Hinman, Peter G. (2005). *Fundamentals of Mathematical Logic*. A K Peters.
https://github.com/kmi-ne/Math-MyNotes

Chapter 1

Propositional Logic

connective, sentence symbol, L-symbol, L-expression:

Definition 1.1 [a] [b]

1. $$\mathsf{Connec} := \{\neg, \vee, \wedge, \rightarrow, \leftrightarrow\}$$
2. $$\mathsf{SentSymb} := \{p_n \mid n \in \omega\}$$
3. $$\mathsf{Symb}_L := \mathsf{Connec} \cup \mathsf{SentSymb}$$
4. $$\mathsf{Expr}_L := \mathsf{Sent}_L^{<\omega}$$

— label: `dfn_Connec` `dfn_SentSymb` `dfn_Symb` `dfn_Expr`

[a] $\neg, \vee, \wedge, \rightarrow, \leftrightarrow, p_n$ may be defined as $0, 1, 2, 3, 4, n+5$ (resp.).
[b] $X^{<\omega} := \{f \mid \exists N \in \omega\ (f\colon N \to X)\}$: the set of all finite sequences on X

concatenation:

Definition 1.2

$$f \frown g := \{\langle n + \mathrm{dom}(f), g(n)\rangle \mid n \in \mathrm{dom}(g)\}$$

— label: `dfn_conc`

Convention 1.3 $[s_0 \ldots s_{\mathbf{n}}]$ / $s_0 \ldots s_{\mathbf{n}}$ — Syn. for $\{\langle 0, s_0\rangle, \ldots, \langle n, s_{\mathbf{n}}\rangle\}$

L-atomic sentence:

Definition 1.4 [a]

$$\mathsf{AtSent}_L := \mathsf{SentSymb}^1$$

See: SentSymb

— label: `dfn_AtSent`

[a] $X^n := \{f \mid f\colon n \to X\}$: the set of all n-term sequences on X

$\mathsf{AtSent}_L = \{[s] \mid s \in \mathsf{SentSymb}\}$
$\mathsf{AtSent}_L \subseteq \mathsf{Expr}_L$

definition by recursion:

Theorem 1.5

$$\begin{cases} z \in Z \\ G\colon Z \times \omega \to Z \end{cases} \to \exists! F\colon \omega \to Z \begin{cases} F(0) = z \\ \forall n \in \omega\ F(n^+) = G(F(n), n) \end{cases}$$

— label: `thm_recdfn`

$(X, A, \mathcal{H})$ is an induction system:

Definition 1.6

$$\mathrm{Ind}(X, A, \mathcal{H}) :\leftrightarrow \begin{cases} A \subseteq X \neq \varnothing \\ \forall H \in \mathcal{H}\ \exists n \in \omega\ (H\colon X^n \to X) \end{cases}$$

— label: `dfn_Ind`

$!n \in \omega\ (H\colon X^n \to X)$. Define $k_{H,X}$ as n:

$$\exists n \in \omega\ (H\colon X^n \to X) \to \begin{cases} k_{H,X} \in \omega \\ H\colon X^{k_{H,X}} \to X \end{cases}$$

. Thus

$$\mathrm{Ind}(X, A, \mathcal{H}) \leftrightarrow \begin{cases} A \subseteq X \neq \varnothing \\ \forall H \in \mathcal{H} \begin{cases} k_{H,X} \in \omega \\ H\colon X^{k_{H,X}} \to X \end{cases} \end{cases}$$

. Write meta-$k_{H,X}$ as $\mathbf{k}_{H,X}$.

Theorem 1.7

$$\mathrm{Ind}(X, A, \mathcal{H}) \to \exists! F\colon \omega \to \wp(X) \begin{cases} F(0) = A \\ \forall n \in \omega\ F(n^+) = F(n) \cup \left\{ H(x_{\mathbf{1}}, \dots, x_{\mathbf{k}_{H,X}}) \;\middle|\; \begin{cases} H \in \mathcal{H} \\ x_{\mathbf{1}}, \dots, x_{\mathbf{k}_{H,X}} \in F(n) \end{cases} \right\} \end{cases}$$

See: $\mathrm{Ind}(X, A, \mathcal{H})$ — label: `thm_rec_suc`

Definition 1.8 Define: $(X, A, \mathcal{H})$ as F in Theorem 1.7:

$$\mathrm{Ind}(X, A, \mathcal{H}) \to \begin{cases} (X, A, \mathcal{H})\colon \omega \to \wp(X) \\ (X, A, \mathcal{H})(0) = A \\ \forall n \in \omega\ (X, A, \mathcal{H})(n^+) = (X, A, \mathcal{H})(n) \cup \left\{ H(x_{\mathbf{1}}, \dots, x_{\mathbf{k}_{H,X}}) \;\middle|\; \begin{cases} H \in \mathcal{H} \\ x_{\mathbf{1}}, \dots, x_{\mathbf{k}_{H,X}} \in (X, A, \mathcal{H})(n) \end{cases} \right\} \end{cases}$$

See: $\mathrm{Ind}(X, A, \mathcal{H})$ — label: `dfn_recmap`