Modeling notes for Costa Rica Project
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Fishery growth with catch

Let B be biomass, r and K be the usual parameters for the growth function, and Y catch, with $t = 0, \ldots, T_{cert}, \ldots, \bar{T}$ indexing time. Certification takes place at time T_{cert}. A general difference equation given that harvest occurs after growth is:

$$B_{t+1} - B_t = g(B_t; r, K) - Y_t \tag{1}$$

For any given level of catch, equilibrium is described by $B_t = B_{t+1}$, giving the following function that may be implicitly defined $B_{eq}(Y)$. In the Costa Rica certification application, logistic growth is assumed such that $g(B; r, K) = rB(1 - \frac{B}{K})$.

Harvest

Let q be a catch ability coefficient (fraction of population fished per unit effort), $f_{c,t}$ be effort in communities that become certified, and $f_{u,t}$ be effort in communities that do not become certified. The derivation of f will descried more below. Catch is equal to the weighted sum of catch in each village type:

$$Y_t = q\gamma f_{c,t}^{\alpha} B_t + q(1-\gamma) f_{u,t}^{\alpha} B_t \tag{2}$$

In Equation (2), the relative size of the villages that become certified is given by $\gamma \in [0, 1]$; if these villages harvest 15% of the fish ex ante, then $\gamma = 0.15$. The other important parameter is α. This governs how quickly diminishing returns to effort take effect, but can also be viewed as something of a biological mixing parameter.[1] In order for equilibrium aggregate harvest to be effected by certification (effort restrictions) in a limited number of fisheries, it is necessary that $\alpha < 1$.

Profits and Entry

Let p be ex-vessel price and c be cost per unit effort (think fuel and wage costs); both do not vary between village types. Total revenue is equal to pY_t, and profit π_t in each community type is given by:

$$\pi_{c,t} = \gamma(pqf_{c,t}^{\alpha} B_t - cf_{c,t}) \tag{3}$$
$$\pi_{u,t} = (1-\gamma)(pqf_{u,t}^{\alpha} B_t - cf_{u,t}) \tag{4}$$

Suppose that profits lead to changes in effort in the fishery, and that we can model this to depend on a parameter $\varphi \geq 0$. This parameter is the same for both village types, but is

[1] For α near 1, there is a lot of stock mixing between village types, and decreases in effort by one village are compensated by increases in others. As α decreases, it is more like each village harvests its own stock. This is not a clean interpretation, but helps with intuition.

scaled by ex ante catch. If entry is tightly controlled, then $\varphi \to 0$.[2] The difference equations for effort are:

$$\gamma(f_{c,t+1} - f_{c,t}) = \varphi\gamma(pq\gamma f_{c,t}^{\alpha} B_t - c f_{c,t}) \tag{5}$$

$$(1-\gamma)(f_{u,t+1} - f_{u,t}) = \varphi(1-\gamma)(pq\gamma f_{u,t}^{\alpha} B_t - c f_{u,t}) \tag{6}$$

Of course, for Equations 5-6, the proportions simplify out, giving:

$$f_{c,t+1} - f_{c,t} = \varphi(pq f_{c,t}^{\alpha} B_t - c f_{c,t}) \tag{7}$$

$$f_{u,t+1} - f_{u,t} = \varphi(pq f_{u,t}^{\alpha} B_t - c f_{u,t}) \tag{8}$$

Certification

Certification enters as an effort restriction for the certified villages. That is $f_{c,t} = \bar{f}_c$ for $t \geq T_{cert}$. Note that ex ante, it should be the case $f_{c,eq} = f_{u,eq}$. We've been talking about defining $\bar{f}_c = 0.9 \times f_{c,eq}$. We are also interested in $f'_{u,eq}$, the effort level of the unrestricted areas post certification.

Ex-ante Equilibrium

Before certification, it can be shown that equilibrium effort is the same between the two fisheries (scaled by γ), so that $f_{c,eq} = f_{u,eq}$. Because of the way this problem is set up, this delivers fairly standard open access equilibrium results for fishing effort:

$$f_{c,eq} = f_{u,eq} = \left(\frac{pqB_{eq}}{c}\right)^{\frac{1}{1-\alpha}} \tag{9}$$

For biomass using the logistic assumption on growth, $\alpha \neq 1$ means that biomass is now defined implicitly:

$$r\left(1 - \frac{B_{eq}}{K}\right) = q\left(\frac{pqB_{eq}}{c}\right)^{\frac{1}{1-\alpha}} \tag{10}$$

but this is very similar to the standard open access case. Notably, γ enters nowhere in either Equations 9 or 10 (though it may play a role in dynamics if $f_{c,0} \neq f_{u,0}$).

Ex-post Equilibrium

Now, turn attention to the post-certification period, where $f_{c,t} = \bar{f}_c$. It will turn out that the effect of the effort restriction depends on whether $\bar{f}_c < f_{c,eq}$. First, note that $f_{u,eq}$ has a similar expression as before, but will depend on the new equilibrium biomass $\tilde{B}_{eq}$:

$$\tilde{f}_{u,eq} = \left(\frac{pq\tilde{B}_{eq}}{c}\right)^{\frac{1}{1-\alpha}} \tag{11}$$

[2] This parameter could also be asymmetrical.

That is, all changes in uncertified effort are mediated through shifts in biomass. The new equilibrium biomass is implicitly defined by:

$$r\left(1-\frac{\tilde{B}_{eq}}{K}\right)=\gamma q\bar{f}_c^{\alpha}+(1-\gamma)q\left(\frac{pq\tilde{B}_{eq}}{c}\right)^{\frac{1}{1-\alpha}} \tag{12}$$

The math here gets a bit messy, but the following statements can be shown to hold:

- If $\bar{f}_c < f_{c,eq}$, then increases in γ lead to increases in $\tilde{B}_{eq}$.
- Thus, if $\bar{f}_c < f_{c,eq}$, then increases in γ lead to increases in $\tilde{f}_{u,eq}$.
- Increases in $\bar{f}_c$ lead to decreases in $\tilde{B}_{eq}$.
- Thus, increases in $\bar{f}_c$ lead to decreases in $\tilde{f}_{u,eq}$.
- If $\bar{f}_c < f_{c,eq}$, then $\tilde{B}_{eq} < B_{eq}$.[3]

There is not an easy characterization of the effect of certification on revenue for the certified village, though if equilibrium effort and biomass both increase, then the revenue effect will be positive for the uncertified villages.

Outcomes

There are a number of outcomes of interest. Let $\lambda \geq 0$ be a parameter the percentage of revenue that goes into the community investment fund (so $\lambda = 0.1$ here). Further, let P be the price that the retailer can charge for the fish without certification, and ρP be the new price after certification (with $\rho \geq 1$). Assume that P is set in a competitive environment such that the retailer cannot alter P (they can alter ρ). Finally assume the discard rate of fish not good enough to make it to the retailed after certification is $1-\delta$ (so that δ make it). Then outcomes of interest at any time t are:

Outcome	Certified Villages	Uncertified Villages
Harvest, ex ante	$\gamma q f_{c,t}^{\alpha} B_t$	$(1-\gamma)q f_{u,t}^{\alpha} B_t$
Harvest, ex post	$\gamma q \bar{f}_c^{\alpha} B_t$	$(1-\gamma)q f_{u,t}^{\alpha} B_t$
Exported harvest	$\delta\gamma q \bar{f}_c^{\alpha} B_t$	0
Fisher revenue, ex ante	$p\gamma q f_{c,t}^{\alpha} B_t$	$p(1-\gamma)q f_{u,t}^{\alpha} B_t$
Fisher revenue, ex post	$p\gamma q \bar{f}_c^{\alpha} B_t$	$p(1-\gamma)q f_{u,t}^{\alpha} B_t$
Community fund revenue	$\lambda p\delta q \bar{f}_c^{\alpha} B_t$	0
Retailer revenue less input costs	$(\rho P-(1+\lambda)p)\delta q \bar{f}_c^{\alpha} B_t$	

To calculate the PV effect on any of these items, assume a discount factor β (perhaps 0.92 or something under 8% discounting). Next, determine the type of counterfactual comparison. Here are three possibilities:

[3] To see this, note that Equation 12 with $\gamma = 0$ is just Equation 10 (ex ante biomass). This combined with the first bullet point shows that ex post biomass increases.

- Equilibrium comparison: Compare old and new equilibrium values, interpret as equilibrium change per year.
- Equilibrium change with transition path: Model change from current equilibrium values to new equilibrium values, to PDV calculation for duration. Gives NPV of impacts.
- Current state change: Model change from current biomass and effort levels to new equilibrium values, to PDV calculation for duration. Gives NPV of impacts.

Parameters

Name	Symbol	Current Value	Value under Cert	Source
Biological parameters				
bio param	r	model	no change	
bio param	K	model	no change	
Fishery parameters				
CPUE	q			
Cost	c			
Entry	φ			
Diminishing returns to effort	α		same	
Discard to local market	δ	1	0.2	
Price parameters				
Ex-vessel price	p			
Retail price	P		no change	Market
Cert markup	ρ			
Community fund	λ	0	0.1	Cert Requirements