Durham University
MATH1541 Statistics
Exercise Sheet 13

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0.1 Q1

0.1.1 a)

The probability of a student passing the exam is not constant, as a student's understanding is dependent on many other factors, thus a binomial distribution is not appropriate in this case.

0.1.2 b)

Again the probability of getting a correct answer is not constant or independent (later questions are dependent on good understanding of earlier material), thus a binomial distribution will not give a good answer.

0.2 c)

The differing temperatures likely affect the probability of material failure, and the fact the same material is being used for this test means the experiment is not independent - once again, a binomial distribution will not be conducive to correct calculations.

0.3 Q4

0.3.1 a)

P = the amount of ways to ignore two jean-wearers over the amount of ways to choose a sample of seven workers.
$\mathrm{P} = \frac{\binom{5}{2}}{\binom{9}{7}} = \frac{5}{18} = 0.2778$

0.3.2 b)

$X \sim \mathrm{B}(7, \frac{4}{9})$
$\mathrm{P}(X = 4) = 0.2342$

0.3.3 c)

In this population, selecting a sample where more than 4 people wear suits is impossible - however, the Binomial approximation ignores this, and has non-zero probability values for $X > 4$.

0.4 Q5

0.4.1 a)

If we assume the probability, 0.4, of a person being a Conservative voter, and that each person's probability of voting that way is independent, then we also have a binary choice (Tory voter, or not), thus allowing us to model Y as binomial.

0.4.2 b)

y	0	1	2	3	4	5	6
$\mathrm{P}(Y = y)$	0.04666	0.1866	0.3110	0.2765	0.1382	0.03686	4.096×10^{-3}

0.4.3 c)

$\mathrm{E}(Y) = \sum_{i=0}^{6} y \cdot \mathrm{P}(Y = y)$
$\mathrm{E}(Y) = 2.3998$
$\mathrm{Var}(Y) = \left(\sum_{i=0}^{6} y^2 \cdot \mathrm{P}(Y = y)\right) - \mathrm{E}(Y)^2$
$\mathrm{Var}(Y) = 7.1993 - 2.3998^2 = 1.4402$

0.4.4 d)

$np = 2.4$
$np(1-p) = 1.44$

0.4.5 e)

$Y \sim \mathrm{B}(900, 0.4)$
$Z \sim \mathrm{N}(360, (6\sqrt{6}^2))$
$\mathrm{P}(333 \leq Y \leq 378) \approx \mathrm{P}(333 \leq Z \leq 378)$
$\mathrm{P} = 0.8566$

0.4.6 f)

$P = \frac{Y}{900}$
Assume $P \sim \mathrm{N}(0.4, 2.67 \times 10^{-4})$
$\mathrm{P}(0.37 \leq P \leq 0.42) = 0.8563$

0.4.7 g)

$\mathrm{MoE} = 2\sqrt{\frac{p(1-p)}{n}} = 0.01$
$n = \frac{0.4\times0.6}{0.02^2} = 600$

0.4.8 h)

The margin of error is dependent on both n and p, and therefore quoting a margin of error based on a value of p that is different from the true value means that your quoted margin of error does not correspond to the actual population, and is not useful in calculations.