Your machine shop has five jobs to do today. Each job must be done by exactly one of your three machines, but a machine can do more than one job, one after another. For each machine, there is a setup cost for the regular workday. Once this cost is paid, you can use the machine for up to 8 hours; if you do not pay the setup cost, you cannot use the machine today. If you set up a machine for the regular workday, you may also elect to set up the machine for overtime, in which case there is an additional cost, and you may use the machine for up to 2 more hours. The costs are as follows:

	Setup cost ($)	Overtime cost ($)
Machine 1	600	190
Machine 2	585	150
Machine 3	700	200

Thus, if you pay $600, you can use machine 1 for up to 8 hours, and if you pay $600+$190=$790, you may use the machine 1 for up to 8+2=10 hours. The number of hours each job takes on each machine is as follows:

	Job 1	Job 2	Job 3	Job 4	Job 5
Machine 1	2.0	2.0	4.5	1.0	4.1
Machine 2	2.5	2.5	4.5	1.5	4.7
Machine 3	2.1	2.1	4.0	2.5	4.5

a) Formulate a mixed-integer linear optimization (MILP) model to find the cheapest way to get all 5 jobs done today.

minimize

$$600m_1 + 585m_2 + 700m_3 + 190o_1 + 150o_2 + 200o_3$$

subject to

$$\sum_i x_{i,1} = 1, \sum_i x_{i,2} = 1, \sum_i x_{i,3} = 1, \sum_i x_{i,4} = 1, \sum_i x_{i,5} = 1$$

$$2x_{1,1} + 2x_{1,2} + 4.5x_{1,3} + 1x_{1,4} + 4.1x_{1,5} \leq 8m1 + 2o1$$

$$2.5x_{2,1} + 2.5x_{2,2} + 4.5x_{2,3} + 1.5x_{2,4} + 4.7x_{2,5} \leq 8m2 + 2o2$$

$$2.1x_{3,1} + 2.1x_{3,2} + 4x_{3,3} + 2.5x_{3,4} + 4.5x_{3,5} \leq 8m3 + 2o3$$

$$m_i \leq o_i$$

$$m_i \in \{0,1\}, \quad o_i \in \{0,1\}, \quad x_{i,j} \in \{0,1\}$$

where,

m_i – 1 if machine number i $\{1-3\}$ is used, 0 if it is unused

o_i – 1 if machine number i $\{1-3\}$ is used for overtime, 0 if it is unused for overtime

$x_{i,j}$ – 1 if machine $i\{1-3\}$ does job $j\{1-5\}$, 0 if not used