CW1 – Navier Stokes
Group 1

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Question 1 –Dimensionless analysis

Continuity equation:

$$\frac{\partial u_i}{\partial x} = 0 \quad [1]$$

Momentum equations:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} \quad [2]$$

$$\frac{p}{\rho} = \hat{p} \quad [3]$$

Non-dimensional form:

1.	$u_i, x_i, t, \hat{p}$
2.	$[u_i] = LT^{-1}$ $[x_i] = L$ $[t] = T$ $[\hat{p}] = L^2T^{-2}$
3.	L, T
4.	Length scale: Square cylinder size ``D´´ Time scale: $\frac{D}{U_B}$
5.	$u_i = u_i^* U_B \Leftrightarrow u_i^* = \frac{u_i}{U_B}$ $t = t * \frac{D}{U_B} \Leftrightarrow t^* = \frac{tU_B}{D}$ $x_i = x_i^* D \Leftrightarrow x_i^* = \frac{x_i}{D}$ $\hat{p} = \hat{p}^* U_B^2 \Leftrightarrow \hat{p}^* = \frac{\hat{p}}{U_B^2}$
6.	$\frac{\partial u_i}{\partial x_i} = 0 \rightarrow \frac{\partial u_i^* U_B}{\partial x_i D} = 0 \rightarrow \left(\frac{U_B}{D}\right)\frac{\partial u_i^*}{\partial x_i^*} = 0 \Rightarrow \frac{\partial u_i^*}{\partial x_i^*} = 0$ $\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial \hat{p}}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j}$

$$\rightarrow \frac{\partial u_i^* U_B}{\partial t^* D/U_B} + u_j^* U_B \frac{\partial u_i^* U_B}{\partial x_j^* D}$$

$$= -\frac{\partial \hat{p}^* U_B^2}{\partial x_i^* D} + \nu \frac{\partial^2 u_i^* U_B}{\partial x_j^* D \partial x_j^* D}$$

$$\rightarrow \frac{U_B^2}{D}\frac{\partial u_i^*}{\partial t^*} + u_j^* \frac{\partial u_i^*}{\partial x_j^*} = -\frac{\partial \hat{p}^*}{\partial x_i^*} + \frac{\nu}{U_B D}\frac{\partial^2 u_i^*}{\partial x_j^* \partial x_j^*}$$

$$\rightarrow \frac{\partial u_i^*}{\partial t^*} + u_j^* \frac{\partial u_i^*}{\partial x_j^*} = -\frac{\partial \hat{p}^*}{\partial x_i^*} + \frac{\nu}{U_B D}\frac{\partial^2 u_i^*}{\partial x_j^* \partial x_j^*}$$

$$\frac{\nu}{U_B D} = \frac{1}{Re_D}$$

7. At walls: $u_i = 0 \quad \Leftrightarrow \quad u_i^* U_B = 0$

$u_i^* = 0$

Flow: $\frac{U_B}{U_B} = 1$

Through the non-dimensional analysis, the continuity equation becomes $\frac{\partial u_i^*}{\partial x_i^*} = 0$ and the momentum equation becomes $\frac{\partial u_i^*}{\partial t^*} + u_j^* \frac{\partial u_i^*}{\partial x_j^*} = -\frac{\partial \hat{p}^*}{\partial x_i^*} + \frac{\nu}{U_B D}\frac{\partial^2 u_i^*}{\partial x_j^* \partial x_j^*}$ as seen in step 6 above. The non-dimensional parameter that appears is found to be $\frac{\nu}{U_B D} = \frac{1}{Re_D}$. This shows that the diffusive term in the momentum equation is affected inversely to the Reynolds number. With the Reynolds number being simplified to the inertial forces over viscous forces, the diffusion in this example will increase with an increase in viscous force or a decrease in inertial force. Or the diffusion will decrease with a decrease in viscous force or an increase in inertial force.

Question 2 – Streamline plots

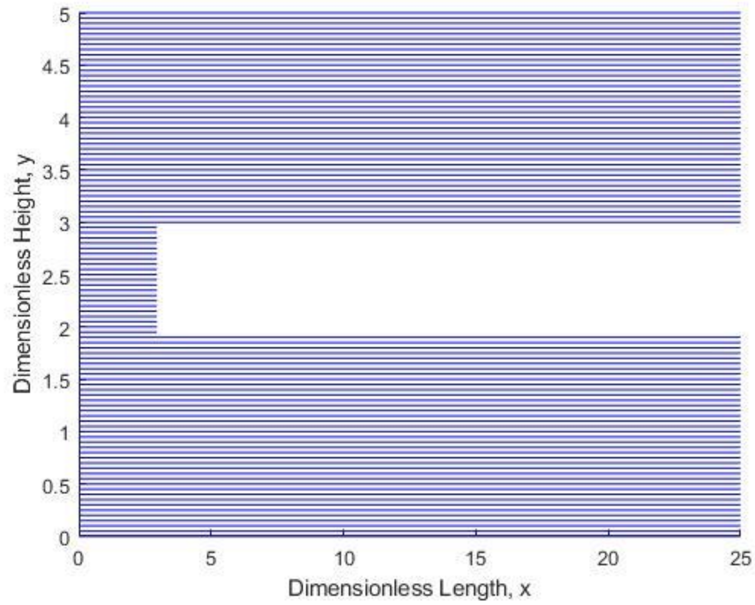

Figure 1: Streamline plot at t=0 seconds

Figure 1 shows the initial set up for the stream line plot. The only velocity seen is in the u direction and is therefor blocked by the square cylinder in and behind it. There is zero V velocity and this can be seen by the lines being purely horizontal.

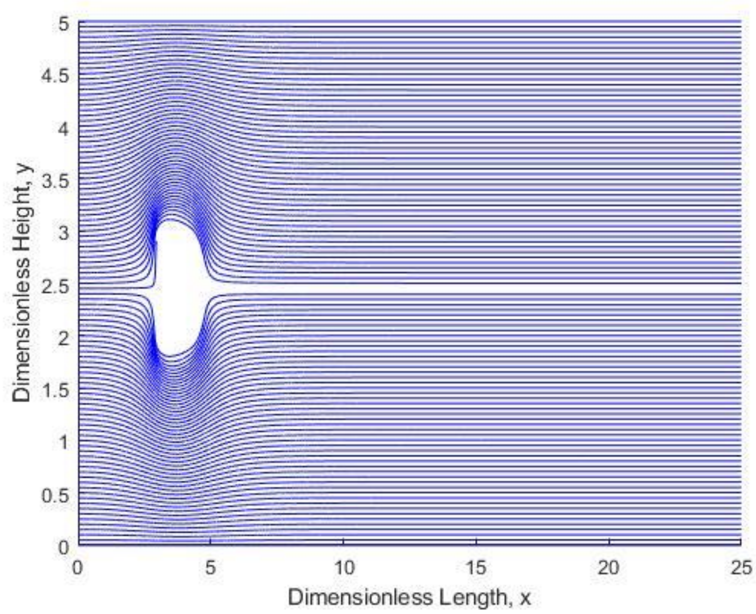

Figure 2: Streamline plot at t=5 seconds

Figure 2 shows the flow approaches from the left and forms a stagnation point at the middle of the front face of the cylinder, this is where the maximum pressure coefficient occurs. The flow then bifurcates into separate streams towards the top and bottom edges of the object. Flow separation occurs at the leading edge of the cylinder and the flow is reattached at x=5. High concentration of streamlines indicate increased velocities at the leading and rear edge corners of the object, although at the leading edge they are more pronounced. This can be derived from mass conservation, which dictates an increase in speed for decreasing area, assuming constant density.

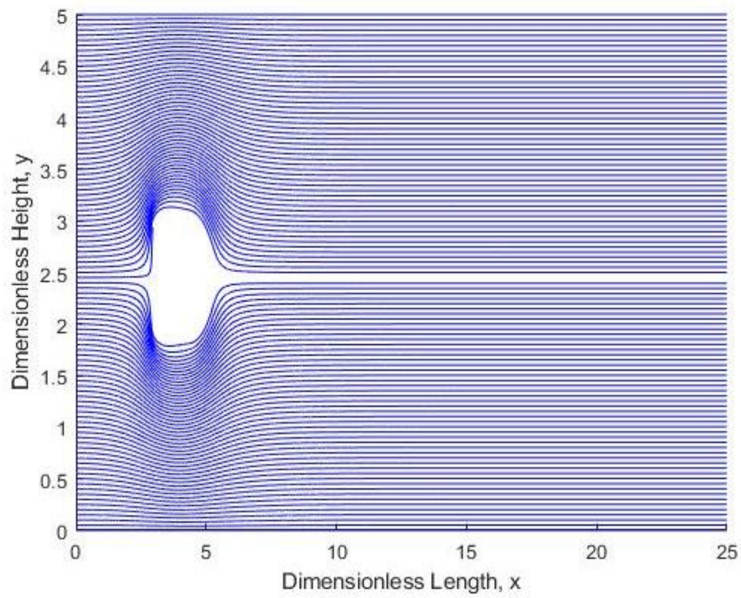

Figure 3: Streamline plot at t=10 seconds

Figure 3 shows the streamline concentration at the leading edge remains constant while at the rear it is decreased due to the growth of the wake in the flow past the cylinder.

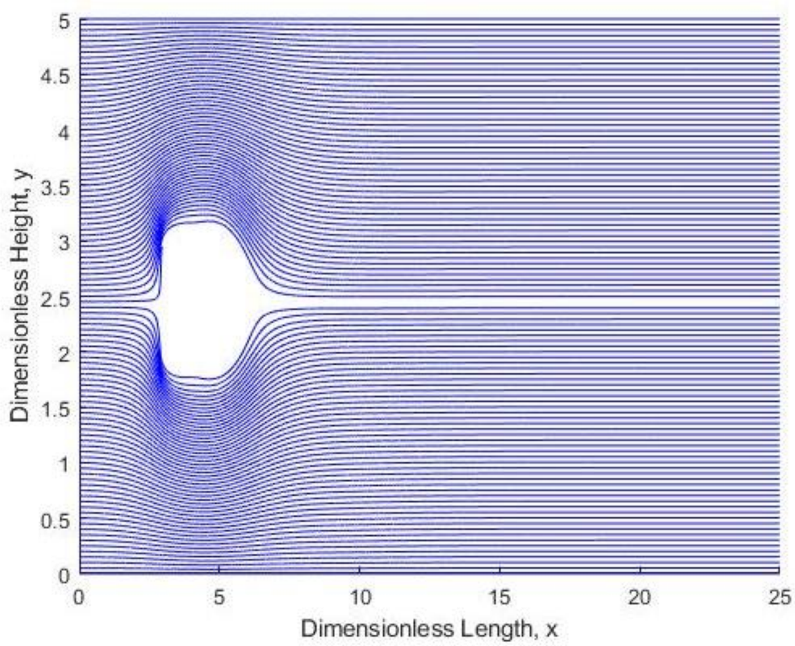

Figure 4: Streamline plot at t=20 seconds

Figure 4 shows the size of the wake is almost twice as large in the x-direction compared with the reading at 5 seconds. At the rear edge of the cylinder, the boundary layer height seems to be increasing slightly.

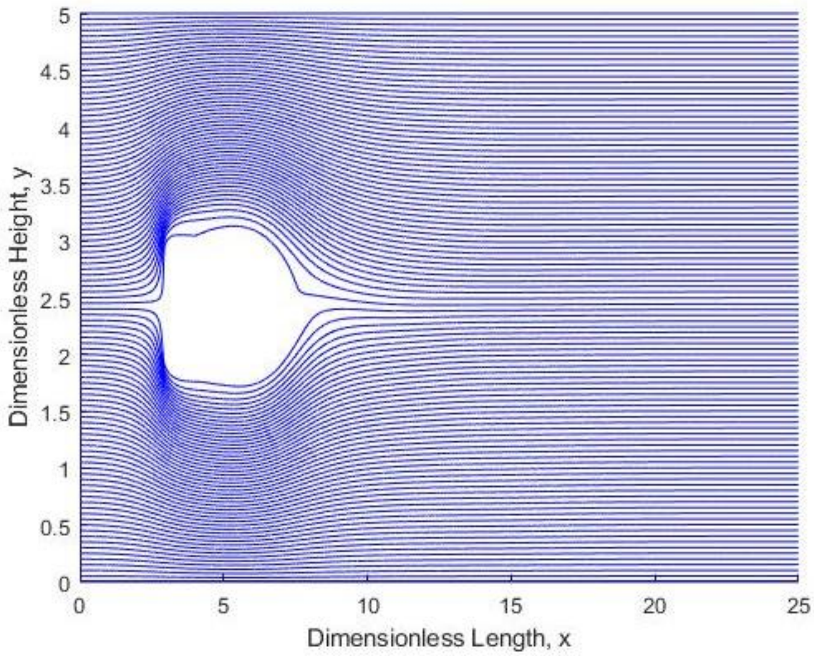

Figure 5: Streamline plot at t=40 seconds

Figure 5 shows the wake of the flow past the cylinder is still growing in length, although at a slower rate than before. A significant increase in flow separation is observed, and appears to be occurring closer to the leading edge with increasing time.

At t = 60 seconds vortex formation can be observed in the wake of the object. Flow separation is significantly increased and reattachment happens approximately at x=12.

At t=70 seconds the streamlines show the continued creation of vortices originating from the top and bottom streams, in alternating fashion. This pattern of oscillating flow is known as vortex shedding, and occurs when a fluid flows past a blunt body, such as a square cylinder.

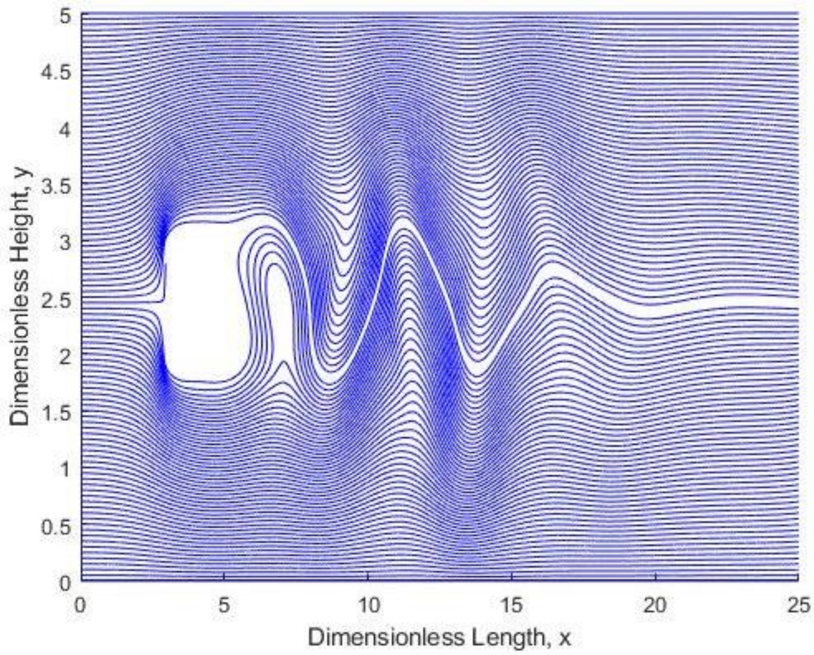

Figure 6: Streamline plot at t=100 seconds

Figure 6 shows the frequency at 100 seconds in, the flow and vortex shedding has fully developed and shows a strong occillation. An increase in velocity with passing time is observed between figure 5 and 6, as indicated by the reduction in distance between streamlines in the wake of the flow

Question 3 – Strouhal number

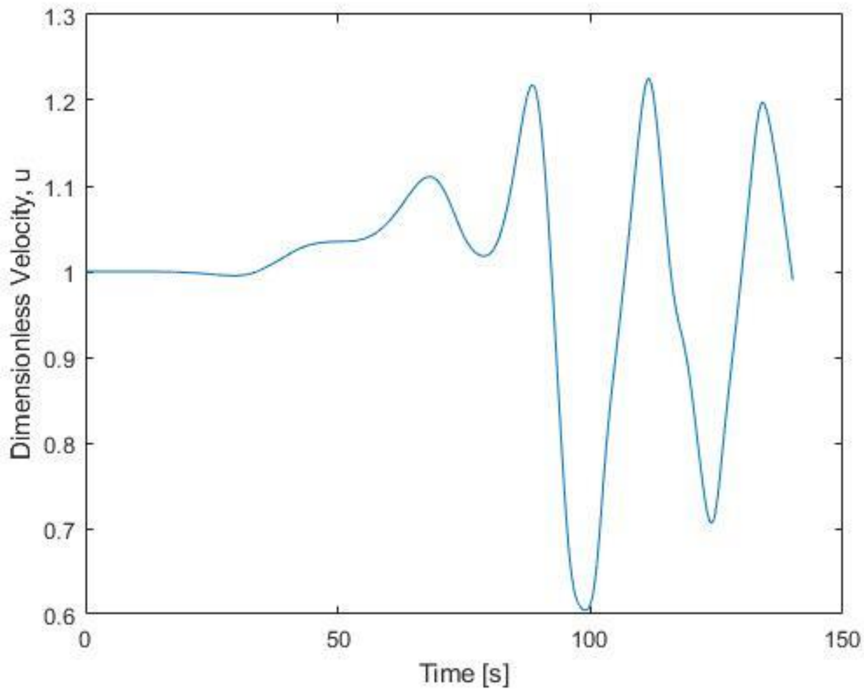

Figure 7. Dimensionless velocity u fluctuation over simulation time.

Figure 7 shows the velocity *u* changing over time at the wake of a square cylinder. The velocity *u* fluctuates in a sinusoidal wave starting **76** seconds into the simulation. According to (Okajima, 1982), such behaviour is a sign of vortex shredding. Strouhal number is a dimensionless number that can be used to describe a behaviour of vortex shredding in a flow. In the case of a flow over a square cylinder, the Strouhal number St can be found using the following equation:

$$St = \frac{fD}{U} \tag{4}$$

where f is the vortex shredding frequency [Hz], D is the length of a side of the square cylinder [m], U is the flow velocity. However, since dimensionless values for time, velocity and length are used in the estimations, the estimated frequency will equal to Strouhal number since both values are dimensionless. (Equation 5)

In order to find the Strouhal number related to vortex shredding, the vortex shredding frequency needs to be estimated using the fast Fourier transform (FFT) method. Initially, the range of absolute values of FFT velocities is found, where the vortex shredding frequency is extracted at a peak FFT velocity value. As shown in section 1, all parameters are non-dimensionalised thus making the formula for Strouhal number:

$$St = f(N) \tag{5}$$

where N is the peak FFT velocity value[-].

The FFT values used for the estimation of the Strouhal number were taken from **76**th second of the simulation(19 t*) onwards to **140**th second. This value was picked by observation and is based on the streamlines from Figure 6 entering a vortex pattern and the velocity *u* fluctuation initialising a sinusoidal form. This means that the FFT velocity was taken from the y-coordinate of the simulation box equal to **36**. (j=36)

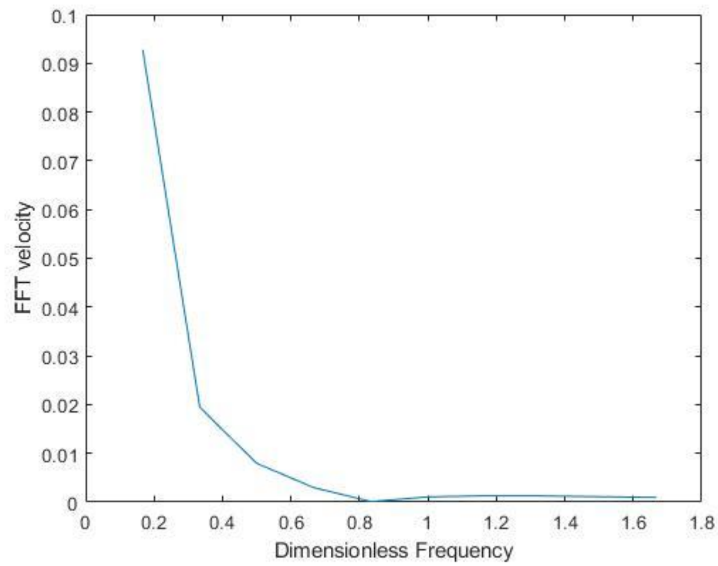

Figure 8. FFT Velocity against Dimensionless frequency

Figure 8 demonstrates the variation of FFT Velocity with frequency giving the very left point as the peak maximum *N* that corresponds to the vortex shredding frequency. Therefore, the result for the Strouhal number estimated for the given flow is **0.1667**. When compared to (Okajima, 1982) study on the Strouhal number of square cylinders at *Re = 100*, the estimated result of Strouhal number for this project is higher by approximately 15% percent.

References

Okajima, A. (1982). Strouhal numbers of rectangular cylinders. *Journal of Fluid Mechanics*, 123, pp.379-398.