1:5){
if(i==1 && j==1){
      z=as.vector(means_mat_ref)
      #z=rbind(z,z)
else {
     means_mat=new_matrx+((as.matrix(new_pca$x[,p]))%*%t(as.matrix(new_pca$x[p,])))
     mean_face_t=t(means_mat)
      p=p+1
      face_vec=as.vector(mean_face_t)
      z=cbind(z,face_vec)
final_matrix2=rbind(final,z)
```