

A System-of-Equations Approach to Modeling Age-Structured Fish Populations: The Case of Alaskan Red King Crab, *Paralithodes camtschaticus*

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Greenberg, J. A., S. C. Matulich, and R. C. Mittelhammer. 1991. A system-of-equations approach to modeling age-structured fish populations: the case of Alaskan red king crab, *Paralithodes camtschaticus*. Can. J. Fish. Aquat. Sci. 48: 1613–1622.

This paper presents a simultaneous system-of-equations approach to modeling age-structured populations using trawl survey age/size frequency data. The analysis builds upon a Ricker spawner–recruit structure and provides a cohort-based estimation method that retains the underlying dynamic properties of a delay-difference model. The framework shares a common spawner–recruit function across age-class equations. This exploits the commonality among cohort members and serves as an instrumental variable, lessening the effect of measurement errors in estimation. The dynamic features of the underlying age-structured population are retained through age-specific net survivability and growth parameters that link age-classes. The technique uses multiple observations on a cohort to further mitigate the effect of measurement error and improve overall estimation efficiency. A seemingly unrelated regression estimation method is required to address contemporaneous correlation of errors across age-classes. This framework is applied to trawl survey data for adult male Alaskan king crab, *Paralithodes camtschaticus*.

Les auteurs présentent un système d'équations simultanées pour modéliser des populations structurées selon l'âge à l'aide de données sur la fréquence des âges selon l'âge à l'aide de données sur la fréquence des âges selon la longueur recueillies au cours de relevés par chalutage. L'analyse repose sur une structure géniteurs–recrues élaborée par Ricker et constitue une méthode d'évaluation basée sur les cohortes qui retient les propriétés dynamiques sous-jacentes d'un modèle de délai des différences. La structure fondamentale partage une fonction commune géniteur–recrue dans les équations relatives aux classes d'âge. Ceci tire parti du facteur commun entre les membres des cohortes et sert de variable contributive qui réduit l'incidence des erreurs de quantification dans les estimations. Les caractéristiques dynamiques de la population sous-jacente structurée selon l'âge sont retenues par l'entremise du taux de survie net spécifique à l'âge et les paramètres de croissance qui lient les classes d'âge. La méthode utilise des observations multiples d'une cohorte pour réduire encore plus les erreurs de quantification et améliorer l'efficacité globale des estimations. Une méthode d'estimation de la régression, apparemment sans rapport, est nécessaire pour étudier la corrélation contemporaine des erreurs parmi les classes d'âge. Cette structure fondamentale est appliquée à des données de relevés par chalutage relatives à des mâles adultes du crabe royal rouge (*Paralithodes camtschaticus*).

Received June 26, 1990
Accepted March 13, 1991
(JA628)

Reçu le 26 juin 1990
Accepté le 13 mars 1991

An extensive literature on age/size-structured fish population models has developed in recent years in part because age and/or size frequency catch data are readily available. Modern age/size analytical methods are rooted in virtual population analysis (Gulland 1965; Murphy 1965; Pope 1972; Ricker 1975). As the limitations of virtual population analysis became apparent, it evolved into variants of catch-at-age analysis (Doubleday 1976; Paloheimo 1980; Fournier and Archibald 1982; Collie and Sissenwine 1983; Deriso et al. 1985, 1989). More recently, size-structured frameworks have been proposed for situations where either catch-at-age data do not exist, but size frequency data do exist, or where the accuracy of size-at-age estimates comes into question (Schnute 1987; Schnute et al. 1989a, 1989b; Sullivan et al. 1990; Parma and Deriso 1990).

Modeling age-structured Alaskan red king crab, *Paralithodes camtschaticus*, stocks in the Bristol Bay portion of the Bering Sea unfortunately cannot benefit much from this rich

literature. Commercial catch statistics do not report the distribution of catch by size or age. Rather gross harvest weight and numbers are reported in this males-only fishery. Also, grading by size occurs during secondary processing outside of Alaska (primarily in Japan and Seattle). The National Marine Fisheries Service (NMFS), however, conducts annual trawl surveys to estimate stock abundance. These trawl survey data are reported both in terms of a size frequency distribution and an age frequency distribution that is calculated from a length–age key. Male and female stocks are reported separately.

There are two obvious approaches to modeling age-class abundance from the trawl survey data. First, a delay-difference structure like that proposed by Deriso (1980) could be used. A recruitment function would be estimated for, say, 5-yr-olds, and advanced to subsequent age-classes with age-specific mortality and growth parameters. This approach has the advantage of yielding an exact representation of age-structure effects on biomass dynamics, but relies on the accuracy of age-specific

estimates of stock. Alternatively, one could estimate a series of separate and independent age-class recruitment functions. Parameter estimates would differ across age-class equations reflecting altered recruitment trajectories due to cumulative growth and mortality. This procedure has the distinct advantage of basing the estimates of individual age-class abundance on estimates of aggregate parental stocks, but does not explicitly model the underlying age-specific growth and mortality parameters as age-class advances.

Both approaches are theoretically sound and should, in the absence of measurement errors, yield virtually identical predictions. But fisheries data tend to be measured with error, and this certainly is the case with the king crab trawl survey data. The NMFS data suffer from two predominant types of error: survey error and errors related to converting carapace size to estimates of crab age. Errors in the crab data are obvious, since there are a number of cases where the population of a cohort (in numbers) increases as it advances in age. For example, the reported population of male 5-yr-olds in 1978 was 22.0 million, while the 1979 population of male 6-yr-olds was 20% greater, 26.5 million.

Walters and Ludwig (1981) and Ludwig and Walters (1981) initially addressed this problem of errors in variables when estimating stock-recruitment relationships. They approached the problem by incorporating independent assessments of environmental and measurement error variances. Ludwig and Walters (1989) extended their earlier analysis, comparing the generalized Ricker (1954) family of recruitment functions with those of Deriso (1980) and Schnute (1985). They concluded that in the presence of large data errors, the trade-off between bias and variance requires one to "give up fidelity of the estimation model to the underlying dynamics in order to improve estimation performance" (p. 142). The Ricker class of models was found to outperform the Deriso-Schnute family of models.

This paper takes an alternative approach — one that exploits beneficial attributes of both Ricker- and Deriso-type models. To do so, however, requires addressing not only measurement errors, but also requires adopting a cohort-based estimation scheme. The measurement error problem is addressed with an instrumental variables approach similar to that of Ludwig and Walters (1989). That is, individual right-hand-side age-class variables are replaced by more aggregated (less errant) "instruments" that are correlated with the variable of interest, but measured with less error. A Ricker-type spawner-recruit model is used for this purpose. However, unlike Ludwig and Walters (1989), individual age-class equations are not estimated independently. Rather, the model developed in this paper views each age-class within a system of simultaneous equations that share a common spawner-recruit function. Each age-class equation is linked through age-specific net survivability and growth parameters, retaining the dynamic features of the underlying age-structured population.

The "systems" approach introduces contemporaneous correlation of errors across age-classes. Accounting for inter-age-class correlations that occur within each time step of a cohort requires the use of an estimation technique known as seemingly unrelated regression.

The framework developed here is applied to modeling age-structured, adult male red king crab stocks in the Bristol Bay portion of the Bering Sea. The analysis uses NMFS published size/age distribution data developed from the 1968–89 trawl

surveys.¹ We limit the analysis to adult male red king crab for several reasons. First, king crab recruit into the parental stock at approximately age 5 whereas they recruit into the fishery at ages 8 or 9 and can live to 14+ yr of age. Second, this is a males-only fishery. Third, trawl survey data for juveniles are grossly inaccurate because of survey gear design.

Methods

Conceptual Framework

The conceptual basis for modeling age-class recruitment is illustrated in the system-of-equations (1) for 5- to 8-yr-old male crab:

$$(1) \quad \begin{aligned} M_{5,t} &= f(F_{t-6}, M_{t-6}; \beta) + \mu_{5,t} \\ M_{6,t+1} &= \tau_6 M_{5,t} + \mu_{6,t+1} \\ M_{7,t+2} &= \tau_7 M_{6,t+1} + \mu_{7,t+2} \\ M_{8,t+3} &= \tau_8 M_{7,t+2} + \mu_{8,t+3} \end{aligned}$$

The variables in (1) defined as $M_{a,t}$, for ages $a = 5, 6, 7, 8$, are age-specific recruitment measured in biomass; $f(F_{t-s}, M_{t-s};$

$\beta)$ represents a spawner-recruit function; $F_{t-s} = \sum_{a=5}^{14} F_{a,t-s}$ and

$M_{t-s} = \sum_{a=5}^{14} M_{a,t-s}$ represent parental stock biomass in year

$t-s$, the year of spawning, where $F_{a,t-s}$ and $M_{a,t-s}$ represents the biomass of females and males of age-class a in year $t-s$; β represents the parameter vector of the spawner-recruit function; τ_6, τ_7 , and τ_8 represent 5- to 6-, 6- to 7-, and 7- to 8-yr-old net survivability/growth rates, where $\tau_a = \exp(z_a)$ with z_a representing the survivability/growth parameter for age-class a ; and $\mu_{5,t}, \dots, \mu_{8,t+3}$ represent the respective process errors in the progression of the cohort.

The dynamic features of the underlying age-structured population are retained via a longitudinal view of a cohort. That is, the system emulates a delay-difference equation similar to that proposed by Deriso (1980); age-specific net survivability and growth parameters link the age-classes.

If the population values in (1) were observed without error, and assuming the classical process error properties were maintained, the usual least squares approach to estimating the parameters in (1) could be pursued. However, given that the population observations are subject to measurement error, the least squares approach applied to (1) would yield biased and inconsistent parameter estimates. In order to clarify the estimation problem involved, let

$$(2) \quad M_{a,t}^o = M_{a,t} + v_{a,t}, \quad a = 5, \dots, 8$$

represent the deviation between an observed population level of crab ($M_{a,t}^o$) and the actual population level ($M_{a,t}$), where $v_{a,t}$ represents measurement error. Furthermore, assume for the moment that the observed level of aggregate parental stock is

¹Age and size categories are used interchangeably throughout this paper. Following NMFS age/carapace length composition data, age-specific adult male crab (M_a) conform to the following size distribution: $M_5 = 95\text{--}109$ mm, $M_6 = 110\text{--}119$ mm, $M_7 = 120\text{--}129$ mm, $M_8 = 130\text{--}139$ mm, and $M_{9-14} \geq 140$ mm.

accurate. Then, representing (1) in terms of observable variables yields (3):

$$(3) \quad \begin{aligned} M_{5,t}^o &= f(F_{t-6}^o, M_{t-6}^o; \beta) + (\mu_{5,t} + v_{5,t}) \\ M_{6,t+1}^o &= \tau_6 M_{5,t}^o + (\mu_{6,t+1} + v_{6,t+1} - \tau_6 v_{5,t}) \\ M_{7,t+2}^o &= \tau_7 M_{6,t+1}^o + (\mu_{7,t+2} + v_{7,t+2} - \tau_7 v_{6,t+1}) \\ M_{8,t+3}^o &= \tau_8 M_{7,t+2}^o + (\mu_{8,t+3} + v_{8,t+3} - \tau_8 v_{7,t+2}). \end{aligned}$$

It is clear from (2) and (3) that the explanatory variables are correlated with the (composite) disturbance terms of the estimable equations; unbiased and consistent least squares estimates of τ_6 , τ_7 , and τ_8 will not be forthcoming.

This problem can be mitigated by replacing the explanatory variables with closely correlated instrumental variables, so long as the instruments are not closely correlated with the error terms. A suitable candidate to replace $M_{a,t}^o$ is a common spawner-recruit function adjusted for cumulative growth and mortality, which allows (3) to be rewritten as (4):

$$(4) \quad \begin{aligned} M_{5,t}^o &= f(F_{t-6}^o, M_{t-6}^o; \beta) + (\mu_{5,t} + v_{5,t}) \\ M_{6,t+1}^o &= \tau_6 f(F_{t-6}^o, M_{t-6}^o; \beta) + (\mu_{6,t+1} + v_{6,t+1} + \tau_6 \mu_{5,t}) \\ M_{7,t+1}^o &= (\tau_7 \tau_6) f(F_{t-6}^o, M_{t-6}^o; \beta) \\ &\quad + (\mu_{7,t+2} + v_{7,t+2} + \tau_7 \mu_{6,t+1} + \tau_7 \tau_6 \mu_{5,t}) \\ M_{8,t+3}^o &= (\tau_8 \tau_7 \tau_6) f(F_{t-6}^o, M_{t-6}^o; \beta) \\ &\quad + (\mu_{8,t+3} + v_{8,t+3} + \tau_8 \mu_{7,t+2} + \tau_8 \tau_7 \mu_{6,t+1} + \tau_8 \tau_7 \tau_6 \mu_{5,t}). \end{aligned}$$

Equation (4) illustrates two facets of commonality among individual age-classes within a cohort that requires a "system-of-equations" perspective. First, male 5-yr-old crab in year t , male 6-yr-olds in year $t+1$, male 7-yr-olds in year $t+2$, and male 8-yr-olds in year $t+3$ are all spawned by the same parental stock. The practical implication of this commonality is that each member of the cohort shares a common spawner-recruit trajectory from year $t-6$ (the time period in which the cohort was spawned) to year t (the time period in which the cohort was 5 yr old). This is represented in (4) by the identical spawner-recruit function $f(F_{t-6}^o, M_{t-6}^o; \beta)$ shared among all age-class equations. Second, age-class advancement requires only an accumulation of age-specific survivability/growth effects. This is represented in (4) by the age-specific survivability/growth coefficients, τ_a , which are shared across equations and multiplied together as age-class advances to represent the cumulative effect of growth and mortality.

An alternative representation of the system in (4) that expresses the observations on dependent variables in identical time periods is shown in (5):

$$(5) \quad \begin{aligned} M_{5,t}^o &= f(F_{t-6}^o, M_{t-6}^o; \beta) + (\mu_{5,t} + v_{5,t}) \\ M_{6,t}^o &= \tau_6 f(F_{t-7}^o, M_{t-7}^o; \beta) + (\mu_{6,t} + v_{6,t} + \tau_6 \mu_{5,t-1}) \\ M_{7,t}^o &= (\tau_7 \tau_6) f(F_{t-8}^o, M_{t-8}^o; \beta) \\ &\quad + (\mu_{7,t} + v_{7,t} + \tau_7 \mu_{6,t-1} + \tau_7 \tau_6 \mu_{5,t-2}) \\ M_{8,t}^o &= (\tau_8 \tau_7 \tau_6) f(F_{t-9}^o, M_{t-9}^o; \beta) + (\mu_{8,t} + v_{8,t} \\ &\quad + \tau_8 \mu_{7,t-1} + \tau_8 \tau_7 \mu_{6,t-2} + \tau_8 \tau_7 \tau_6 \mu_{5,t-3}). \end{aligned}$$

Assuming the process and measurement errors have zero means and finite variances, nonlinear least squares yields consistent and asymptotically unbiased parameter estimates. (For the sake of simplifying notation, the superscript o , which indicates observed, is dropped from the remainder of the paper.)

This system-of-equations approach to age-class estimation raises the issue of estimation efficiency gains that can be achieved by accounting for the contemporaneous correlation of the systemwide error structure. Survey errors, size-at-age errors, and unmeasured environmental perturbations are likely to be systematically related across equations in any given time period. That is, disturbances across equations may exhibit some degree of common movement and, hence, correlation. Zellner (1962) first recognized the gains in efficiency that can be achieved by incorporating information regarding contemporaneous correlation of the error terms into the estimation process. He suggested applying generalized least squares jointly to the system-of-equations. This approach, known as Seemingly Unrelated Regression (SUR), simultaneously estimates all parameters in the system-of-equations, rather than equation by equation. Additional detail on the SUR process is described in the Appendix.

In cases where sample sizes are large, even higher gains in estimation efficiency might be obtained by incorporating information regarding autocorrelation among the error terms of the various age-class equations within a cohort (recall the composite error terms of (4)). However, for small sample sizes often found in the analyses of commercial fisheries, the use of these more complicated estimation procedures is not recommended. The more complicated error covariance structure cannot, in general, be accurately estimated, and the theoretical gains in estimation efficiency are not obtained in practice (Judge et al. 1985, p. 496–497).

Complete specification of adult male stocks requires estimation of 9- to 14-yr-old males. These six age-classes are modeled as a single, aggregate age-class, $M_{9-14,t}$. There are three principal reasons for adopting this aggregate age-class approach for male 9- to 14-yr-olds. First, the age-class is managed as a composite class of legal crab; management does not, nor is it expected to, discriminate among these larger size categories. Second, individual age/size-class models similar to $M_{5,t}$ to $M_{8,t}$ would exhaust too many degrees of freedom. The lagging structure of parental stock within a spawner-recruit function results in a loss of one additional observation for each successive age-class. Specification of a 14-yr-old equation would require a 15-yr lag between parental stock and recruits. Third, isolating the transition from 8- to 9-yr-olds would require a tenuous assumption concerning the age/size distribution of harvest, for which there are no data. Accordingly, $M_{9-14,t}$ is specified in (6) as a simple function of unharvested male 8- to 14-yr-old crab in the previous period ($M_{8-14,t-1} - \text{Harvest}_{t-1}$), adjusted by a composite survivability/growth coefficient (z_{9-14}):

$$(6) \quad M_{9-14,t} = (M_{8-14,t-1} - \text{Harvest}_{t-1}) \exp(z_{9-14}) + \mu_{9-14,t}.$$

The $M_{9-14,t}$ equation (6) does not share parameters with other age-class equations. However, contemporaneous correlation of errors links this equation with the system specified in (4) or (5). Accordingly, the aggregate $M_{9-14,t}$ equation should be included in the SUR estimation process.

Conceptual Framework Benefits

In general, the systems approach to modeling age-structured fish populations offers a variety of conceptual advantages. For one, it retains the aggregation benefits of a spawner-recruit function. The shared spawner-recruit function anchors age-class population estimates to observations on the aggregate

parental stock, rather than to inherently less accurate observations on a single prerecruit age-class. Aggregation lessens the effect of size/age errors in explanatory variables. The accuracy of the aggregate observed parental stock of 5- to 14-yr-olds is not affected if, for example, a 7-yr-old crab is mistakenly classified as an 8- or 6-yr-old. Aggregation can also mitigate other randomly distributed errors in the measurement of the 10 individual age-class populations from which the aggregate 5- to 14-yr-old observation is derived.

A second benefit is that the systems approach incorporates observations across age-classes within a cohort to estimate age-class recruitment. For example, four observations per cohort would be simultaneously considered in (4) or (5). Use of several observations on a cohort lessens the influence of any single observation on the estimation of the spawner-recruit function, or on any of the survivability/growth parameters. In essence, the systems approach reduces the incentive for closely fitting an aberrant observation. It does so by considering the additional errors between the estimated and the observed populations of other members of that cohort.

The cohort framework offers other conceptual advantages. The number of observations available for estimation is greatly increased by pooling cross-sectional and time series data. This benefit is especially important to this research, since there are only 22 yr of trawl survey data. Pooling information across four age-classes effectively quadruples the number of observations. Moreover, the number of equation-specific parameters decreases relative to a single, independent equation approach in which separate spawner-recruit functions, each with its own parameters, are estimated for each age-class. Degrees of freedom are gained with the system-of-equations approach, which allows tighter confidence intervals to be constructed around parameter estimates.

Empirical Estimation

Empirical estimation of the system conceptualized in (5) and (6) hinges on specifying the common spawner-recruit function. A variety of alternative specifications were examined, ranging from the simple Ricker models in which parental stock is aggregated to specifications that attempt to differentiate between the contribution of male and female parental stocks. Decomposition of parental stocks by sex yielded a significantly better overall statistical fit. The common spawner-recruit specification given in (7) provided the best statistical fit:

$$(7) \quad M_{a,t} = \beta_0 F_{t-(a+1)}^{\beta_1} \exp(\beta_2 F_{t-(a+1)} + \beta_3 FM_{t-(a+1)} + \beta_4 M_{t-(a+1)}) \exp(z_a) + \mu_{a,t}^*$$

where $a = 5, 6, 7$, and 8 ; $\exp(z_a) = \prod_{j=5}^a \tau_j$, $\tau_5 = 1.0$; and

$\mu_{a,t}^*$ represents the composite disturbance term. Male recruitment into the 5-yr-old age-class in period t is a function of composite adult female stock ($F_{t-(a+1)}$), adult male stock ($M_{t-(a+1)}$), and the product of adult females and adult males ($FM_{t-(a+1)}$), all lagged to the year of conception ($t-(a+1)$ years). All variables are measured in millions of pounds of biomass. β_0 through β_4 are parameters to be estimated, where β_1 is a flexibility parameter included to accommodate the possibility of increasing as well as decreasing average reproductive productivity.

Attempts to represent either the leading term or the exponential expression in terms of lagged aggregate parental stock

($F_{t-(a+1)} + M_{t-(a+1)}$) failed. This representation performed very poorly in estimating the level of recruits, which points to the importance of the female broodstock. Females can be bred only once a year, while a single male can breed many females. Nevertheless, contribution of adult males to reproductive success cannot be overlooked and is captured in the exponential term. This spawner-recruit functional specification suggests that breeding success, and thus recruitment, may be a function of not only the individual stock of adult males and females, but also the relative sex composition of adult crab.

The system implied by (6) and (7), or equivalently (5) and (6), was modified for empirical analysis in a fundamental way. Differential mortality was hypothesized for the period beginning in 1981. The Alaskan king crab fishery experienced a boom-bust phenomena reminiscent of many commercial fisheries. Statewide harvests more than tripled between 1968 and 1980, culminating in a record catch of 180 million pounds. The Bristol Bay harvest rose from 8.7 million pounds (1 pound = 0.454 kg) in 1968 to a peak of 130 million pounds in 1980. In 1983, the Bristol Bay fishery was closed — statewide harvests plummeted to 17 million pounds by 1985, and remain depressed today. Otto (1986) attributed the collapse, at least in part, to an increase in mortality. Accordingly, a structural break commencing in 1981 was investigated.

Mortality shifts were accommodated in the male 6-, 7-, 8-, and 9- to 14-yr-old equations by shifting the survivability/growth coefficients. In order to reflect a change in mortality affecting juvenile crab, all parameters of the Ricker function were allowed to change. Separate recruitment functions were specified for the period prior to 1981 and following 1980. A variety of less flexible mortality adjustments also were investigated, as well as different terminal periods of increased mortalities.

Results

The model ultimately used to estimate age-class recruitment is presented in (8):

$$\begin{aligned} M_{5,t} &= (\beta_0 + \beta'_0 I_0) F_{t-6}^{\beta_1 + \beta'_1 I_0} \exp((\beta_2 + \beta'_2 I_0) F_{t-6} \\ &\quad + (\beta_3 + \beta'_3 I_0) FM_{t-6} + (\beta_4 + \beta'_4 I_0) M_{t-6}) + \mu_{5,t}^* \\ M_{6,t} &= (\beta_0 + \beta'_0 I_1) F_{t-7}^{\beta_1 + \beta'_1 I_1} \exp((\beta_2 + \beta'_2 I_1) F_{t-7} \\ &\quad + (\beta_3 + \beta'_3 I_1) FM_{t-7} + (\beta_4 + \beta'_4 I_1) M_{t-7}) \\ &\quad \exp(z_6 + z'_6 I_0) + \mu_{6,t}^* \\ (8) \quad M_{7,t} &= (\beta_0 + \beta'_0 I_2) F_{t-8}^{\beta_1 + \beta'_1 I_2} \exp((\beta_2 + \beta'_2 I_2) F_{t-8} \\ &\quad + (\beta_3 + \beta'_3 I_2) FM_{t-8} + (\beta_4 + \beta'_4 I_2) M_{t-8}) \\ &\quad \exp(z_6 + z'_6 I_1 + z_7 + z'_7 I_0) + \mu_{7,t}^* \\ M_{8,t} &= (\beta_0 + \beta'_0 I_3) F_{t-9}^{\beta_1 + \beta'_1 I_3} \exp((\beta_2 + \beta'_2 I_3) F_{t-9} \\ &\quad + (\beta_3 + \beta'_3 I_3) FM_{t-9} + (\beta_4 + \beta'_4 I_3) M_{t-9}) \\ &\quad \exp(z_6 + z'_6 I_2 + z_7 + z'_7 I_1 + z_8 + z'_8 I_0) + \mu_{8,t}^* \\ M_{9-14,t} &= (M_{8-14,t-1} - \text{Harvest}_{t-1}) \exp(z_{9-14} + z'_{9-14} I_4) \\ &\quad + \mu_{9-14,t}^* \end{aligned}$$

The parameters z'_6 , z'_7 , z'_8 , and z'_{9-14} were added to the existing growth/mortality parameters in the 6-, 7-, 8-, and 9- to 14-yr-old equations to accommodate the mortality break. All parameters in the spawner-recruit function were allowed to change by incorporating $\beta'_0, \dots, \beta'_4$. Cross-equation restric-

tions in the form of identical parameters (β_0 through β_4 and β'_0 through β'_4) are induced by the spawner-recruit function being shared across age-classes; individual survivability/growth coefficients (z_a and z'_a) appearing in more than one equation also are restricted to being identical. The I_j are indicator variables equalling 1.0 if $t = 1980 + j$ through 1989, for $j = 0, 1, 2$, and 3, otherwise $I_j = 0.0$. I_4 is an indicator variable equalling 1.0 if $t = 1981$ through 1985, otherwise $I_4 = 0.0$. The time subscript $t = 1977, 1978, \dots, 1989$. The error structure is specified $E(\mu_{ht}^*, \mu_{kt}^*) = \sigma_{hk}$ for $h \neq k$ and $h, k = 5, 6, 7, 8$, and 9–14. All other variables are as previously defined. The SAS Institute Inc. (1984) nonlinear SUR procedure, SYSNLIN, was used for model estimation.

The first year of data used in model estimation is 1977 for two reasons. First, the SAS-SUR procedure requires that the entire system span a common time period. Second, the 9-yr lag defining parental stock of 8-yr-olds established 1977 as the first year for which a complete set of observations on all age-classes is available.

Three observations in the data series are treated as outliers: the 1977 observations on 5- and 6-yr-olds and the 1988 observation on 9- to 14-yr-olds. The 5- and 6-yr-old stock observations appeared suspicious because their values were nearly twice the adjacent 1976 and 1978 observations. Regarding the 9- to 14-yr-old observation, the number of 8- to 14-yr-olds in 1988, net of harvest, represents an upper bound to the abundance of 9- to 14-yr-olds the following year. However, the 1989 NMFS population estimate of 9- to 14-yr-olds exceeds this upper bound by 42%. Outlier tests confirmed the status of these observations as outliers at the 0.05 level of Type I error.² Rather than eliminating the 1977 and 1988 observations by marking them with indicator variables, cubic splines were fit to the respective age-class data with the outlying observations removed. Observed biomass values of 80.71, 71.82, and 29.05 were replaced in the data set with 50.25, 51.48, and 56.02 for male 5-, 6-, and 9- to 14-yr-olds, respectively, as indicated by the respective cubic splines.

Model estimates are presented in Table 1 — the checks indicate age-classes that share the corresponding parameter estimates. The t -ratios indicate whether a particular parameter is significantly different from 0.0, except for β_0 and β_1 which were tested for a significant difference from 1.0.

All parameters, with the exception of β'_0 , z_6 , z'_7 , and z_{9-14} are significant at the 0.02 level. The marginal Type I error probabilities at which the hypotheses β'_0 , z_6 , and z'_7 equal zero would be rejected are 0.35, 0.07, and 0.25, respectively. The parameters are, nonetheless, retained at their estimated values to prevent the possibility of committing a Type II error from omitting relevant variables, which would result in model bias. Interest in subsequent use of the model estimates for forecasting crab stocks suggests that model bias be avoided. The survivability/growth coefficient z_{9-14} is not statistically different from zero; a zero value indicates that growth exactly offsets mortality. The coefficient is retained at its estimated value rather than imposing the restriction that growth exactly offsets mortality.

All parameter signs conform to a priori expectations. Goodness-of-fit statistics indicate that the predictive accuracy of each age-class is quite good. R^2 increases as age-class advances, ranging from 0.76 for the 5-yr-old equation to 0.99 for the 9-

TABLE 1. SUR dynamic recruitment model with shared parameters (indicated by \checkmark) for 5-, 6-, 7-, 8-, and 9- to 14-yr-old male Alaskan red king crab (millions of pounds, 1968–89).

Parameter	Estimate	t -ratio	M_5	M_6	M_7	M_8	M_{9-14}
β_0	0.036	42.96	\checkmark	\checkmark	\checkmark	\checkmark	
β'_0	1.607	0.94	\checkmark	\checkmark	\checkmark	\checkmark	
β_1	3.184	7.30	\checkmark	\checkmark	\checkmark	\checkmark	
β'_1	-2.309	-4.56	\checkmark	\checkmark	\checkmark	\checkmark	
β_2	-0.075	-9.61	\checkmark	\checkmark	\checkmark	\checkmark	
β'_2	0.053	4.09	\checkmark	\checkmark	\checkmark	\checkmark	
β_3	0.138	5.74	\checkmark	\checkmark	\checkmark	\checkmark	
β'_3	-0.102	-3.39	\checkmark	\checkmark	\checkmark	\checkmark	
β_4	-0.016	-7.02	\checkmark	\checkmark	\checkmark	\checkmark	
β'_4	0.014	4.78	\checkmark	\checkmark	\checkmark	\checkmark	
z_6	-0.125	-1.87		\checkmark	\checkmark	\checkmark	
z'_6	-0.274	-3.18		\checkmark	\checkmark	\checkmark	
z_7	0.193	2.83			\checkmark	\checkmark	
z'_7	-0.151	-1.16			\checkmark	\checkmark	
z_8	0.188	3.77				\checkmark	
z'_8	-0.429	-3.42				\checkmark	
z_{9-14}	0.010	0.50					\checkmark
z'_{9-14}	-0.707	-5.81					\checkmark

Goodness-of-fit statistics

R^2	0.759	0.891	0.972	0.964	0.988
U	0.254	0.175	0.107	0.130	0.078

NOTE: t -statistics on β_0 and β_1 test the null hypothesis that the coefficient = 1.0. U is the Theil inequality coefficient (see text footnote 3 for details). Goodness-of-fit statistics were simulated over the estimated model. The 1977 observation of 5- and 6-yr-olds and the 1988 observation of 9- to 14-yr-olds were replaced with cubic spline estimates. System degrees of freedom = 47.

to 14-yr-old equation. Theil inequality coefficients range from 0.25 for the 5-yr-old equation to 0.08 for the 9- to 14-yr-old equation, suggesting that the model predicts turning points quite well.³ Note, also, that the system degrees of freedom are 47. A single, independent equation approach would yield from 3 to 11 degrees of freedom, and consequently, much wider confidence intervals.

Discussion

Figures 1 and 2 further illustrate the excellent systemwide fit. The predictive accuracy of the systems approach is evident by comparing observed versus predicted estimates on the basis of crab numbers rather than biomass, at least for the single age-class models (Fig. 1). This avoids any confusion due to partially compensatory growth and mortality, as age-class advances. The aggregate 9- to 14-yr-old results (Fig. 2) are illustrated in biomass because of an inability to disaggregate this composite age-class.

Figure 1 brings attention to the principal strength of this type of cohort analysis. An independent equation approach imposes a single, and possibly misguided, standard of predictive accuracy — closeness of fit relative to only those individual observations on the particular age-class. Such a singular criterion is

²The observations in question were marked with indicator variables; t -tests indicated that the parameters were significantly different from 0.0 at the 0.05 level.

³The U -inequality ($0 \leq U \leq 1$) was developed by Theil (1961) to evaluate the turning point accuracy of simulation forecasts. A coefficient of zero is associated with a perfect forecast whereas a coefficient of 1.0 implies that the model is no better than a status quo forecast of zero change in the observed value. Pindyck and Rubinfeld (1981) provided a good discussion of the various Theil forecast error statistics.

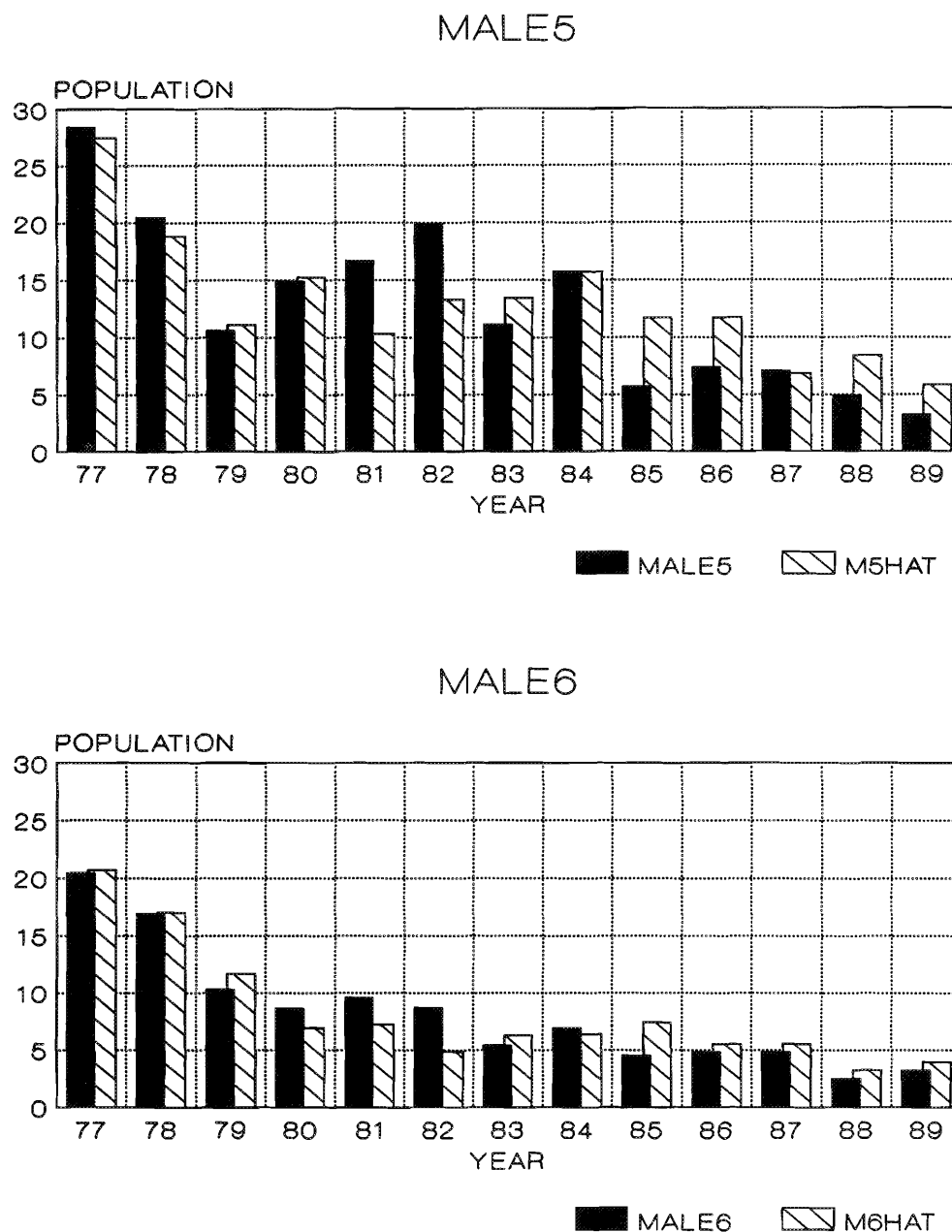


FIG. 1. SUR predicted versus observed population estimates (millions) of 5- to 8-yr-old males. (Fig. 1 concluded next page)

inherently flawed when the underlying trawl survey data are known to contain errors.

The systems approach to stock estimation imposes a second criterion on parameter estimation and model performance. Closeness of fit relative to an individual observation within any age-class is tempered by the influence of that estimate on prior and/or subsequent age-classes. For example, consider Fig. 3. The estimate of the 1982 population of 5-yr-olds underestimates the observed population of 6.4 million crab. However, examination of the cohort reveals that the 1983–85 estimates of male 6-, 7-, and 8-yr-olds, respectively, are close to the observed populations. Comparing survey data observations suggests an extremely high, probably unrealistic, mortality rate of 73% affecting male 5-yr-olds in 1982. A strength of the cohort framework is that it smooths these types of extremes. Consistency across a cohort may necessitate substantial devia-

tions between estimated and observed population for a single age-class.

The systems approach avoids the aforementioned problem of increasing cohort population over time provided that mortality rates are positive. Although mortality rates are not estimated directly, they are recoverable from the growth/survivability parameters z_a . NMFS estimates of weight by age, $W_{t,a}$, can be used to convert the z_a to an explicit mortality estimate, as shown in (9)⁴:

$$(9) \quad m_{a,t} = 1 - \exp(z_a + z'_a I_0)(W_{t,a}/W_{t,a+1}), \quad a = 6, 7, 8$$

where $m_{a,t}$ is the mortality rate of age a crab in time t ; all other variables are as previously defined. The term $(W_{t,a}/W_{t,a+1})$ can-

⁴NMFS estimates that male 5-, 6-, 7-, and 8-yr-old crab weigh 1.77, 2.52, 3.31, and 4.27 pounds (0.81, 1.14, 1.50, and 1.94 kg), respectively.

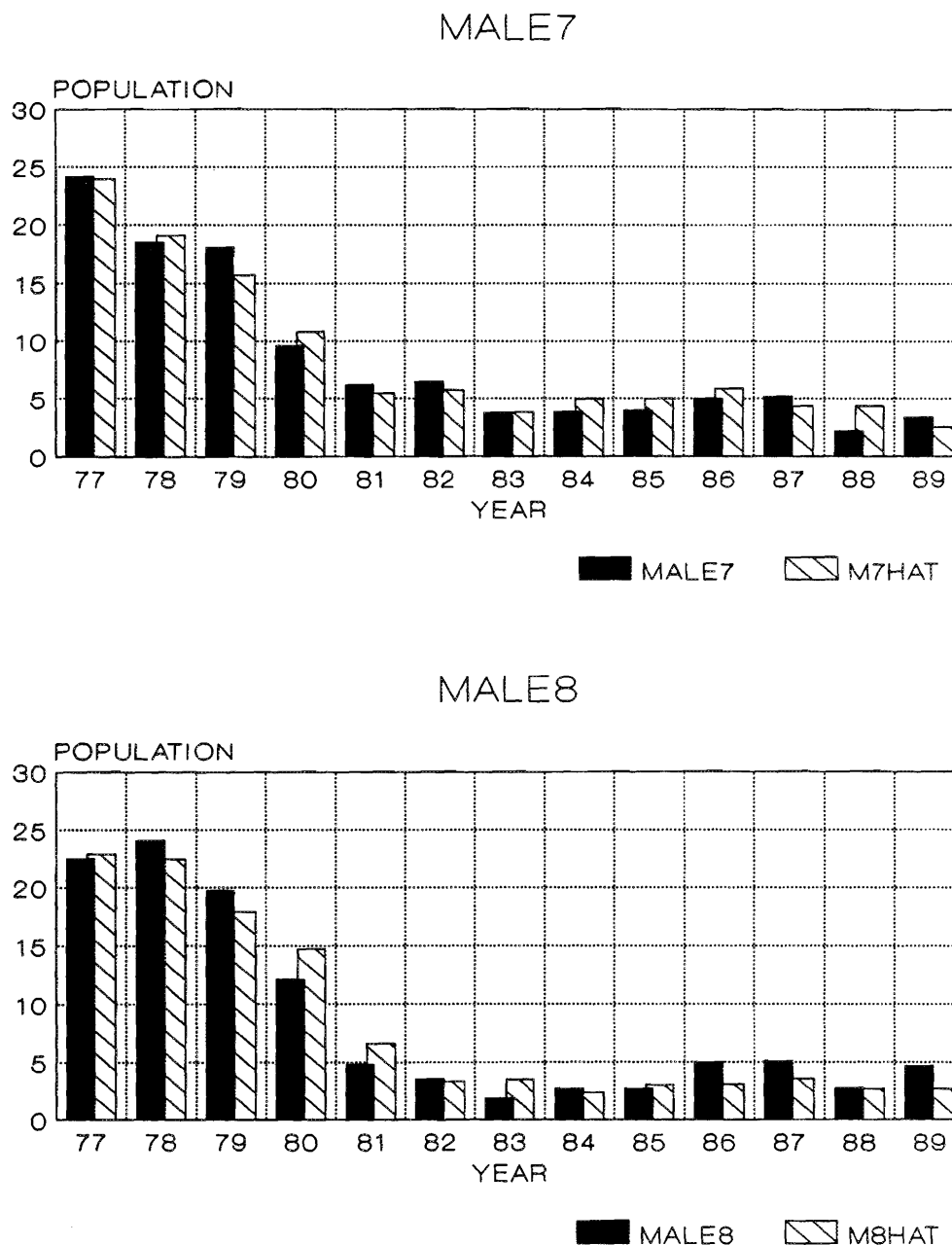


FIG. 1. (Concluded)

cels the implicit growth component in the growth/survivability rate.

Average annual natural mortality rates by age are provided in Table 2 for the time periods 1977–80 and 1981–89. Comparison of the rates for the two time periods reveals a pronounced structural shift in mortality beginning in 1981. The 5-yr-old mortality rate increases from 38.0 to 52.9%, the 6-yr-old rate increases from 7.6 to 20.6%, and the 7-yr-old rate increases from 6.5 to 39.1%.

While the aggregate survivability/growth coefficient for male 9- to 14-yr-olds cannot be converted to a pure mortality estimate, a considerable increase in mortality beginning in 1981 can be inferred. The survivability/growth rate for this age-class decreased from 101% (growth slightly exceeding mortality) to 49.9% (mortality twice the rate of growth).

An increase in mortality played a substantial role in the decline of the king crab stocks in the early 1980's. However, mortality for the aggregate 9- to 14-yr-old age-class appears to have returned to "normal" by 1985. In contrast, the results suggest that younger age-classes continue to suffer high mortality rates. Why a structural shift in mortality for the two groups would span different time periods is unclear. The results, however, are consistent with the observed trawl survey data and fishery performance, as reflected by harvest and weight per unit effort.

Conclusion

The framework presented here takes a systems approach to population estimation based on cohort analysis. The systems approach estimates a single, shared spawner–recruit function

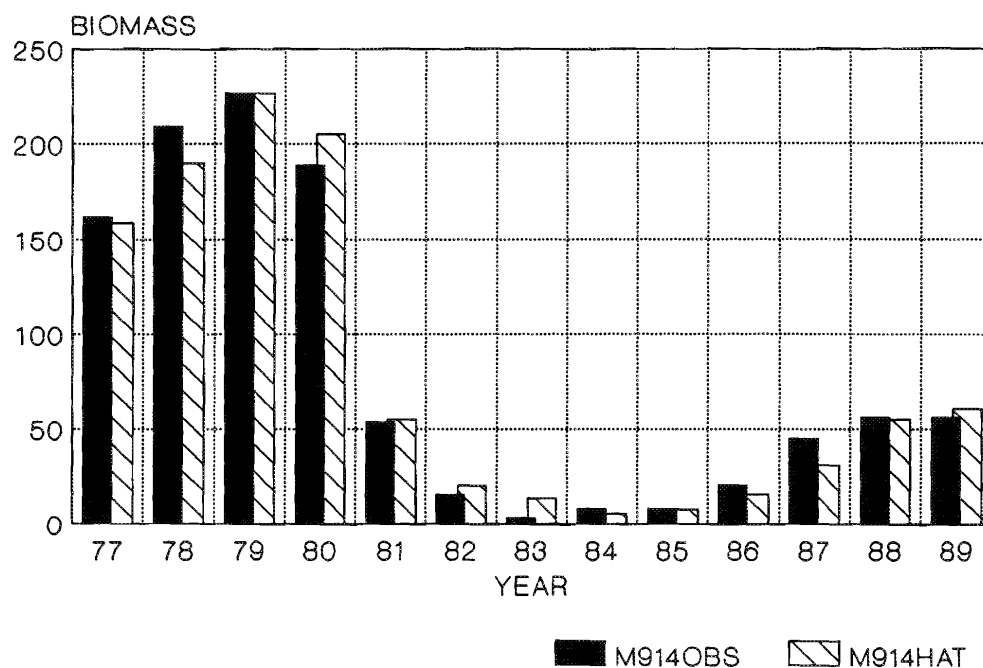


FIG. 2. SUR predicted versus observed biomass estimates (millions of pounds) of 9- to 14-yr-old males.

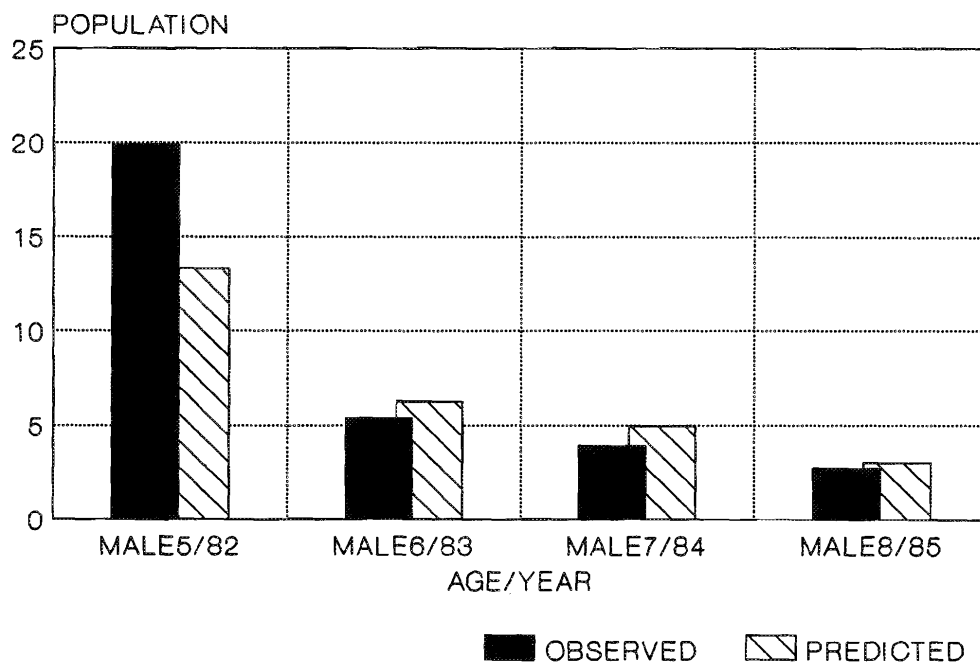


FIG. 3. SUR predicted versus observed population estimates (millions) of the 1982 cohort.

TABLE 2. Estimated age-specific male annual mortality rates by time period.

Age (yr)	Mortality rate	
	1977-80	1981-89
5-6	0.380	0.529
6-7	0.076	0.206
7-8	0.065	0.391

which explicitly recognizes that all members of a cohort derive from a common parental stock. Thus, all observations on a cohort are used simultaneously in estimating the spawner-recruit function. This approach contrasts with the traditional Ricker framework in which individual age-class populations are modeled as unique, independent recruitment functions.

The systems approach offers a variety of operational features which mitigate problems endemic to population modeling of fisheries, e.g. brief data series, inconsistent or errant survey observations on size/age-structured populations, and difficulties in measuring mortalities. The brief data series available for

the Bristol Bay king crab fishery, 1968–89, coupled with lengthy time lags required by the spawner–recruit framework, provide few observations for model estimation. However, the systems approach simultaneously utilizes all information on a cohort by pooling cross-sectional and time series data, thereby greatly increasing the number of effective observations. This permits more detailed, parameter-intensive specifications while mitigating the usual associated degrees of freedom problems, and it can lead to gains in parameter estimation efficiency.

The systems approach advances the findings of Ludwig and Walters (1989) by retaining fidelity with the underlying biomass dynamics while addressing the errors in variables problem endemic to much fisheries data. It does so by blending features of a spawner–recruit model with that of a delay-difference model. The spawner–recruit component subsequently serves as an instrumental variable which is estimated based on several successive observations of a cohort, as opposed to a single observation per year. This feature is particularly advantageous in that the NMFS trawl survey data contain a variety of inconsistencies due to a combination of survey error and problems inherent in using carapace size to estimate crab age. Instances of extremely high, low, and even negative mortalities (cohort population increasing over time) are present in the data series. By pooling all observations on a cohort, the influence of any single observation on the estimated spawner–recruit function is reduced. This implicit smoothing feature acts to produce biologically consistent age-class estimates which could not be assured with either an independent equation approach to spawner–recruit function estimation or with a delay-difference approach. The SUR estimation method also allows one to address the problem of contemporaneous correlation among age-class errors.

Acknowledgements

This research was funded in part by the National Marine Fisheries Service, Washington Sea Grant, and the College of Agriculture and Home Economics at Washington State University.

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Appendix

The SUR process developed by Zellner (1962) can be illustrated with a generalized regression equation. Suppose the i th equation in a system of m equations is

$$y_i = X_i \beta_i + \mu_i, \quad i = 1, \dots, m$$

where y_i is a $n \times 1$ vector, X_i is a $n \times k$ matrix, β_i is a $k \times 1$ vector of parameters, and μ_i is a $n \times 1$ disturbance term vector. The system of m equations may be written as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} X_1 & & \\ & X_2 & \\ & & \ddots \\ & & & X_m \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{bmatrix} + \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_m \end{bmatrix}$$

or, in matrix notation,

$$y = X\beta + \mu$$

where y is a $(mn) \times 1$ vector, X is a $(mn) \times (km)$ matrix, β is a $(km) \times 1$ vector, and μ is a $(mn) \times 1$ vector.

Letting μ_{it} be the error term for the i th equation in the t th time period, the existence of contemporaneous error correlation, but no autocorrelation, implies that $E[\mu_{it}\mu_{js}] = \sigma_{ij}$ if $t = s$, but 0 if $t \neq s$. The covariance matrix for any pair of equations is $E[\mu_i\mu_j'] = \sigma_{ij}I_n$, where I_n is an $n \times n$ identity matrix. The covariance matrix for the entire system-of-equations is

$$\boldsymbol{\theta} = E[\boldsymbol{\mu}\boldsymbol{\mu}'] = \begin{bmatrix} \sigma_{11}I_n & \sigma_{12}I_n & \cdot & \cdot & \cdot & \sigma_{1m}I_n \\ \sigma_{21}I_n & \sigma_{22}I_n & \cdot & \cdot & \cdot & \sigma_{2m}I_n \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \sigma_{m1}I_n & \sigma_{m2}I_n & \cdot & \cdot & \cdot & \sigma_{mm}I_n \end{bmatrix} = \boldsymbol{\Sigma} \otimes I_n.$$

The symbol \otimes denotes Kronecker multiplication of matrices, and

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdot & \cdot & \cdot & \sigma_{1m} \\ \sigma_{21} & \sigma_{22} & \cdot & \cdot & \cdot & \sigma_{2m} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \sigma_{m1} & \sigma_{m2} & \cdot & \cdot & \cdot & \sigma_{mm} \end{bmatrix}$$

where the matrix $\boldsymbol{\Sigma}$ is positive definite and symmetric so that $\sigma_{ij} = \sigma_{ji}$. The generalized least squares estimate of $\boldsymbol{\beta}$, \mathbf{b} , is

$$\mathbf{b} = (\mathbf{X}'\boldsymbol{\theta}^{-1}\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\theta}^{-1}\mathbf{y}.$$

The problem encountered in obtaining parameter estimates from the previous equation is that $\boldsymbol{\Sigma}$ is unknown. Zellner suggested that ordinary least squares be applied to each equation to obtain the least squares residuals. These residuals are then used to obtain an estimate of $\boldsymbol{\Sigma}$, so that $\hat{\boldsymbol{\Sigma}}$ replaces $\boldsymbol{\Sigma}$ and $\hat{\boldsymbol{\theta}}$ replaces $\boldsymbol{\theta}$ in the preceding equation. In the case where y_i is a nonlinear function of $\boldsymbol{\beta}_i$, i.e. $y_i = f_i(X_i; \boldsymbol{\beta}_i) + \mu_i$, for $i = 1, \dots, m$, iterative numerical techniques are used to find the estimates of $\boldsymbol{\beta}_i$ that minimize $\hat{\boldsymbol{\mu}}'(\hat{\boldsymbol{\Sigma}} \otimes I_n)^{-1}\hat{\boldsymbol{\mu}}$, where $\hat{\boldsymbol{\Sigma}}$ is estimated from estimated residuals based on nonlinear least squares estimates of the equations in the system.