Bayesian Logistic Regression Problems

ELG 5218 - Uncertainty Evaluation in Engineering Measurements and Machine Learning

Instructor: Miodrag Bolic, University of Ottawa

Date: January 28, 2026

PART A: CONCEPTUAL QUESTIONS (Simple to Intermediate)

A1. What is the fundamental difference between classical logistic regression and Bayesian logistic regression?

A2. Why is the logistic regression posterior non-conjugate?

A3. What are the three main steps of the Laplace Approximation?

A4. Explain the physical interpretation of the Hessian in one dimension.

A5. What happens to the Laplace approximation when the posterior is multimodal?

PART B: MATHEMATICAL DERIVATIONS

B1. Derive the gradient of the log-posterior for Bayesian logistic regression.

Problem. Given

$$p(\mathbf{w} \mid X, \mathbf{y}) \propto p(\mathbf{y} \mid X, \mathbf{w})\, p(\mathbf{w}),$$

where

$$p(\mathbf{y} \mid X, \mathbf{w}) = \prod_{n=1}^{N} \sigma(\mathbf{w}^\top \mathbf{x}_n)^{y_n} \big(1 - \sigma(\mathbf{w}^\top \mathbf{x}_n)\big)^{1-y_n}, \quad p(\mathbf{w}) = \mathcal{N}(\mathbf{w} \mid 0, \alpha^{-1} I),$$

derive $\nabla_{\mathbf{w}} \log p(\mathbf{w} \mid X, \mathbf{y})$.

B2. Derive the Hessian matrix for Bayesian logistic regression.

Problem. Compute $\nabla^2_{\mathbf{w}} \log p(\mathbf{w} \mid X, \mathbf{y})$.

PART C: PARAMETRIC ANALYSIS (What if we change parameters?)

C1. What happens to the posterior covariance if we increase the regularization parameter α?

C2. How does the predictive uncertainty change as we move away from the training data region?

PART D: IMPLEMENTATION AND PRACTICAL QUESTIONS

D1. In the NumPyro notebook, what is the role of `numpyro.plate`?

D2. How would you modify the component failure prediction model if you had an imbalanced dataset (e.g., 95% functional, 5% failed)?

D3. What differences would you expect between Laplace Approximation and MCMC (HMC) sampling for this problem?

PART E: OTHER PROBLEMS

E2: Posterior Mode for Logistic Regression

Problem. Consider the logistic regression model with one data point $x = 2$, $y = 1$, and prior $w \sim \mathcal{N}(0, 1)$. Find the posterior mode of w analytically and show it as a formula that can can be solved then numerically.