Topic :- Superposition Theorem

Superposition Theorem

- The superposition theorem extends the use of Ohm's Law to circuits with multiple sources.
- **Definition** :- *The current through, or voltage across, an element in a linear bilateral network equal to the algebraic sum of the currents or voltages produced independently by each source.*
- The Superposition theorem is very helpful in determining the voltage across an element or current through a branch when the circuit contains multiple number of voltage or current sources.

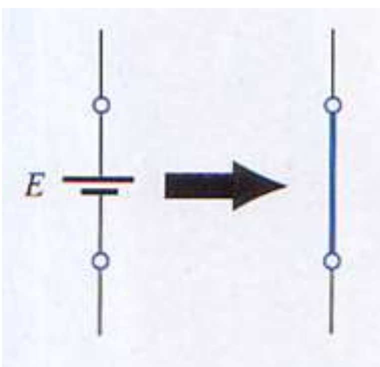
Superposition Theorem

- In order to apply the superposition theorem to a network, certain conditions must be met :

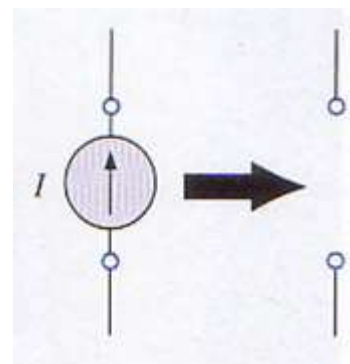
1. All the components must be linear, for e.g.- the current is proportional to the applied voltage (for resistors), flux linkage is proportional to current (in inductors), etc.
2. All the components must be bilateral, meaning that the current is the same amount for opposite polarities of the source voltage.
3. Passive components may be used. These are components such as resistors, capacitors, and inductors, that do not amplify or rectify.
4. Active components may not be used. Active components include transistors, semiconductor diodes, and electron tubes. Such components are never bilateral and seldom linear.

Procedure for applying Superposition Theorem

- ➢ Circuits Containing **Only Independent Sources**
- Consider only one source to be active at a time.
- Remove all other **IDEAL VOLTAGE SOURCES** by **SHORT CIRCUIT** & all other **IDEAL CURRENT SOURCES** by **OPEN CIRCUIT.**



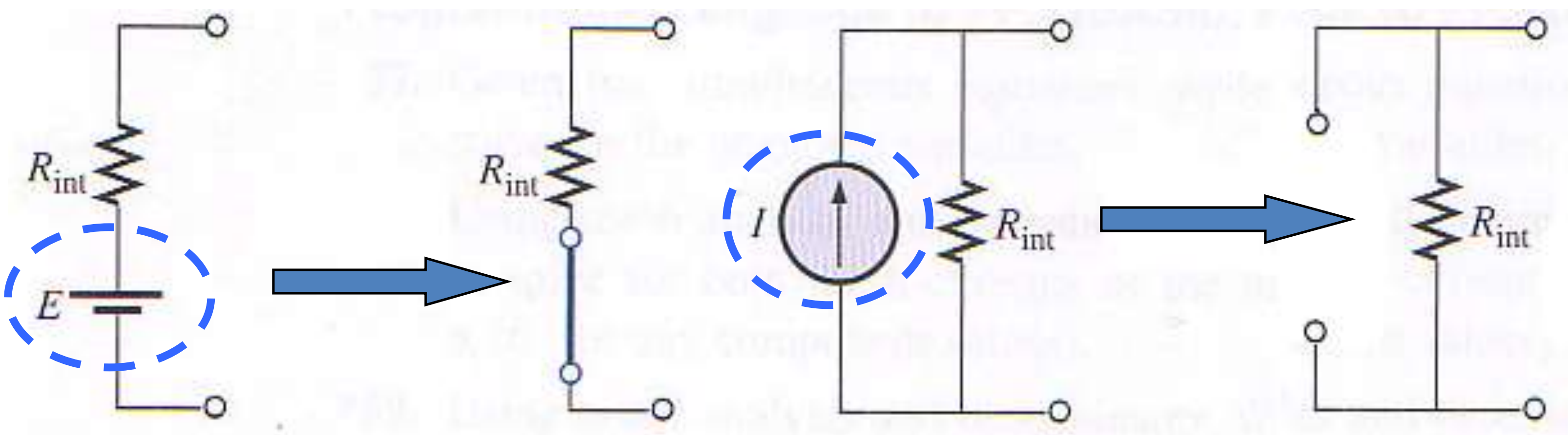
Voltage source is replaced by a Short Circuit



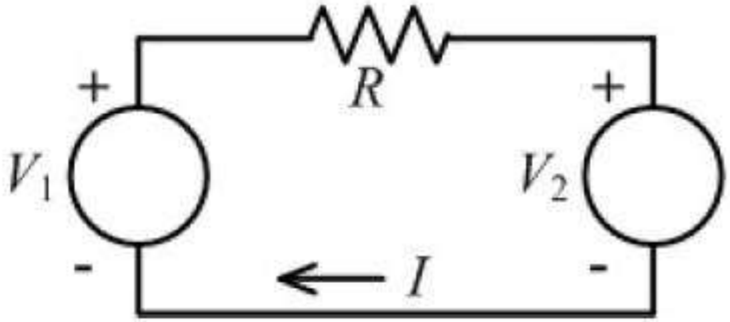
Current source is replaced by a Open Circuit

Procedure for applying Superposition Theorem

- If there are practical sources, replace them by the combination of **ideal source** and an **internal resistances** (as shown in figure).
- After that, **short circuit** the **ideal voltage source** & **open circuit** the **ideal current source** (as shown in figure).

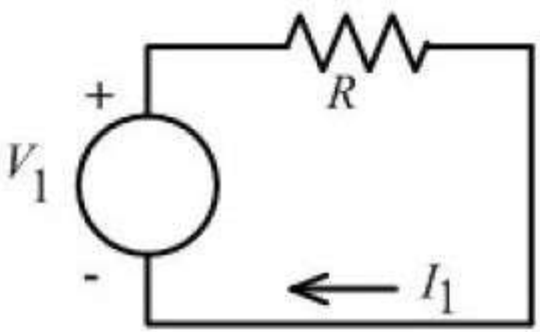


Example : 1



Find the current flowing through R.

1.

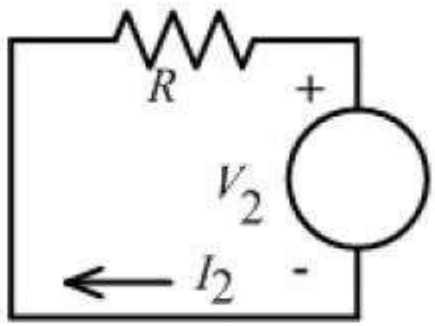


Short Circuiting Voltage source V_2 & finding the current I_1

$$I_1 = \frac{V_1}{R}$$

Example : 1

2.



Short Circuiting Voltage source V_1 & finding the current I_2

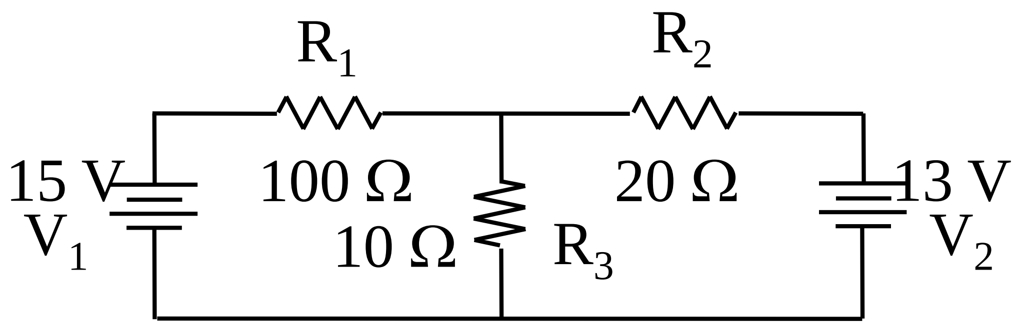
$$I_2 = -\frac{V_2}{R}$$

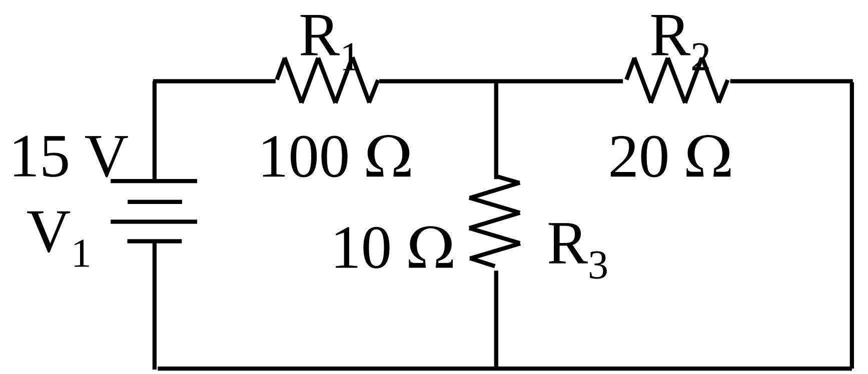
The net current is :- $I = I_1 + I_2 = \frac{V_1}{R} - \frac{V_2}{R}$

Same answer is obtained by another method (shown below) which would turn out to be tedious when applied to bigger circuits as in next example….

$$V_R = V_1 - V_2 \ (1),\ \ I = \frac{V_R}{R} = \frac{V_1 - V_2}{R} \ \ (2),\ \ I = \frac{V_1}{R} - \frac{V_2}{R} \ \ (3)$$

Example :2

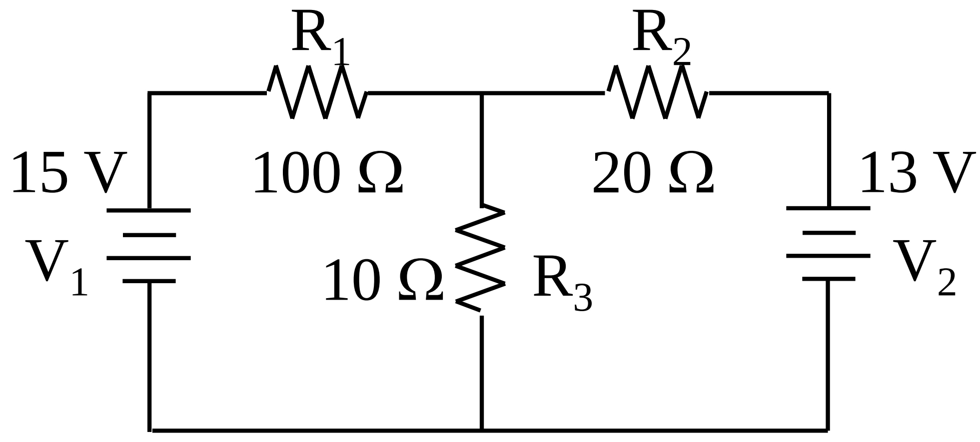




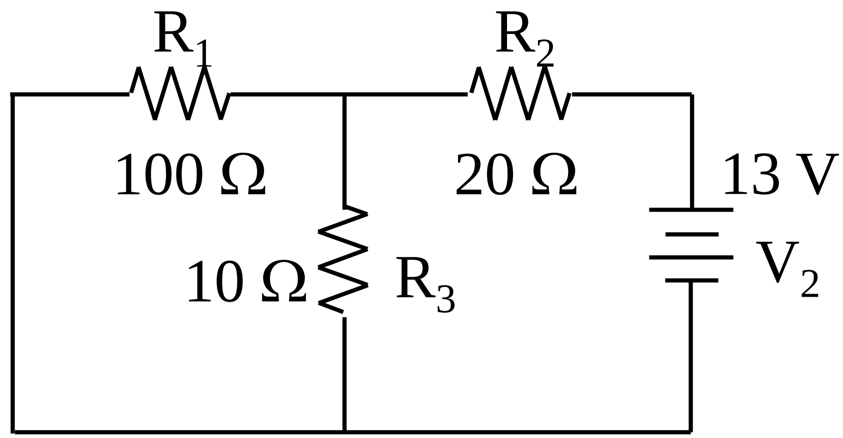
V_2 shorted

R_{EQ} = 106.7 Ω, I_T = 0.141 A and I_{R_3} = 0.094 A

Example :2

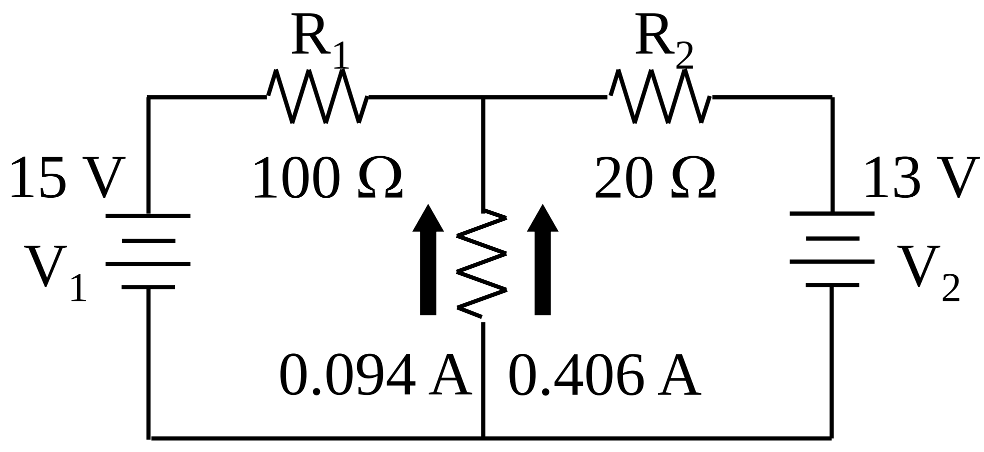


V1 shorted



$R_{EQ} = 29.09\ \Omega$, $I_T = 0.447$ A and $I_{R_3} = 0.406$ A

Example :2



With V_2 shorted

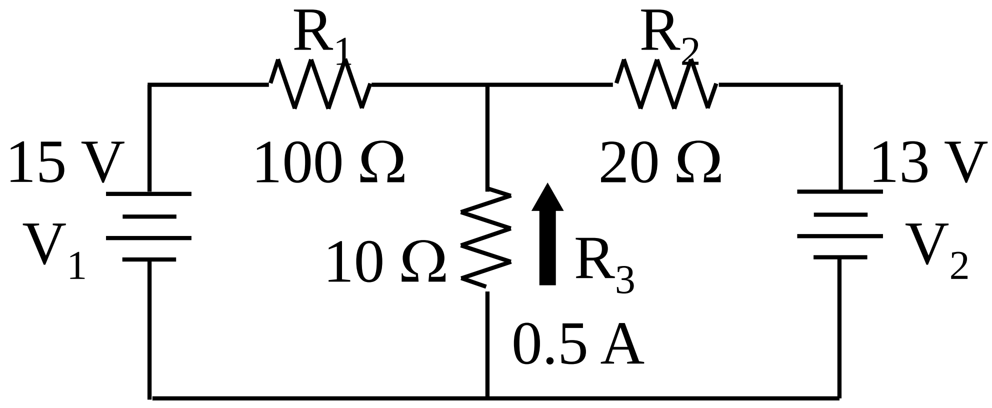
$R_{EQ} = 106.7\ \Omega$, $I_T = 0.141$ A and $I_{R_3} = 0.094$ A

With V_1 shorted

$R_{EQ} = 29.09\ \Omega$, $I_T = 0.447$ A and $I_{R_3} = 0.406$ A

Adding the currents gives $I_{R_3} = 0.5$ A

Example :2



With 0.5 A flowing in R_3, the voltage across R_3 must be 5 V (Ohm's Law). The voltage across R_1 must therefore be 10 volts (KVL) and the voltage across R_2 must be 8 volts (KVL). Solving for the currents in R_1 and R_2 will verify that the solution agrees with KCL.

$$I_{R_1} = 0.1 \text{ A and } I_{R_2} = 0.4 \text{ A}$$

$$I_{R_3} = 0.1 \text{ A} + 0.4 \text{ A} = 0.5 \text{ A}$$

Procedure for applying Superposition Theorem

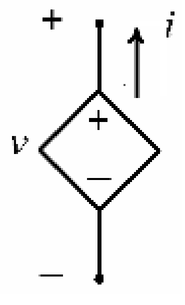
- Circuits Containing **Independent as well as Dependent Sources**

- Consider only one source to be active at a time.
- Remove all other **IDEAL INDEPENDENT VOLTAGE SOURCES** by **SHORT CIRCUIT** & all other **IDEAL INDEPENDENTCURRENT SOURCES** by **OPEN CIRCUIT -** as per the original procedure of superposition theorem
- **BUT NEITHER SHORT CIRCUIT NOR OPEN CIRCUIT THE DEPENDENT SOURCE. LEAVE THEM INTACT AND AS THEY ARE**

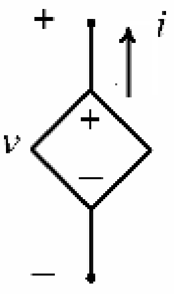
Procedure for applying Superposition Theorem

(a) Dependent Voltage Source

A voltage source whose parameters are controlled by voltage/current else where in the system



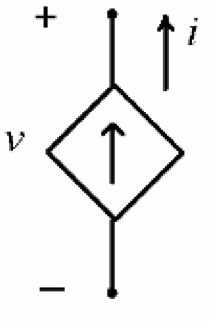
v = *μVx*
VDVS
(Voltage Dependent
Voltage source)



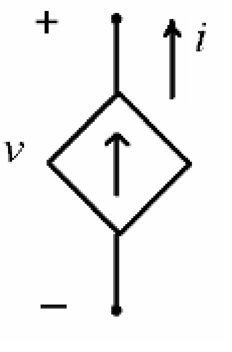
v = *ρix*
CDVS
(Current Dependent
Voltage source)

(b) Dependent Current Source

A current source whose parameters are controlled by voltage/current else where in the system



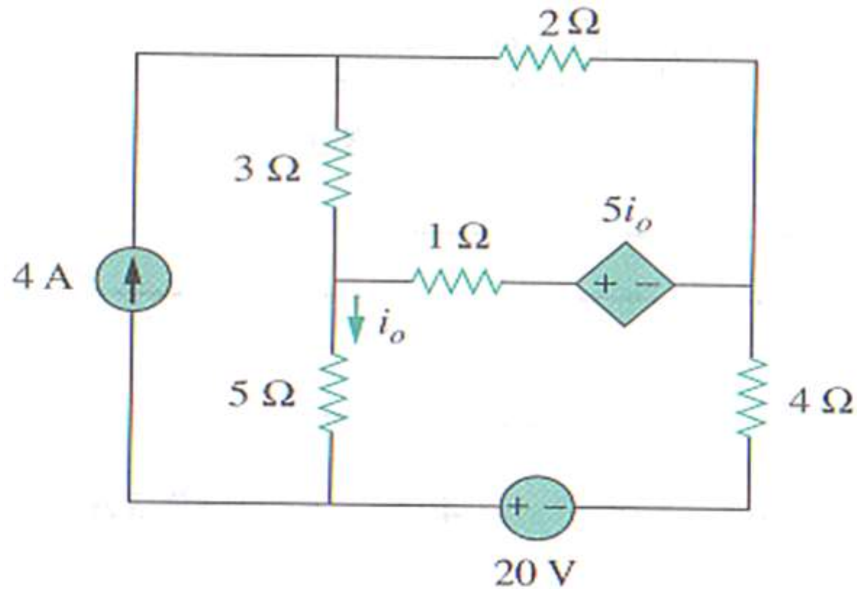
v = *αVx*
VDCS
(Voltage Dependent
Current source)



v = *βix*
CDCS
(Current Dependent
Current source)

Example : 3

Find i_0 in the circuit shown below. The circuit involves a dependent source. The current may be obtained as by using superposition as :

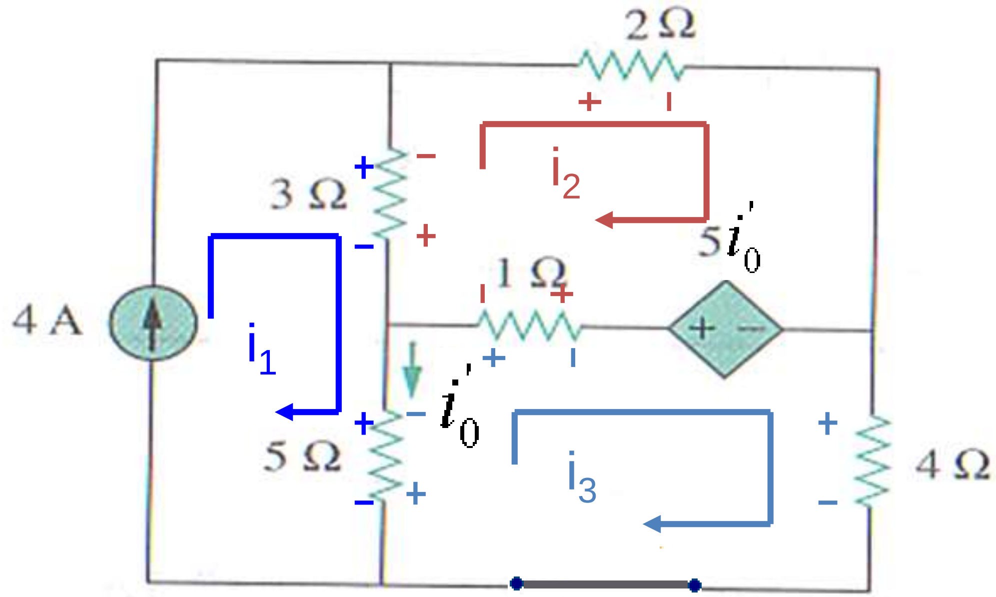


i'_0 is current due to 4A current source

i''_0 is current due to 20V voltage source

$$\therefore i_0 = i_0^{'} + i_0^{''}$$

To obtain i'_0 we short circuit the 20V sources



For loop 1

$$i_1 = 4\,A.$$

For loop 2

$$3(i_2 - i_1) + 2i_2 - 5i'_0 + 1(i_2 - i_3) = 0$$

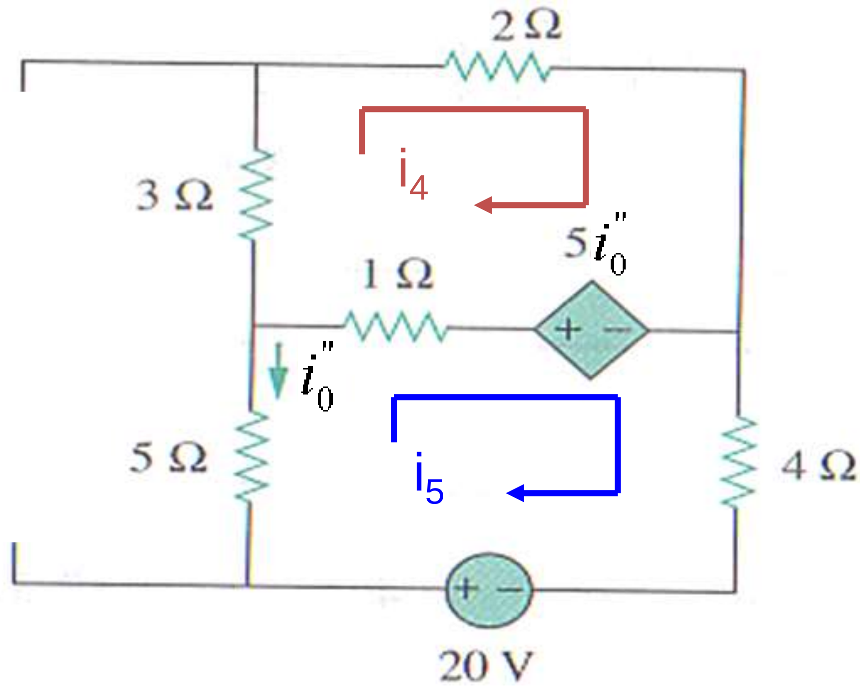
For loop 3

$$5(i_3 - i_1) + 1(i_3 - i_2) + 5i'_0 + 4i_3 = 0$$

$$i'_0 = i_1 - i_3$$

For solving i_1, i_2, i_3 $\Longrightarrow$ $i'_0 = \dfrac{52}{17}\,A$

To obtain i''_0, we open circuit the 4A sources



For loop 4

$$6i_4 - i_5 - 5i_0'' = 0$$

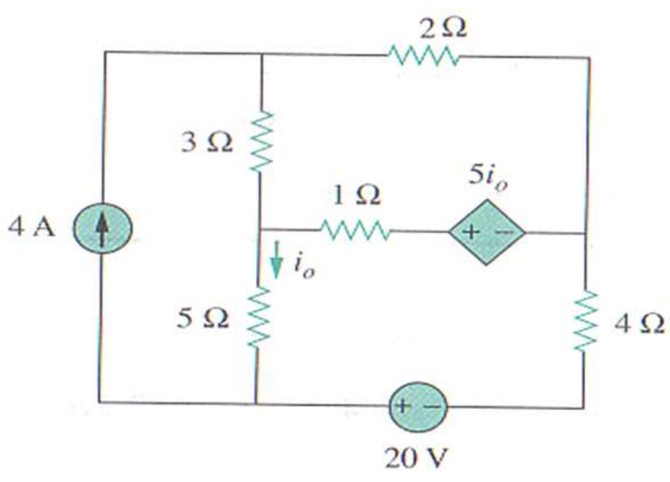
For loop 5

$$-i_4 + 10i_5 - 20 + 5i_0'' = 0$$

$$i_0'' = -i_5$$

For solving i_4 and i_5

$$i_0'' = -\frac{60}{17}\text{A}$$



$$\text{Therefore, } i_0 = i_0' + i_0''$$

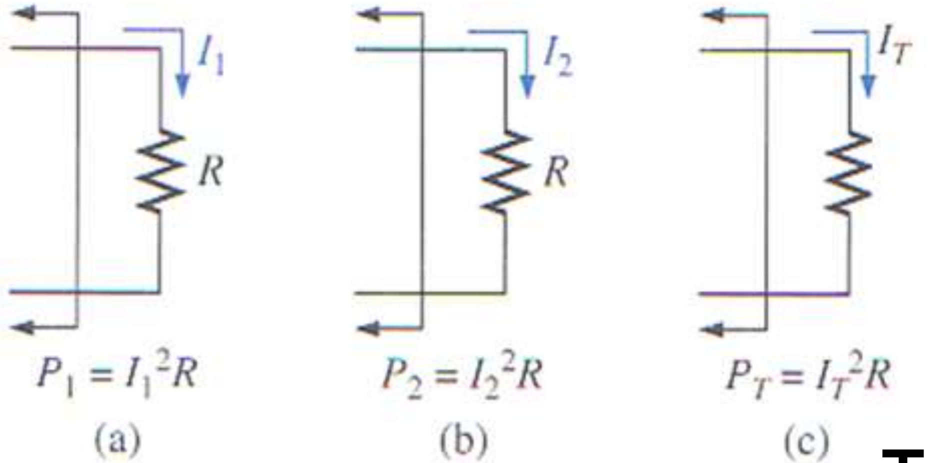
$$= \frac{52}{17} - \frac{60}{17}$$

$$= -\frac{8}{17}\text{A}$$

A LIMITATION :- Superposition is not applicable to

Power

The superposition theorem does not apply to power calculations as the power is proportional to current squared or voltage squared. Consider the following :



$$I_T = I_1 + I_2$$

The power delivered to the circuits are

$$\mathrm{P}_1 = I_1^2 R, \mathrm{P}_2 = I_2^2 R, \textit{and } \mathrm{P}_\mathrm{T} = I_T^2 R$$

$$P_T = (I_1 + I_2)^2 R = I_1^2 R + I_2^2 R + 2 I_1 I_2 R$$

$$\neq P_1 + P_2 \neq I_1^2 R + I_2^2 R$$

The total power must be determined using the total current not by superposition

THANK YOU