Chevalley, Claude. (1956). *Fundamental concepts of algebra.* New York: Academic Press.
https://github.com/kmi-ne/Math-MyNotes

Chapter 1

Monoids

Convention 1.1 $a \,\tau\, b$ — Syn. for $\tau(a, b)$

τ is associative:

Definition 1.2

$$\mathrm{Assoc}(\tau, A) :\leftrightarrow \begin{cases} \tau\colon A \to A \\ \forall a, b, c \in A \ (a \,\tau\, b) \,\tau\, c = a \,\tau\, (b \,\tau\, c) \end{cases}$$

— label: `dfn_Assoc`

Example: [a]

1. $$\mathrm{Assoc}(\tau_{+}, \mathbb{Z})$$
2. $$\mathrm{Assoc}(\tau_{\cdot}, \mathbb{Z})$$
3. $$\mathrm{Assoc}\left(\tau_{\circ}, {}^{S}S\right)$$
4. $$\neg \mathrm{Assoc}(\tau_{-}, \mathbb{Z})$$

[a] $\tau_{\mathbf{f}}$: the function corresponding to a function symbol $\mathbf{f}$

neutral element:

Definition 1.3

$$\mathrm{Neut}(e, \tau, A) :\leftrightarrow \begin{cases} \tau\colon A \to A \\ \forall a \in A \ (a \,\tau\, e = e \,\tau\, a = a) \end{cases}$$

— label: `dfn_Neut`

Example:

1. $$\mathrm{Neut}(0, \tau_{+}, \mathbb{Z})$$
2. $$\mathrm{Neut}(1, \tau_{\cdot}, \mathbb{Z})$$
3. $$\mathrm{Neut}\left(\mathrm{id}, \tau_{\circ}, {}^{S}S\right)$$
4. $$\forall x \in \mathbb{Z} \ \neg \mathrm{Neut}(x, \tau_{-}, \mathbb{Z})$$

Theorem 1.4

$$!e \in A \, \mathrm{Neut}(e, \tau, A)$$

— label: `thm_neut_unq`