

# Homework Statistical Physics NCBJ - set 1

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## Problem 2

Consider both ideal and real gas equations. Calculate the following quantities:

b).

$$\frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_{p,n}$$

c).

$$\left( \frac{\partial U}{\partial T} \right)_{V,n}$$

**b).**

(FOR IDEAL GAS)

First let us notice that:

(1)

$$pV = nRT \Rightarrow V = \left( \frac{nR}{p} \right) T.$$

For the sake of legibility let us denote  $A = \left( \frac{nR}{p} \right)$ .

Having established the formula for  $V$  we can proceed with solving:

(2)

$$\frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_{p,n} = \frac{A}{V} \left( \frac{\partial T}{\partial T} \right)_{p,n} = \frac{A}{V} = \frac{nR}{pV}.$$

(FOR REAL GAS - VAN DER WAALS)

For Van der Waals gas the relation between thermodynamic parameters  $(p, V, T)$  is described by the following expression:

(3)

$$p = \frac{nRT}{(V - nb)} - \frac{an^2}{V^2}.$$

We are supposed to calculate the following expression.

(4)

$$\frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_{p,n} = \frac{1}{V} \left( \frac{1}{\left( \frac{\partial T}{\partial V} \right)_{p,n}} \right).$$

The above equation is much easier to calculate if we write:

(5)

$$T = \left( p + \frac{an^2}{V^2} \right) (V - nb) \frac{1}{nR}.$$

Now, we need to calculate the partial derivative of (5) which yields: (6)

$$\left( \frac{\partial T}{\partial V} \right)_{p,n} = \frac{\partial}{\partial V} \left[ \frac{pV}{nR} + \frac{an}{VR} - \frac{pb}{R} - \frac{abn^2}{V^2R} \right] = \left[ \frac{p}{nR} - \frac{an}{V^2R} + \frac{abn^2}{V^3R} \right].$$

Finally, if we join (6) and (4) we will end up with:

(7)

$$\frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_{p,n} = \frac{1}{V} \left( \frac{1}{\left( \frac{\partial T}{\partial V} \right)_{p,n}} \right) = \frac{1}{\left( \frac{pV}{nR} - \frac{an}{VR} + \frac{abn^2}{V^2R} \right)}$$

**c).**

(ENERGY FOR IDEAL GAS)

(8)

$$U = \frac{t}{2}nRT - \frac{an^2}{V},$$

where  $t$  marks the number of degrees of freedom.

(ENERGY FOR REAL GAS-VAN DER WAALS)

(9)

$$U = \frac{t}{2}nRT$$

(SOLUTION FOR IDEAL GAS)

(10)

$$\left( \frac{\partial U}{\partial T} \right)_{V,n} = \frac{\partial}{\partial T} \left( \frac{t}{2}nRT \right)_{V,n} = \frac{t}{2}nR$$

(SOULTION FOR REAL GAS)

(11)

$$\left( \frac{\partial U}{\partial T} \right)_{V,n} = \frac{\partial}{\partial T} \left( \frac{t}{2}nRT - \frac{an^2}{V} \right)_{V,n} = \frac{t}{2}nR$$

## Problem 5

One mole of ideal gas (monoatomic  $t=3$ ) was heated in an isochoric way. How much did the energy of the gas change?

According to the first law of thermodynamics:

(12)

$$dU = \delta Q - pdV.$$

Isochoric transformation can be expressed as:

$$(p_1, V, T_1) \rightarrow (p_2, V, T_2).$$

Since  $\Delta V = V - V = 0$ , there is no work done.

The heat transfer, on the other hand, can be written as:

(13)

$$\delta Q = nC_V dT = \frac{3}{2}nRdT.$$

The solution to this problem, where  $n = 1$  is thus simply:

(14)

$$\Delta U = \frac{3}{2}R(T_2 - T_1) = \frac{3}{2}R\Delta T.$$

## Problem 6

One mole of ideal gas was compressed adiabatically from  $(p_0, V_0)$  state to  $(p_1, V_1)$  state. Describe how  $p$ ,  $V$  and  $U$  changed during the process. Use the first law of thermodynamics and eq. of the adiabatic process.

Equation of the adiabatic process:

(15)

$$p_0 V_0^\gamma = p_1 V_1^\gamma = \text{const},$$
$$pV^\gamma = \text{const},$$

where  $\gamma = \frac{C_p}{C_V} = \frac{t+2}{t}$ .

Let us denote *const* from eq (15) as  $K$  so that

(16)

$$pV^\gamma = K.$$

An adiabatic process is one in which no heat is gained or lost by the system and so the change in internal energy is only in the form of work. Consequently,  $\Delta U = W$ .

(17)

$$W = \int_{V_0}^{V_1} p dV = \int_{V_0}^{V_1} \frac{K}{V^\gamma} dV = \frac{1}{\gamma - 1} \left[ \frac{K}{V_1^{\gamma-1}} - \frac{K}{V_0^{\gamma-1}} \right] = \frac{1}{\gamma - 1} [p_1 V_1 - p_0 V_0].$$

Answer:

(18)

$$\Delta U = W = \frac{1}{\gamma - 1} [p_1 V_1 - p_0 V_0]$$

(19)

$$\Delta p = p_1 - p_0 = \frac{K}{V_1^\gamma} - \frac{K}{V_0^\gamma}$$

(20)

$$\Delta V = V_1 - V_0 = \left( \frac{K}{p_1} \right)^{\frac{1}{\gamma}} - \left( \frac{K}{p_0} \right)^{\frac{1}{\gamma}}$$

We can express (18), (19), (20) in terms of temperature  $T$  by using the following relations (derived using the ideal gas equation):

(21)

$$TV^{\gamma-1} = \text{const} = B,$$

(22)

$$p^{1-\gamma} T^\gamma = \text{const} = A.$$

Consequently we have:

(23)

$$\Delta p = p_1 - p_0 = \left( \frac{A}{T_1} \right)^{\frac{\gamma}{1-\gamma}} - \left( \frac{A}{T_0} \right)^{\frac{\gamma}{1-\gamma}},$$

(24)

$$\Delta V = V_1 - V_0 = \left( \frac{B}{T_1} \right)^{\frac{1}{\gamma-1}} - \left( \frac{B}{T_0} \right)^{\frac{1}{\gamma-1}}.$$

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