Homework Statistical Physics NCBJ - set 1

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Problem 2

Consider both ideal and real gas equations. Calculate the following quantities:

b).

$$\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{p,n}$$

c).

$$\left(\frac{\partial U}{\partial T}\right)_{V,n}$$

1 b).

(FOR IDEAL GAS)

First let us notice that:

(1)
$$pV = nRT \Rightarrow V = \left(\frac{nR}{p}\right)T.$$

For the sake of legibility let us denote $A = \left(\frac{nR}{p}\right)$.

Having established the formula for V we can procede with solving:

(2)
$$\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{p,n} = \frac{A}{V}\left(\frac{\partial T}{\partial T}\right)_{p,n} = \frac{A}{V} = \frac{nR}{pV}.$$

(FOR REAL GAS - VAN DER WAALS)

For Van der Waals gas the relation between thermodynamic parameters (p, V, T) is described by the following expression:

(3)
$$p = \frac{nRT}{(V - nb)} - \frac{an^2}{V^2}.$$

We are supposed to calculate the following expression.

(4)

$$\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{p,n} = \frac{1}{V}\left(\frac{1}{\left(\frac{\partial T}{\partial V}\right)_{p,n}}\right).$$

The above equation is much easier to calculate if we write:

(5)

$$T = \left(p + \frac{an^2}{V^2}\right)(V - nb)\frac{1}{nR}.$$

Now, we need to calculate the partial derivative of (5) which yields: (6)

$$\left(\frac{\partial T}{\partial V}\right)_{p,n} = \frac{\partial}{\partial V}\left[\frac{pV}{nR} + \frac{an}{VR} - \frac{pb}{R} - \frac{abn^2}{V^2R}\right] = \left[\frac{p}{nR} - \frac{an}{V^2R} + \frac{abn^2}{V^3R}\right].$$

Finally, if we join (6) and (4) we will end up with:

(7)

$$\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{p,n} = \frac{1}{V}\left(\frac{1}{\left(\frac{\partial T}{\partial V}\right)_{p,n}}\right) = \frac{1}{\left(\frac{pV}{nR} - \frac{an}{VR} + \frac{abn^2}{V^2R}\right)}$$

2 c).

(ENERGY FOR IDEAL GAS)

(8)

$$U = \frac{t}{2}nRT - \frac{an^2}{V},$$

where t marks the number of degrees of freedom.

(ENERGY FOR REAL GAS-VAN DER WAALS)

(9)

$$U = \frac{t}{2}nRT$$

(SOLUTION FOR IDEAL GAS)

(10)

$$\left(\frac{\partial U}{\partial T}\right)_{V,n} = \frac{\partial}{\partial T}\left(\frac{t}{2}nRT\right)_{V,n} = \frac{t}{2}nR$$

(SOULTION FOR REAL GAS)

(11)

$$\left(\frac{\partial U}{\partial T}\right)_{V,n} = \frac{\partial}{\partial T}\left(\frac{t}{2}nRT - \frac{an^2}{V}\right)_{V,n} = \frac{t}{2}nR$$

Problem 5

One mole of ideal gas (monoatomic t=3) was heated in an isochoric way. How much did the energy of the gas change?

According to the first law of thermodynamics:

(12)
$$dU = \delta Q - pdV.$$

Isohoric transformation can be expressed as:

$$(p_1, V, T_1) \rightarrow (p_2, V, T_2).$$

Since $\Delta V = V - V = 0$, there is no work done.

The heat transfer, on the other hand, can be written as:

(13)
$$\delta Q = nC_V dT = \frac{3}{2}nRdT.$$

The solution to this problem, where $n = 1$ is thus simply:

(14)
$$\Delta U = \frac{3}{2}R(T_2 - T_1) = \frac{3}{2}R\Delta T.$$

Problem 6

One mole of ideal gas was compressed adiabatically from (p_0, V_0) state to (p_1, V_1) state. Describe how p, V and U changed during the process. Use the first law of thermodynamics and eq. of the adiabatic process.

Equation of the adiabatic process:

(15)

$$p_0 V_0^\gamma = p_1 V_1^\gamma = const,$$
$$pV^\gamma = const,$$

where $\gamma = \frac{C_p}{C_V} = \frac{t+2}{t}$.

Let us denote *const* from eq (15) as K so that

(16)
$$pV^\gamma = K.$$

An adiabatic process is one in which no heat is gained or lost by the system and so the change in internal energy is only in the form of work. Consequently, $\Delta U = W$.

(17)

$$W = \int_{V_0}^{V_1} pdV = \int_{V_0}^{V_1} \frac{K}{V^\gamma}dV = \frac{1}{\gamma - 1}\left[\frac{K}{V_1^{\gamma-1}} - \frac{K}{V_0^{\gamma-1}}\right] = \frac{1}{\gamma - 1}[p_1 V_1 - p_0 V_0].$$

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Answer:

(18)

$$\Delta U = W = \frac{1}{\gamma - 1} \left[p_1 V_1 - p_0 V_0 \right]$$

(19)

$$\Delta p = p_1 - p_0 = \frac{K}{V_1^\gamma} - \frac{K}{V_0^\gamma}$$

(20)

$$\Delta V = V_1 - V_0 = \left(\frac{K}{p_1} \right)^{\frac{1}{\gamma}} - \left(\frac{K}{p_0} \right)^{\frac{1}{\gamma}}$$

We can express (18), (19), (20) in terms of temperature T by using the following relations (derived using the ideal gas equation):

(21)

$$TV^{\gamma - 1} = const = B,$$

(22)

$$p^{1-\gamma} T^\gamma = const = A.$$

Consequently we have:

(23)

$$\Delta p = p_1 - p_0 = \left(\frac{A}{T_1} \right)^{\frac{\gamma}{1-\gamma}} - \left(\frac{A}{T_0} \right)^{\frac{\gamma}{1-\gamma}},$$

(24)

$$\Delta V = V_1 - V_0 = \left(\frac{B}{T_1} \right)^{\frac{1}{\gamma - 1}} - \left(\frac{B}{T_0} \right)^{\frac{1}{\gamma - 1}}.$$

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