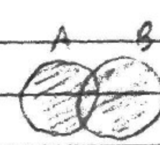
Stor 215 HW 3 Kathleen McShane

1. $A = \{x \in \mathbb{Z}_+ : x^2 < 25\}$

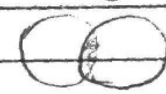
$B = \{x \in \mathbb{N} : x^2 < 3x + 5\}$

$A = \mathbb{Z}_+ = \{0, 1, 2, 3, 4\}$

$B = \mathbb{N} = \{1, 2, 3, 4\}$

$A \cup B$ 

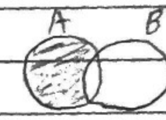
$A \cup B$ a) $\{0, 1, 2, 3, 4\}$

$A \cap B$ 

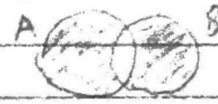
$A \cap B$ b) $\{1, 2, 3, 4\}$

$A - B$ c) $\{0\}$

b/c $A: 0-4$ $B = 1-4$

$A - B$ 

$A \Delta B$ d) $\{0\}$

b/c 

3. A and B are sets and $A \cap B$, and $A \cap B^c$ are not disjoint

$\Rightarrow$ There exists some element x such that $(x \in A \cap B) \wedge (x \in A \cap B^c)$

$x \in A \cap B \quad \wedge \quad x \in A \cap B^c$

$x \in A \wedge x \in B \wedge x \in A \wedge x \in B^c$

" "$\Leftrightarrow x \in B$ " " $\wedge \neg(x \in B)$

$\Rightarrow$ False

b/c

$(x \in A \wedge x \in B) \wedge (x \in A \wedge \neg(x \in B))$

$x \in A \wedge (x \in B \wedge \neg(x \in B))$

$x \in A \wedge \underbrace{(x \in B \wedge x \in B^c)}_{\text{False}}$

$\Rightarrow$ False

$\blacksquare$

2. For any two sets A and B, $(A \cap B)^c = A^c \cup B^c$

$S \in R$

$x \in S \rightarrow A^c \cup B^c$

$\Rightarrow \neg(x \in A) \vee \neg(x \in B)$

$\Rightarrow \neg(x \in A \wedge B)$

$\Rightarrow x \in \mathbb{R} = x \in (A \wedge B)^c$

and $\mathbb{R} \in S$

$x \in \mathbb{R} \Rightarrow x \in (A \wedge B)^c$

$\Rightarrow \neg x \in A \vee \neg x \in B$

$\Rightarrow x \in A^c \vee x \in B^c$

$\Rightarrow x \in S \Rightarrow x \in A^c \vee B^c$

$\boxtimes$

4. Function f given: f(red) = blue, f(green) = yellow, f(blue) = green, f(yellow) = red

a) Domain A: D: {red, green, blue, yellow}

Range B: R: {blue, yellow, green, red}

b) As a function from A to B, f is one-to-one and f is onto.

c) f^{-1} exists b/c the function is a bijection.

f^{-1}(blue) = red, f^{-1}(yellow) = green,

f^{-1}(green) = blue, f^{-1}(red) = yellow

5. function is increasing if $x \geq y \Rightarrow f(x) \geq f(y)$

if $f(x) \geq f(y)$

$g(x) \geq g(y)$

therefore: $f(x) + g(x) \geq f(y) + g(y)$