Durham University
MATH1541 Statistics
Exercise Sheet 17

Kamil Hepak
Tutorial Group 4

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1 Q4

1.1 a)

1.1.1 i)

Let subscript 1 denote fathers of schizophrenic children, and subscript 2 denote fathers of "normal" children. Let $\mu_d = \mu_1 - \mu_2$.
$H_0 : \mu_d = 0$, $H_a : \mu_d \neq 0$
$n_1 = 10, n_2 = 6$
$t_{\min(n_1-1,n_2-1)} = t_5$
$t^*_{99\%} = 4.032$
$\bar{x_1} = 35.70, \bar{x_2} = 9.67$
$\bar{x_1} - \bar{x_2} = 26.03$
$s_1^2 = 18.68, s_2^2 = 33.47$
Thus, the 99% confidence interval can be constructed with:

$$26.03 \pm 4.032\sqrt{\frac{18.68}{10} + \frac{33.47}{6}}$$

Therefore:

$$\mu_d \in [15.03, 37.03]$$

And since $0 \notin$ the C.I., there is sufficient evidence to not accept the null hypothesis at the 99% significance level.

1.1.2 ii)

Let subscript 1 denote fathers of schizophrenic children, and subscript 2 denote fathers of "normal" children. Let $\mu_d = \mu_1 - \mu_2$.
$H_0 : \mu_d = 0$, $H_a : \mu_d \neq 0$
$n_1 = 10, n_2 = 6$
$t_{n_1+n_2-2} = t_{14}$
$t^*_{99\%} = 2.977$
$\bar{x_1} = 35.70, \bar{x_2} = 9.67$
$\bar{x_1} - \bar{x_2} = 26.03$
$s_1^2 = 18.68, s_2^2 = 33.47$

$s_p = \frac{9 \cdot 18.68 + 5 \cdot 33.47}{14} = 23.96$
Thus, the 99% confidence interval can be constructed with:

$$26.03 \pm 2.077 \cdot 23.96 \cdot \sqrt{\frac{1}{10} + \frac{1}{6}}$$

Therefore:

$$\mu_d \in [0.33, 52.73]$$

And since $0 \notin$ the C.I., there is sufficient evidence to not accept the null hypothesis at the 99% significance level.

1.2 b)

Let subscript 1 denote mothers of schizophrenic children, and subscript 2 denote mothers of "normal" children. Let $\mu_d = \mu_1 - \mu_2$.
$H_0 : \mu_d = 0$, $H_a : \mu_d \neq 0$
Assuming the variances are not equal and noting that they are not known:
$n_1 = 10, n_2 = 6$
$t_{\min(n_1-1, n_2-1)} = t_5$
$t^*_{99\%} = 4.032$
$\bar{x_1} = 30.2, \bar{x_2} = 17.0$
$\bar{x_1} - \bar{x_2} = 13.2$
$s_1^2 = 34.4, s_2^2 = 38.8$
Thus, the 99% confidence interval can be constructed with:

$$13.2 \pm 4.032 \sqrt{\frac{34.4}{10} + \frac{38.8}{6}}$$

Therefore:

$$\mu_d \in [0.51, 25.89]$$

And since $0 \notin$ the C.I., there is sufficient evidence to not accept the null hypothesis at the 99% significance level.