# Lab 1: Wave Equations

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Adapted from Physics 141 lab manual by Blake Hipsley and Wolfgang Lorenzon

## 1 Introduction

In this lab session we will study wave mechanics. A **wave** is defined as any periodic disturbance that propagates through a medium. For example, a sound wave is a disturbance in pressure that propagates through air (or water, or anything else). At different parts of the wave, the molecules of air are closer together or farther apart, creating areas of higher and lower pressure. A sinusoidal sound wave is considered a wave, rather than just a disturbance, because these areas of high and low pressure are **periodic**: they occur at regular intervals of position and time.

Why study waves? This course is designed to give you an introduction to the strange world of elementary particle physics. Later on, we will show that elementary particles have the properties of **both** a wave and a particle. For this to make any sense, we need to understand the properties of waves first.

## 2 Wave Equations: Interactive Animation

The first part of the lab is meant to familiarize you with wave equations and their properties. Begin by opening the link: `https://www.desmos.com/calculator/gquttoa59i`

You should see a screen like this:

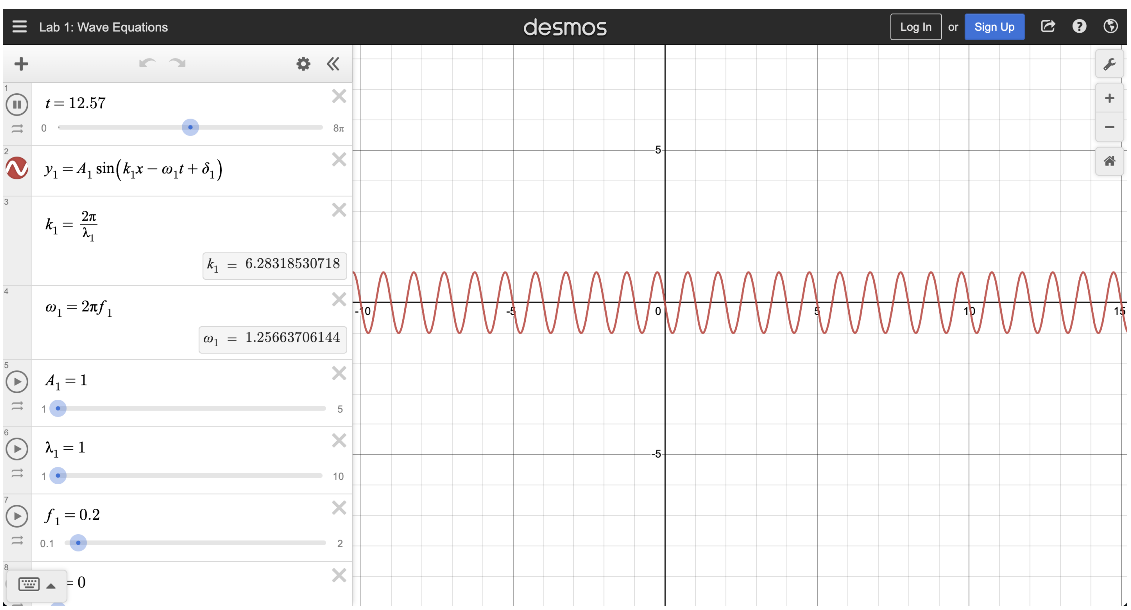


Figure 1: Animation webpage with no modifications.

The following is important to know before we move forward:

1. Do NOT press any of the "X" in the left side bar, which will delete one of the variables in the equation. If you accidentally do, you might need to re-load the page.

2. Each "parameter" (Amplitude $A$, wavelength $\lambda$, frequency $f$, phase shift $\delta$, time $t$) has a slider and a play button. At the start, time is running in a loop and the wave should be moving to the right. You can press "play" on any other parameter and multiple parameters may be "playing" at a time.

3. Each "function" has a on/off switch. For example, see the red circle with the wavy line through it next to "$y_1$" in the top left corner. By clicking this circular button, functions can be turned on and off.

4. Play around with these features to familiarize yourself before moving forward. Once you are ready, set the parameters $A_1$, $\lambda_1$, and $f_1$ to the initial values shown in Figure 1.

## 2.1 Anatomy of a Wave Equation

All waves discussed today are *sinusoidal* waves. The wave in Figure 1 is an example of a sinusoidal wave. The displacement $y$ from the x-axis at a point $x$ and time $t$ is given mathematically by the expression:

$$y(t) = A\sin(kx - \omega t + \delta) \tag{1}$$

This (with extra subscripts like $A_1$ instead of $A$) is the equation displayed in the top left of your window. There are many parameters in this equation, in addition to the usual variables $x$ and $y$, and the time $t$:

- **Amplitude**: The amplitude $A$ controls how "tall" the wave is, or the distance between the top "crest" and the x-axis.

- **Wavelength/Wave number**: The wave number $k$ is related directly to the wavelength $\lambda$, which is the distance between two neighboring wave peaks or "crests." The relation is $k = 2\pi/\lambda$. Physicists interchangeably discuss either wavelength or wave number, depending on the context. For this lab, we will discuss the wavelength $\lambda$ to keep things simple.

- **Frequency**: Again, there are two types of frequency that can be used depending on the context. Angular frequency $\omega$ is the frequency in units of radians, and therefore is exactly $2\pi$ times "regular" frequency, which is the number of full oscillations per unit time. If you've ever turned a ratio dial to a specific number like 101.1 MHz, you are tuning in to a specific *frequency* (not angular frequency, but just frequency). In this case, that frequency is of a radio wave oscillating 101.1 million times per second.

- **Phase shift**: The phase shift $\delta$ is usually used to measure the relative phase shift between two waves. There's a certain sense in which the phase shift is arbitrary for a single wave, and it can be "absorbed" into a redefinition of other parameters.

**Experiment 1** Try moving each slider around and verify how each parameter changes the wave animated in the graph on the right hand side of your screen.

**Question 1** *Units* are an important part of understanding physical equations. Given that $x$ and $y$ have units of length and anything put into the sine function must be unit-less, can you determine the units of the following parameters:

1. Amplitude $A$

2. Wave length $\lambda$

3. Frequency $f$

4. Phase shift $\delta$

Hint: answers will be length and/or time raised to some power. For example, $x$ has units of length so $x^2$ has units of length$^2$. $t$ has units of time, so $1/t$ has units of time$^{-1}$.

## 2.2 Wave Speed

Wave speed is the rate at which the crests move from left to right.

**Question 2** Try moving the sliders for $A$, $f$, $\lambda$ and $\delta$. Which of these 4 parameters change the wave speed? Do they increase or decrease the speed? Can you make the wave move from right to left instead (don't worry, the answer is no, for now).

The wave speed is determined from the other parameters:

$$v = f\lambda \tag{2}$$

Now it should be obvious which parameters change wave speed: frequency and wavelength. Because the sliders are limited only to positive values, it also makes sense why we couldn't make the wave move from right to left, because that means the wave speed would be negative. To see the wave move the opposite direction, try changing $-\omega_1 t$ to $+\omega_1 t$ in the wave equation. Remember to change this back when you are done.

However, in almost all cases the wave speed is a constant. Wave speed is usually a function of the medium, not the wave. For example, the speed of light is constant and there is really only one free parameter: either frequency or wavelength. Changing one means the other one must change to compensate and keep speed constant. If you double the frequency, the wavelength is divided by 2.

**Experiment 2** Try it out! Turn on the function $y_2$ and set it so that the frequency is double $f_2 = 2f_1$ and the wavelength is half $\lambda_2 = \lambda_1/2$, using the sliders. You should be able to get two waves on top of each other both moving to the right at the same speed, despite their different wavelengths and frequencies. An example is shown in Figure 2.

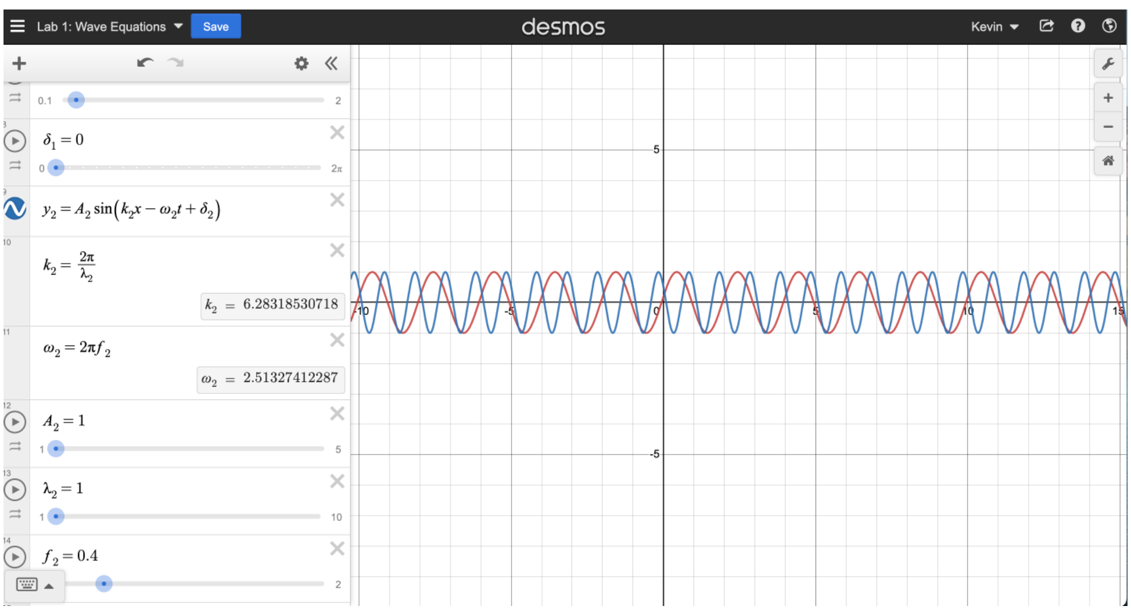


Figure 2: Two waves with different frequencies moving at the same speed.

Since the wave speed is a function of the medium, what is that function? For a string, the speed of a wave is given by:

$$v = \sqrt{\frac{T}{\mu}} \tag{3}$$

where $T$ is the tension on the string (units of *Newtons* or kg$\times$m/s$^2$ for force) and $\mu$ is the string's mass per unit length. Remember, this *also* equals $f\lambda$ and is constant.

**Question 3** Can you verify that Equation 3 gives units of speed, or length over time?

## 2.3 Principle of Linear Superposition and Standing Waves

What happens when two waves pass through the same medium at the same time? At every point in space, we add the value of the two wave functions to get the total displacement. This is called the **Principle of Linear Superposition**. It doesn't *always* apply, but it will in this lab today.

**Experiment 3** Scroll to the bottom and turn on $y_3$, which is a linear superposition (fancy word for adding together) of $y_1$ and $y_2$. Perform the following tests:

1. Set $f_1 = f_2$, $\lambda_1 = \lambda_2$, and $\delta_1 = \delta_2$. Change the amplitudes $A_1$ and $A_2$, or just press the play button on one of the two amplitudes. What do you see? How does the amplitude of the green function change?

2. Set all the properties of both waves to be identical. Now, press the play button on $\delta_2$. Why does it look like the amplitude of the green function is changing, even though the amplitudes of the red and blue functions are clearly not?

3. Play around with as many sliders and animations as you wish, and see what weird waves you can make!

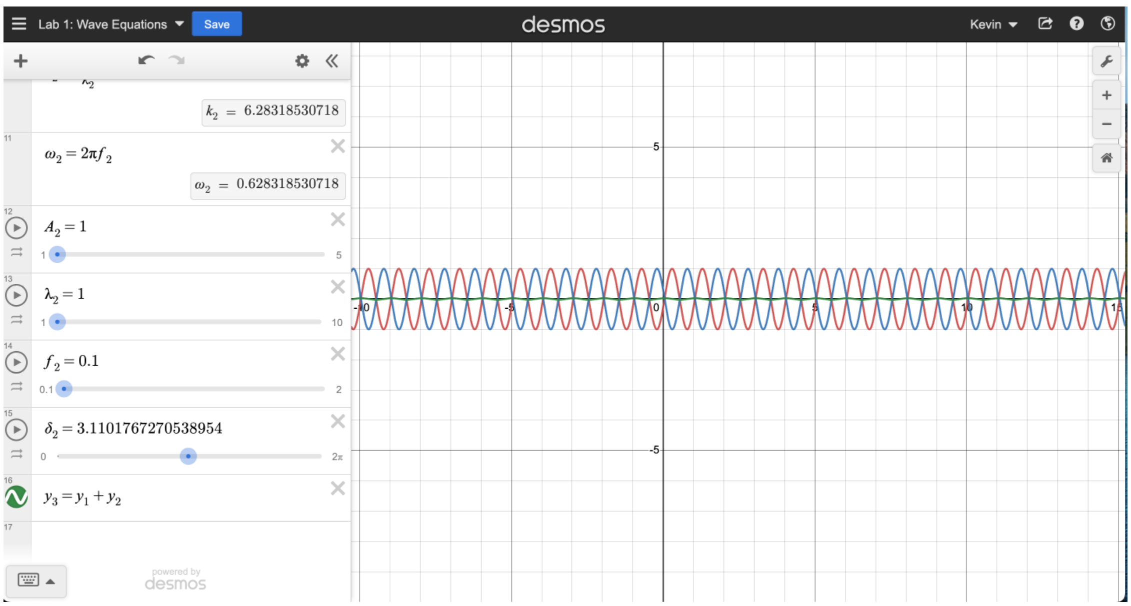


Figure 3: When we add together two functions which are *out of phase*, the result is *destructive interference*.

In parts 1 and 2 of experiment 3 we discovered **constructive interference** and **destructive interference**, respectively. This is an enormously important concept in **quantum mechanics**, where fundamental particles are described by wave equations which can interfere with each other. We will see more of interference in future labs and demonstrations.

## 2.4 Resonance and Standing Waves

A **resonator** is a device or system which oscillates at some characteristic frequency or set of frequencies. These special frequencies are the **resonant frequencies**. When a wave passes through the object at the resonant frequency **resonance** occurs, producing **standing waves**.

Resonators are usually objects who have resonant frequencies because their dimensions are an integer multiple (or an integer plus 1/2) of the wavelength at that frequency (remember, because the speed is constant wavelength and frequency are directly related). For a string fixed at both ends, the resonant wavelengths are given by:

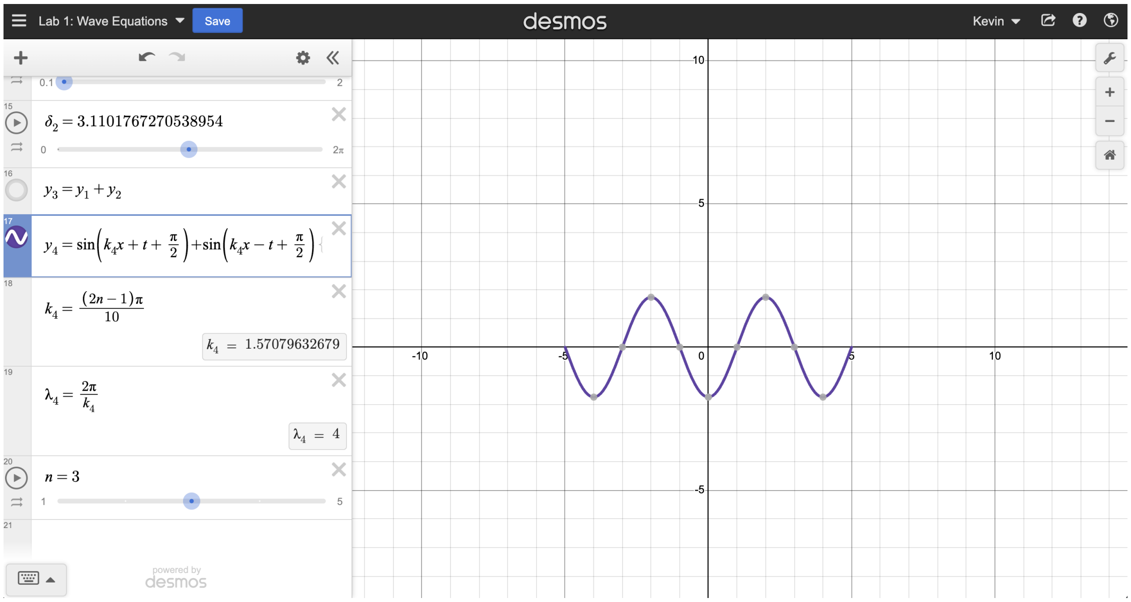


Figure 4: The resonant string animated in desmos. Here, the 3rd resonant frequency is shown.

$$\lambda_n = \frac{2L}{n} \tag{4}$$

where $n = 1, 2, 3...$ and $L$ is the length of the string. There are an infinite number of resonant wavelengths! The resonant frequencies in a string can then be calculated using Equations 2, 3 and 4:

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}} = \frac{v}{\lambda} \tag{5}$$

Note that it may be easier to first calculate $v$ and $\lambda_n$, and then calculate $f_n$.

Turn off $y_1$, $y_2$ and $y_3$ and turn on $y_4$. An example of the animation with $y_4$ turned on is shown in Figure 4. This animation illustrates a standing wave on a string. The string is fixed at $x = \pm 5$. These fixed points are called **nodes** and the crests occur at **anti-nodes**. The wave is called a standing wave because the crests don't move from left to right, but stay in place.

**Experiment 4** Use the slider for $n$ to change the number of anti-nodes between the two fixed nodes. $n$ is the number of the resonant harmonic we are plotting. Notice how the wavelength $\lambda_4$ changes as you change $n$.

**Experiment 5** Previously, all of our waves moved from right to left or left to right, depending on the plus or minus sign in front of $t$ in the wave equation. How do we mathematically describe a standing wave using our wave equation? Check the panel on the left. It looks like the standing wave is the *linear superposition* of a wave moving left and right. Can you turn $y_4$ back off and turn on $y_1$, $y_2$ and $y_3$, and turn $y_3$ into a standing wave by modifying the equation for $y_2$, making it move from right to left? How are standing waves related to constructive and destructive interference?

# 3 Experiment: Standing Waves on a String

A string is attached to a variable frequency mechanical wave driver, which is in turn attached to a sine wave generator capable of adjusting the frequency of the output wave to the nearest tenth of a Hz. The other end of the string passes over a pulley and attaches to a variable hanging mass. The string is fixed at both ends and standing waves in the resonant frequency can be produced. The hanging mass can be varied to adjust the tension in the string and hence the wave speed. A diagram of the setup is shown in Figure 5.

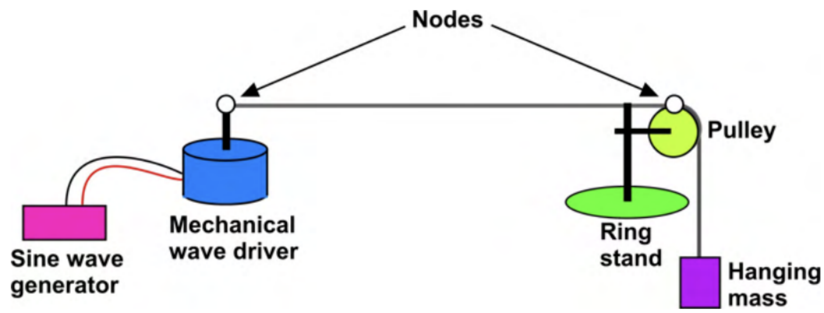


Figure 5: The experimental setup for producing standing waves on a string.

**Question 4** Calculate the first 5 ($n = 1, 2, 3, 4, 5$) resonant frequencies $f_n$ of a $L = 1.2$ meter string using Equation 5. Assume the tension $T = 1$ Newton (which is the tension from a 100 gram hanging mass) and the mass density $\mu =$???. Is there a pattern between $n$ and $f_n$? Can you see this pattern from Equation 5?

**Experiment 6** Next we will measure the first 5 resonant frequencies and compare to the calculated result.

1. The experimental setup should already be assembled for you. Make sure the 100g mass is hanging and the mechanical vibrator is “unlocked” so that the oscillating pin can move freely.
2. Adjust the equipment so that the length of the string between the two nodes is 1.2 meters.
3. Turn the amplitude dial of the sine wave generator all the way down. Turn on the sine wave generator and dial the frequency down to 1 Hz.
4. Turn up the amplitude until the string oscillates visibly.
5. Turn up the frequency gradually until you find the first resonant mode. Note the value of the resonant frequency $f_1$.
6. Continue turning up the frequency until you have observed 5 normal modes. Compare the values to those you calculated.
7. If time permits, repeat the experiment with the 200g mass. What is the expected relationship between the frequencies with twice the tension and the previous frequencies with a 100g mass? Can you verify this relationship?