

Test of error calculations for Ising model in 1D and 2D

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This is short theoretical explanation of the test: **IsingTestError1D.h** and **IsingTestError2D.h**.

1 Standard deviation

In files `IsingTestError1D.h` and `IsingTestError2D.h` we test function `Ising::ERROR(string totalFname, ISING_ERROR_TYPE error_type)` which calculates standard deviation of choosen variable X using bootstrap algorithm.

Standard deviation of variable X can be expressed by:

$$\sigma(X) = \sqrt{\langle (X - \langle X \rangle)^2 \rangle} = \sqrt{\langle X^2 \rangle - \langle X \rangle^2}$$

and it tells as how far a set of numbers is spread out. Low standard deviation indicates that the data points tend to be near the mean value of the set, high - the opposite. Apart from showing dispersion of a data set, standard deviation is usually used as a measure of confidence in statistical conclusions.

The standard deviation of some data set is a square root of its variance $V(X) = \langle X^2 \rangle - \langle X \rangle^2$. A useful property of the standard deviation is that, unlike the variance, it is expressed in the same units as the data.

2 Bootstrap method

If the sample size is insufficient to calculate the standard deviation from definition, we can use for this purpose bootstrap method. Bootstrapping uses approximate distribution to estimate properties of an estimator (like standard deviation). To achieve it we can take set of observed data and (assuming independence of observation) construct a number of resamples. Such resamples have to be of equal size to the observed dataset and be obtained by random sampling with replacement from the original dataset. From a number of resamples we can create approximate distribution of variable and obtain estimator - in our case standard deviation.

The general bootstrap algorithm is as follows: