Determination of the Solubility Product of an Ionic Compound

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1 Data

The following data were collected.

Dilution	Precipitation well
Ca^{2+}	#6
OH^-	#4

2 Calculations

2.1 Calcium Ion Serial Dilution

The first well without a precipitate is well number 6. The concentration of calcium ions per well is

$$[Ca^{2+}] = \frac{0.10}{2^n}\,M \tag{1}$$

where n is the well number. So, the concentration of calcium ions in well number 6 is $\frac{0.10}{2^6}\,M = 1.6 \times 10^{-3}\,M$

For each of the wells in this series, the concentration of added NaOH was a constant $\frac{0.1\,M}{2} = 0.05\,M$ after being added to each well.

The K_{sp} for this ion equilibrium, $Ca(OH)_2 \rightleftharpoons Ca^{2+} + 2OH^-$, is

$$K_{sp} = \left[Ca^{2+}\right]\left[OH^-\right]^2 \tag{2}$$

So, putting in the concentrations for well number 6, which is assumed to have complete dissociation of the calcium hydroxide, $K_{sp} = \left[1.6 \times 10^{-3}\right]\left[0.05\right]^2 = 3.9 \times 10^{-6}$

2.2 Hydroxide Ion Serial Dilution

The first well without precipitate for this series is well number 4. The concentration of hydroxide ions per well is

$$[\text{OH}^-] = \frac{0.10}{2^n}\,\text{M} \tag{3}$$

again, with n being the well number. So, the concentration of hydroxide ions in well number 4 is $\frac{0.10}{2^4}\,\text{M} = 6.3 \times 10^{-3}\,\text{M}$

For each of the wells in this series, the concentration of added calcium nitrate was a constant $\frac{0.1\,\text{M}}{2} = 0.05\,\text{M}$ after being added to each well.

Using the ion equilibrium equation and these concentrations, we can find that $K_{\text{sp}} = [0.05]\left[6.3 \times 10^{-3}\right]^2 = 2.0 \times 10^{-6}$

Then, $\bar{K}_{\text{sp}} = \frac{3.9 \times 10^{-6} + 2.0 \times 10^{-6}}{2} = 3.0 \times 10^{-6}$

The actual K_{sp} is 5.02×10^{-6} and the percent error from this value to the calculated average is $\frac{\left|3.0 \times 10^{-6} - 5.02 \times 10^{-6}\right|}{5.02 \times 10^{-6}} = 40.\%$

3 Discussion

1. The equation for calcium hydroxide dissolving in water is

$$\text{Ca(OH)}_2 \rightleftharpoons \text{Ca}^{2+} + 2\text{OH}^- \tag{4}$$

   and its equilibrium expression is

$$K_{\text{sp}} = \left[\text{Ca}^{2+}\right]\left[\text{OH}^-\right]^2 \tag{5}$$

3. The concentration was determined in first well without precipitate by noting that the concentration in the series decreased geometrically by a factor of two per well. So, in each well, the concentration of the series' ion was half of the previous which makes a nice formula as in (1) and (3). Then, the other ion that was added constantly was the same for each well and was half of the ion's actual concentration since it was being added to an equal amount of solvent. This well was used to determine the solubility product since it was assumed the solute was completely dissociated as no light was being obstructed by a suspension of solute.

4. The values obtained from each trial were very close considering the aggregation of the concentrations in the series. They were at least in the same order of magnitude, and using a deviation newly devised by the author, the average logarithmic deviation, the

percent error for just this experiment is $\frac{\left|\log 3.9\times 10^{-6}-\log 3.0\times 10^{-6}\right|+\left|\log 2.0\times 10^{-6}-\log 3.0\times 10^{-6}\right|}{2}$ which is a 14.5% error in terms of the orders of magnitude.

5. This method gives values that are too low because it looks for the first well without precipitate. This means that, most likely, there is a well with a greater concentration of ions that was skipped over due to the quantization of the series. So, since we are using discreet wells instead of an infinite number with every possible combination of concentrations, we will skip over a well with a higher concentration of ions but still without precipitate. A lower perceived concentration means, with equation (5), a lower K_{sp} will be calculated, as claimed.

6. To make this method more accurate, more concentration combinations are needed to try to fill in the gaps due to the nature of having discreet wells. Another method is to use the followed method but instead of settling with the well without precipitate, the procedure is carried out again but with the first concentration being the same as last well with precipitate and the last well being the concentration of the first well without precipitate. This then fills in the location where the actual first concentration of not having precipitate is. The procedure can be carried out as many times as desired to better find the elusive correct K_{sp}.

7. The results should be better if the concentrations of the last well where precipitation occurred and the first well where there was no precipitate were averaged because the actual point where there is no precipitate occurs between these two points, though it wouldn't be as accurate as doing the search as mentioned in item 6.

Since the concentrations are factors, we should use the geometric mean.

$$K_{\text{sp}} = \left(\sqrt{\frac{0.10}{2^5} \cdot \frac{0.10}{2^6}}\right)(0.05)^2 = 5.5 \times 10^{-6}$$

$$K_{\text{sp}} = (0.05)\left(\sqrt{\frac{0.10}{2^3} \cdot \frac{0.10}{2^4}}\right)^2 = 3.9 \times 10^{-6}$$

And, these are noticeably closer to the accepted K_{sp} than before with a geometric mean of $\sqrt{5.5 \times 10^{-6} \cdot 3.9 \times 10^{-6}} = 4.6 \times 10^{-6}$ which is 8% away from the accepted value.

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