Qualifying Exam 2020 Solution

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1a $(d.a.b.0)\backslash\{d,b\} + (b.(*.0 + c.a.0))$

1a - solution 1 $(d.a.b.0)\backslash\{d,b\}$

$$\frac{}{(d.a.b.0)\backslash\{d,b\} + (b.(*.0 + c.a.0)) \xrightarrow{*} (d.a.b.0)\backslash\{d,b\}}\textbf{Choice}_1$$

1a - solution 2 $(b.(*.0 + c.a.0))$

$$\frac{}{(d.a.b.0)\backslash\{d,b\} + (b.(*.0 + c.a.0)) \xrightarrow{*} (b.(*.0 + c.a.0))}\textbf{Choice}_2$$

1b $(a.b.0||c.a.0)||c.b.0$

1b - solution 1 $(a.b.0||c.a.0)||b.0$

$$\frac{\frac{}{c.b.0 \xrightarrow{c} b.0}\textbf{Prefix}}{((a.b.0||c.a.0)||c.b.0) \xrightarrow{c} ((a.b.0||c.a.0)||b.0)}\textbf{Par}_2$$

1b - solution 2 $(b.0||c.a.0)||c.b.0$

$$\frac{\frac{\frac{}{a.b.0 \xrightarrow{a} b.0}\textbf{Prefix}}{(a.b.0||c.a.0) \xrightarrow{a} (b.0||c.a.0)}\textbf{Par}_1}{((a.b.0||c.a.0)||c.b.0) \xrightarrow{a} (b.0||c.a.0)||c.b.0}\textbf{Par}_1$$

1b - solution 3 $(a.b.0||a.0)||c.b.0$

$$\frac{\dfrac{\dfrac{}{c.a.0 \xrightarrow{c} a.0}\,\textbf{Prefix}}{(a.b.0||c.a.0) \xrightarrow{c} (a.b.0||a.0)}\,\textbf{Par}_\mathbf{2}}{((a.b.0||c.a.0)||c.b.0) \xrightarrow{c} (a.b.0||a.0)||c.b.0}\,\textbf{Par}_\mathbf{1}$$

1b - solution 4 $(a.b.0||a.0)||b.0$

$$\frac{\dfrac{\dfrac{}{c.a.0 \xrightarrow{c} a.0}\,\textbf{Prefix}}{(a.b.0||c.a.0) \xrightarrow{c} (a.b.0||a.0)}\,\textbf{Par}_\mathbf{2} \quad \dfrac{}{c.b.0 \xrightarrow{c} b.0}\,\textbf{Prefix}}{((a.b.0||c.a.0)||c.b.0) \xrightarrow{*} ((a.b.0||a.0)||b.0)}\,\textbf{Par}_\mathbf{3}$$

1c $((a.0 + b.0)||c.0)||e.0$

1c - solution 1 $((a.0 + b.0)||c.0)||0$

$$\frac{\dfrac{}{e.0 \xrightarrow{e} 0}\,\textbf{Prefix}}{(((a.0 + b.0)||c.0)||e.0) \xrightarrow{e} (((a.0 + b.0)||c.0)||0)}\,\textbf{Par}_\mathbf{2}$$

1c - solution 2 $((a.0 + b.0)||0)||e.0$

$$\frac{\dfrac{\dfrac{}{c.0 \xrightarrow{c} 0}\,\textbf{Prefix}}{((a.0+b.0)||c.0) \xrightarrow{c} ((a.0+b.0)||0)}\,\textbf{Par}_\mathbf{2}}{(((a.0 + b.0)||c.0)||e.0) \xrightarrow{c} (((a.0 + b.0)||0)||e.0)}\,\textbf{Par}_\mathbf{1}$$

1c - solution 3 $(a.0||c.0)||e.0$

$$\frac{\dfrac{\dfrac{}{(a.0+b.0) \xrightarrow{*} a.0}\,\textbf{Choice}_\mathbf{1}}{((a.0+b.0)||c.0) \xrightarrow{*} (a.0||c.0)}\,\textbf{Par}_\mathbf{1}}{(((a.0 + b.0)||c.0)||e.0) \xrightarrow{*} ((a.0||c.00)||e.0)}\,\textbf{Par}_\mathbf{1}$$

1c - solution 4 $(b.0||c.0)||e.0$

$$\dfrac{\dfrac{\dfrac{}{(a.0+b.0)\xrightarrow{*}b.0}\,\textbf{Choice}_\mathbf{2}}{((a.0+b.0)||c.0)\xrightarrow{*}(b.0||c.0)}\,\textbf{Par}_\mathbf{1}}{(((a.0+b.0)||c.0)||e.0)\xrightarrow{*}((b.0||c.00)||e.0)}\,\textbf{Par}_\mathbf{1}$$

1d $((b.c.0||a.d.0)||b.0)\backslash\{b,c\}$

1d - solution 1 $((b.c.0||d.0)||b.0)\backslash\{b,c\}$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{a.d.0\xrightarrow{\text{a}}d.0}\,\textbf{Prefix}}{(b.c.0||a.d.0)\xrightarrow{\text{a}}(b.c.0||d.0)}\,\textbf{Par2}}{((b.c.0||a.d.0)||b.0)\xrightarrow{\text{a}}((b.c.0||d.0)||b.0)}\,\textbf{Par}_\mathbf{1}}{(((b.c.0||a.d.0)||b.0)\backslash\{b,c\})\xrightarrow{\text{a}}((b.c.0||d.0)||b.0)\backslash\{b,c\}}\,\textbf{Restrict}$$

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Proof by induction over Q.

$P = 0$ There are no derivations, this case does not matter.

$P = \alpha.P'$. There is one derivation: P'.
$sort(\alpha.P') = (\{\alpha\} - *) \cup sort(P')$.
We must prove that $sort(P') \subseteq (\alpha.P')$.
$sort(P') \subseteq \{\alpha\} \cup sort(P')$.
In the case where $\alpha = *$, then the two sets are equal.
Otherwise, this is naturally evident by the laws of sets.

$P = P_1 + P_2$. There are two derivations: P_1 and P_2.
$sort(P_1 + P_2) = sort(P_1) \cup sort(P_2)$.
For P_1:
We must prove that $sort(P_1) \subseteq sort(P_1 + P_2)$.
$sort(P_1) \subseteq sort(P_1) \cup sort(P_2)$.
This is naturally evident by the laws of sets.
The same holds for P_2.

$P = P_1 || P_2$. There are three derivations: $P_1' || P_2$, $P_1 || P_2'$, $P_1' || P_2'$

The inductive hypothesis is: $sort(P_1') \subseteq sort(P_1)$ and $sort(P_2') \subseteq sort(P_2)$

$sort(P_1 || P_2) = sort(P_1) \cup sort(P_2)$.

For the first derivation $P_1' || P_2$:

We must prove that $sort(P_1' || P_2) \subseteq sort(P_1 || P_2)$.

$sort(P_1') \cup sort(P_2) \subseteq sort(P_1) \cup sort(P_2)$.

This holds by the laws of sets when you apply the inductive hypothesis.

The same method applies to the two other derivations.

$P = P' \backslash X$. There is one derivation: $P'' \backslash X$

The inductive hypothesis is: $sort(P'') \subseteq sort(P')$ and that P'' isnt derived using a label in X.

$sort(P' \backslash X) = sort(P') - X$

$sort(P'' \backslash X) = sort(P'') - X$

We must prove that $sort(P'' \backslash X) \subseteq sort(P' \backslash X)$

$sort(P'') - X \subseteq sort(P') - X$

This holds by the inductive hypothesis, and the laws of sets.

$\square$