Rubin, Jean E. (1967). *Set Theory for the Mathematician*. San Francisco: Holden-Day.
https://github.com/kmi-ne/Math-MyNotes

Chapter 1

Class algebra

1.1 Class

Definition 1.1 — x is a set/proper class

1. $\mathsf{M}(x) :\leftrightarrow \exists y\ (x \in y)$
2. $\mathsf{Pr}(x) :\leftrightarrow \neg\, \mathsf{M}(x)$

— label: `Dfn_M`
`Dfn_Pr`

Proposition 1.2

$x \in y \rightarrow \mathsf{M}(x)$ — label: `Thm_elem_is_set`

Proof:

1. **Show:** $\mathsf{M}(x)$ ———— (2, Def. 1.1.1)
2. $\exists y\ (x \in y)$ ———— (3)
3. **Assume:** $x \in y$

Axiom 1.3 — Axiom of Extensionality

$\forall u\ (u \in x \leftrightarrow u \in y) \rightarrow x = y$ — label: `Axm_ext`

Definition 1.4 — x is a subclass/proper subclass of y

1. $x \subseteq y :\leftrightarrow \forall u\ (u \in x \rightarrow u \in y)$
2. $x \subset y :\leftrightarrow x \subseteq y \wedge x \neq y$

— label: `Dfn_Sbc`
`Dfn_Psbc`

Proposition 1.5

1. $x \subseteq x$
2. $x \subseteq y \subseteq z \rightarrow x \subseteq z$
3. $x \subseteq y \subseteq x \rightarrow x = y$

— label: `Thm_Sbc_is_reflRel`
`Thm_Sbc_is_transRel`
`Thm_Sbc_is_antisymRel`

Proof:

1. 1. $x \subseteq x$ ———— (2, Def. 1.1.1)
   2. $\forall u\ (u \in x \rightarrow u \in x)$
2. 1. **Show:** $x \subseteq z$ ———— (2)
   2. $\forall u\ (u \in x \rightarrow u \in z)$ ———— (3)
   3. $\begin{cases} \forall u\ (u \in x \rightarrow u \in y) \\ \forall u\ (u \in y \rightarrow u \in z) \end{cases}$ ———— (4)
   4. **Assume:** $x \subseteq y \subseteq z$
3. 1. **Show:** $x = y$ ———— (2, Axiom of Extensionality)

2. $\forall u\ (u \in x \leftrightarrow u \in y)$ ——— (3)

3. $\begin{cases} \forall u\ (u \in x \to u \in y) \\ \forall u\ (u \in y \to u \in x) \end{cases}$ ——— (4, Def. 1.4.1)

4. **Assume:** $x \subseteq y \subseteq x$

Proposition 1.6

1. $\neg(x \subset x)$
2. $x \subset y \leftrightarrow \begin{cases} x \subseteq y \\ \exists u\ (u \in y \land u \notin x) \end{cases}$
3. $x \subset y \subseteq z \to x \subset z$
4. $x \subseteq y \subset z \to x \subset z$
5. $x \subset y \subset z \to x \subset z$
6. $x \subseteq y \leftrightarrow x \subset y \lor x = y$

— label: `Thm_Psbc_irreflRel`
`Thm_psbc_has_less_elem`
`Thm_psbc_of_sbc_is_psbc`
`Thm_sbc_of_psbc_is_psbc`
`Thm_psbc_of_psbc_is_psbc`
`Thm_sbc_eqv_psbc_or_Eq`

Proof:

1\. 1. $\neg(x \subset x)$ ——— (2, Def. 1.4.2)
2. $\neg(x \subseteq x \land x \neq x)$ ——— (3)
3. $x = x$

2\. 1. **Show:** $\begin{cases} x \subseteq y \\ \exists u\ (u \in y \land u \notin x) \end{cases} \to x \subset y$
2. **Show:** $x \subset y$ ——— (5, 3)
3. **Show:** $x \neq y$ ——— (4, 6)
4. $x = y \to \neg\exists u\ (u \in y \land u \notin x)$
5. **Assume:** $x \subseteq y$
6. **Assume:** $\exists u\ (u \in y \land u \notin x)$
7. **Show:** $x \subset y \to \begin{cases} x \subseteq y \\ \exists u\ (u \in y \land u \notin x) \end{cases}$
8. **Show:** $\exists u\ (u \in y \land u \notin x)$ ——— (9, Def. 1.4.1)
9. $\neg(y \subseteq x)$ ——— (11, 10, Prop. 1.5.3)
10. $x \neq y$ ——— (12, Def. 1.4.2)
11. **Show:** $x \subseteq y$ ——— (12, Def. 1.4.2)
12. **Assume:** $x \subset y$

3\. 1. **Show:** $x \subset z$ ——— (2, 4, Prop. 1.6.2)
2. **Show:** $x \subseteq z$ ——— (3, Prop. 1.5.2)
3. $x \subseteq y \subseteq z$ ——— (7, Def. 1.4.2)
4. **Show:** $\exists u\ (u \in z \land u \notin x)$ ——— (6, 5)
5. $u \in y \to u \in z$ ——— (7, Def. 1.4.1)
6. $\exists u\ (u \in y \land u \notin x)$ ——— (7, Prop. 1.6.2)
7. **Assume:** $x \subset y \subseteq z$

4\. 1. **Show:** $x \subset z$ ——— (2, 4, Prop. 1.6.2)
2. **Show:** $x \subseteq z$ ——— (3, Prop. 1.5.2)
3. $x \subseteq y \subseteq z$ ——— (7, Def. 1.4.2)
4. **Show:** $\exists u\ (u \in z \land u \notin x)$ ——— (6, 5)
5. $u \in x \to u \in y$ ——— (7, Def. 1.4.1)
6. $\exists u\ (u \in z \land u \notin y)$ ——— (7, Prop. 1.6.2)
7. **Assume:** $x \subseteq y \subset z$

5\. 1. **Show:** $x \subset z$ ——— (2, Prop. 1.6.3)

2. $x \subset y \subseteq z$ (3, Def. 1.4.2)
3. **Assume:** $x \subset y \subset z$

6\. 1. $x \subseteq y \leftrightarrow x \subset y \vee x = y$ (7, 2)

2. **Show:** $\begin{bmatrix} x \subseteq y \wedge x \neq y \\ x \subseteq y \wedge x = y \end{bmatrix} \leftrightarrow x \subset y \vee x = y$ (3, 4)
3. **Show:** $x \subseteq y \wedge x \neq y \leftrightarrow x \subset y$ (Def. 1.4.2)
4. **Show:** $x \subseteq y \wedge x = y \leftrightarrow x = y$ (6, 5)
5. **Show:** $x \subseteq y \wedge x = y \rightarrow x = y$
6. **Show:** $x = y \rightarrow x \subseteq y \wedge x = y$ (Prop. 1.5.1)
7. **Show:** $x \subseteq y \leftrightarrow \begin{bmatrix} x \subseteq y \wedge x \neq y \\ x \subseteq y \wedge x = y \end{bmatrix}$