Rubin, Jean E. (1967). *Set Theory for the Mathematician*. San Francisco: Holden-Day.

https://github.com/kmi-ne/Math-MyNotes

第 1 章

クラス代数

1.1 クラス

定義 1.1

1. $\mathsf{M}(x) :\leftrightarrow \exists y\ (x \in y)$ {dfn_M}
2. $\mathsf{Pr}(x) :\leftrightarrow \neg\, \mathsf{M}(x)$ {dfn_Pr}

定理 1.2

$x \in y \to \mathsf{M}(x)$

証明

1: Show: $\mathsf{M}(x)$ .... by 2, 定義 1.1.1

2: $\exists y\ (x \in y)$ .... by 3 {xy3t7}

3: Assume: $x \in y$ {8afen}

公理 1.3：外延性公理

$\forall u\ (u \in x \to u \in y) \to x = y$

定義 1.4

1. $x \subseteq y :\leftrightarrow \forall u\ (u \in x \to u \in y)$ {dfn_Sbc}
2. $x \subset y :\leftrightarrow x \subseteq y \wedge x \neq y$ {dfn_Psbc}

定理 1.5

1. $x \subseteq x$ {thm_Sbc_is_reflRel}
2. $x \subseteq y \subseteq z \to x \subseteq z$ {thm_Sbc_is_transRel}
3. $x \subseteq y \subseteq x \to x = y$ {thm_Sbc_is_antisymRel}

証明

1. 1: Show: $x \subseteq x$ .... by 2, 定義 1.4.1

   2: $\forall u\ (u \in x \to u \in x)$ {rlo90}

2. 1: Show: $x \subseteq z$ .... by 2, 定義 1.4.1

   2: $\forall u\ (u \in x \to u \in z)$ .... by 3 {fioly}

   3: $\begin{cases} \forall u\ (u \in x \to u \in y) \\ \forall u\ (u \in y \to u \in z) \end{cases}$ .... by 4

   4: Assume: $x \subseteq y \subseteq z$ {yyrfy} {ara7r}

3. 1: Show: $x = y$ .... by 2, 外延性公理

31xs0 2: $\forall u\ (u \in x \leftrightarrow u \in y)$ .... by 3 {31xs0}

3: $\begin{cases} \forall u\ (u \in y \to u \in x) \\ \forall u\ (u \in x \to u \in y) \end{cases}$ .... by 4, 定義 1.4.1

cc53c {cc53c}

d1nw6 4: Assume: $x \subseteq y \subseteq x$ {d1nw6}

定理 1.6

lRel 1. $\neg(x \subset x)$ {thm_Psbc_is_irreflRel}

elem 2. $x \subset y \leftrightarrow \begin{cases} x \subseteq y \\ \exists u\ (u \in y \wedge u \notin x) \end{cases}$ {thm_psbc_has_less_elem}

psbc 3. $x \subset y \subseteq z \to x \subset z$ {thm_psbc_of_sbc_is_psbc}

psbc 4. $x \subseteq y \subset z \to x \subset z$ {thm_sbc_of_psbc_is_psbc}

psbc 5. $x \subset y \subset z \to x \subset z$ {thm_psbc_of_psbc_is_psbc

r_Eq 6. $x \subseteq y \leftrightarrow x \subset y \vee x = y$ {thm_sbc_eqv_psbc_or_Eq}

証明

1. 1: Show: $\neg(x \subset x)$ .... by 2, 定義 1.4.2

uiiks 2: $\neg(x \subseteq x \wedge x \neq x)$ .... by 3 {uiiks}

swxyr 3: $x = x$ {swxyr}

2. 1: Show: $x \subset y$ .... by 5, 3, 定義 1.4.2

k9059 2: $x \neq y$ .... by 4, 3 {k9059}

t31j4 3: $x = y \to \neg\exists u\ (u \in y \wedge u \notin x)$ {t31j4}

s4dtt 4: Assume: $\exists u\ (u \in y \wedge u \notin x)$ {s4dtt}

3z6k9 5: Assume: $x \subseteq y$ {3z6k9}

6: Show: $\exists u\ (u \in y \wedge u \notin x)$ .... by 7, 定義 1.4.1

xibkc 7: $\neg(y \subseteq x)$ .... by 9, 8, 定理 1.5.3 {xibkc}

1t6a4 8: $x \neq y$ .... by 10, 定義 1.4.2 {1t6a4}

i0br2 9: and Show: $x \subseteq y$ .... by 10, 定義 1.4.2 {i0br2}

wzl3r 10: Assume: $x \subset y$ {wzl3r}

3. 1: Show: $x \subset z$ .... by 5, 2, 定理 1.6.2

8791f 2: Show: $\exists u\ (u \in z \wedge u \notin x)$ .... by 4, 3 {8791f}

yed92 3: $u \in y \to u \in z$ .... by 7, 定義 1.4.1 {yed92}

z3cnf 4: $\exists u\ (u \in y \wedge u \notin x)$ .... by 7, 定理 1.6.2 {z3cnf}

4vc6p 5: Show: $x \subseteq z$ .... by 6, 定理 1.5.2 {4vc6p}

vgmmi 6: $x \subseteq y \subseteq z$ .... by 7, 定義 1.4.2 {vgmmi}

lwwv5 7: Assume: $x \subset y \subseteq z$ {lwwv5}

4. 1: Show: $x \subset z$ .... by 5, 2, 定理 1.6.2

dzgv3 2: Show: $\exists u\ (u \in z \wedge u \notin x)$ .... by 4, 3 {dzgv3}

f5h3u 3: $u \in x \to u \in y$ .... by 7, 定義 1.4.1 {f5h3u}

pfimo 4: $\exists u\ (u \in z \wedge u \notin y)$ .... by 7, 定理 1.6.2 {pfimo}

mabal 5: Show: $x \subseteq z$ .... by 6, 定理 1.5.2 {mabal}

q216c 6: $x \subseteq y \subseteq z$ .... by 7, 定義 1.4.2 {q216c}

3e2fz 7: Assume: $x \subseteq y \subset z$ {3e2fz}

5. 1: Show: $x \subset z$ .... by 2, 定理 1.6.3

vuc8r 2: $x \subset y \subseteq z$ .... by 3, 定義 1.4.2 {vuc8r}

vig6r 3: Assume: $x \subset y \subset z$ {vig6r}

6. 1: Show: $x \subseteq y \leftrightarrow x \subset y \lor x = y$ .... by 7, 2

2: Show: $\left[\begin{array}{l} x \subseteq y \land x \neq y \\ x \subseteq y \land x = y \end{array}\right. \leftrightarrow x = y$ .... by 6, 3

or0bx 3: Show: $x \subseteq y \land x = y \leftrightarrow x = y$ .... by 5, 4 {or0bx}
s4mfd {s4mfd}

bj1u4 4: $x \subseteq y \land x = y \to x = y$ {bj1u4}

7grw5 5: $x = y \to x \subseteq y \land x = y$ .... by 定理 1.5.1 {7grw5}

le22s 6: $x \subseteq y \land x \neq y \leftrightarrow x \subset y$ .... by 定義 1.4.2 {le22s}

ktr2d 7: $x \subseteq y \leftrightarrow \left[\begin{array}{l} x \subseteq y \land x \neq y \\ x \subseteq y \land x = y \end{array}\right.$ {ktr2d}

定義 1.7

n_cls $u \in \{v \mid \phi\} \leftrightarrow \phi[u/v] \land \mathsf{M}(u)$

定理 1.8

eq_x 1. $\{u \mid u \in x\} = x$ {thm_cls_of_elem_of_x_eq_

_sbc 2. $(\phi \to \psi) \to \{u \mid \phi\} \subseteq \{u \mid \psi\}$ {thm_imp_wff_yield_sbc}

_cls 3. $(\phi \to \psi) \leftrightarrow \{u \mid \phi\} = \{u \mid \psi\}$ {thm_eqv_wff_yield_eq_cls

it_M 4. $(\phi \to \mathsf{M}(v)) \to (u \in \{v \mid \phi\} \leftrightarrow \phi[u/v])$ {thm_wff_imp_M_then_omit_

_wff 5. $\{u \mid \phi \land \mathsf{M}(u)\} = \{u \mid \phi\}$ {thm_cls_wff_and_M_eq_cls

land 6. $\{u \mid u \in \{v \mid \phi\} \land \psi\} = \{u \mid \phi[u/v] \land \psi\}$ {thm_cls_of_elem_of_cls_l

_lor 7. $\{u \mid u \in \{v \mid \phi\} \lor \psi\} = \{u \mid \phi[u/v] \lor \psi\}$ {thm_cls_of_elem_of_cls_l

証明

1. 1: Show: $\{u \mid u \in x\} = x$ .... by 外延性公理

wg8mj 2: Show: $u \in \{u \mid u \in x\} \leftrightarrow u \in x$ .... by 4, 3 {wg8mj}

ngzg5 3: $u \in x \land \mathsf{M}(u) \leftrightarrow u \in x$ .... by 定理 1.2 {ngzg5}

62eyh 4: $u \in \{u \mid u \in x\} \leftrightarrow u \in x \land \mathsf{M}(u)$ .... by 定義 1.7 {62eyh}

2. 1: Show: $\{u \mid \phi\} \subseteq \{u \mid \psi\}$ .... by 2, 定義 1.4.1

oljua 2: $u \in \{u \mid \phi\} \to u \in \{u \mid \psi\}$ .... by 5, 4, 3 {oljua}

uy8bz 3: $\psi \land \mathsf{M}(u) \to u \in \{u \mid \psi\}$ .... by 定義 1.7 {uy8bz}

3olbo 4: $\phi \land \mathsf{M}(u) \to \psi \land \mathsf{M}(u)$ .... by 6 {3olbo}

gyx9f 5: $u \in \{u \mid \phi\} \to \phi \land \mathsf{M}(u)$ .... by 定義 1.7 {gyx9f}

3entz 6: Assume: $\phi \to \psi$ {3entz}

3. 1: Show: $(\phi \to \psi) \leftrightarrow \{u \mid \phi\} = \{u \mid \psi\}$ .... by 3, 2, 定理 1.5.3

ysj0h 2: $(\phi \to \psi) \to \{u \mid \phi\} \subseteq \{u \mid \psi\}$ .... by 定理 1.8.2 {ysj0h}

tesqy 3: $(\psi \to \phi) \to \{u \mid \psi\} \subseteq \{u \mid \phi\}$ .... by 定理 1.8.2 {tesqy}

4. 1: Show: $u \in \{v \mid \phi\} \leftrightarrow \phi[u/v]$ .... by 3, 2

yvfw3 2: $\phi[u/v] \land \mathsf{M}(u) \leftrightarrow \phi[u/v]$ .... by 4 {yvfw3}

whc4n 3: $u \in \{v \mid \phi\} \leftrightarrow \phi[u/v] \land \mathsf{M}(u)$ .... by 定義 1.7 {whc4n}

la3n2 4: Assume: $\phi \to \mathsf{M}(v)$ {la3n2}

5. 1: Show: $\{u \mid \phi \land \mathsf{M}(u)\} = \{u \mid \phi\}$ .... by 2, 外延性公理

wdhlr 2: Show: $u \in \{u \mid \phi \land \mathsf{M}(u)\} \leftrightarrow u \in \{u \mid \phi\}$ .... by 5, 4, 3 {wdhlr}

tss9b 3: $\phi \land \mathsf{M}(u) \leftrightarrow u \in \{u \mid \phi\}$ .... by 定義 1.7 {tss9b}

4igwa 4: $\phi \land \mathsf{M}(u) \land \mathsf{M}(u) \leftrightarrow \phi \land \mathsf{M}(u)$ {4igwa}

v58z3 5: $u \in \{u \mid \phi \land \mathsf{M}(u)\} \leftrightarrow \phi \land \mathsf{M}(u) \land \mathsf{M}(u)$ .... by 定義 1.7 {v58z3}

定義 1.9

1. _emp $\varnothing := \{u \mid u \neq u\}$ {dfn_emp}
2. univ $\mathbf{U} := \{u \mid u = u\}$ {dfn_univ}
3. russ $\mathbf{Ru} := \{u \mid u \notin u\}$ {dfn_russ}

定理 1.10

1. _emp $u \notin \varnothing$ {thm_nothing_is_elem_of_e
2. univ $\mathsf{M}(u) \to u \in \mathbf{U}$ {thm_set_is_elem_of_univ}
3. hing $\varnothing \subseteq x$ {thm_emp_is_sbc_of_everyt
4. univ $x \subseteq \mathbf{U}$ {thm_everything_is_sbc_of

定理 1.11

is_pr $\mathsf{Pr}(\mathbf{Ru})$

定義 1.12

1. _cup $x \cup y := \{u \mid u \in x \vee u \in y\}$ {dfn_cup}
2. _cap $x \cap y := \{u \mid u \in x \wedge u \in y\}$ {dfn_cap}
3. cdif $x \setminus y := \{u \mid u \in x \wedge u \notin y\}$ {dfn_cdif}
4. cmpl $x^{\complement} := \{u \mid u \notin x\}$ {dfn_cmpl}