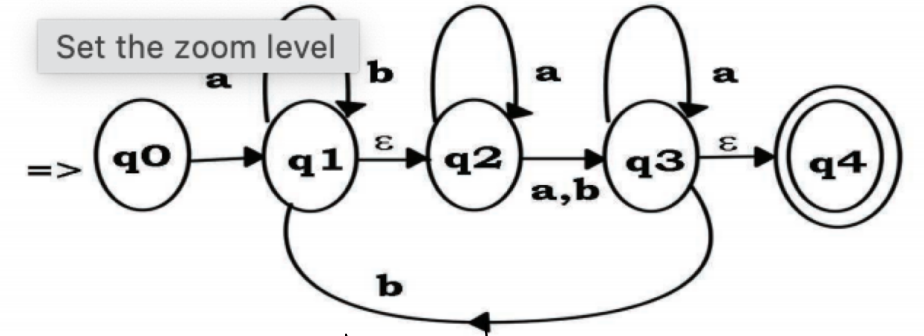
1. [10 points] Convert the given ε-NFA to DFA.



Closure

q_0: $\{q_0\}$

q_1: $\{q_1, q_2\}$

q_2: $\{q_2\}$

q_3: $\{q_3, q_4\}$

q_4: $\{q_4\}$

	ε^*	a	ε^*
q_0	q_0	q_1	$\{q_1, q_2\}$
q_1	q_1 q_2	$\emptyset$ q_2 q_3	$\emptyset$ q_2 $\{q_3, q_4\}$
q_2	q_2	q_2 q_3	q_2 $\{q_3, q_4\}$
q_3	q_3 q_4	q_3 $\emptyset$	$\{q_3, q_4\}$ $\emptyset$
q_4	q_4	$\emptyset$	$\emptyset$

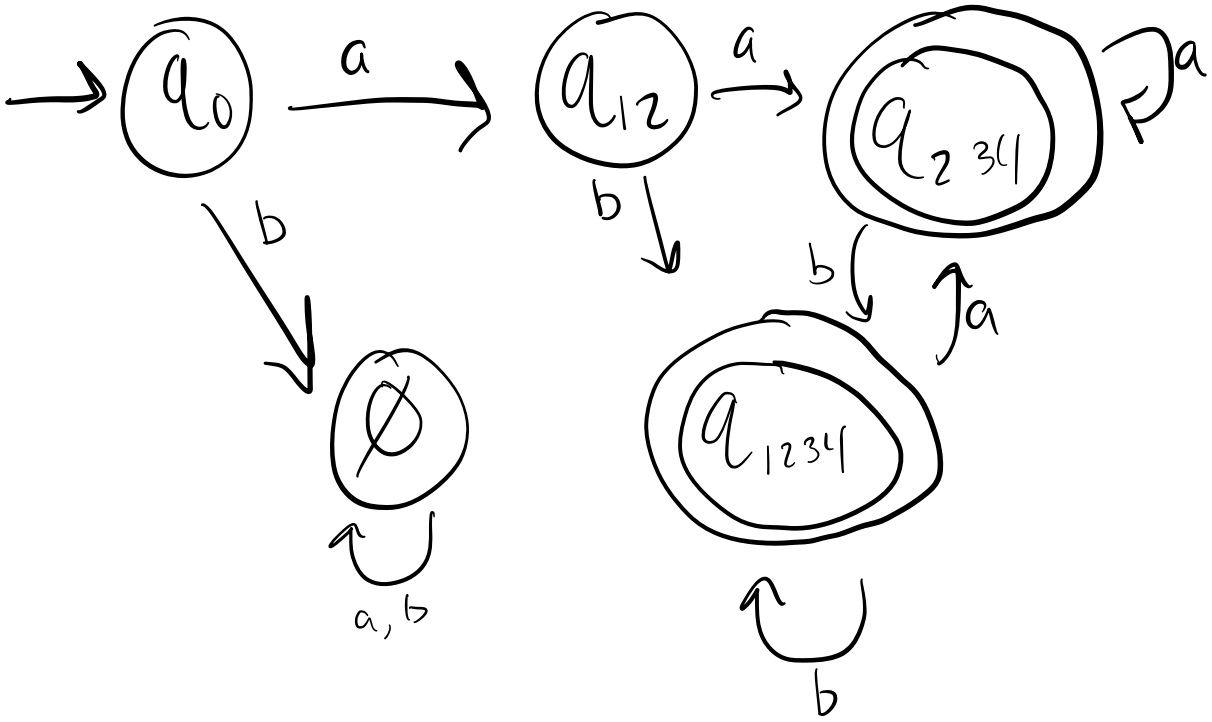
	ε^*	b	ε^*
q_0	q_0	$\emptyset$	$\emptyset$
q_1	q_1 q_2	q_1 q_3	$\{q_1, q_2\}$ $\{q_3, q_4\}$
q_2	q_2	q_3	$\{q_3, q_4\}$
q_3	q_3 q_4	q_1 $\emptyset$	$\{q_1, q_2\}$ $\emptyset$
q_4	q_4	$\emptyset$	$\emptyset$

NFA

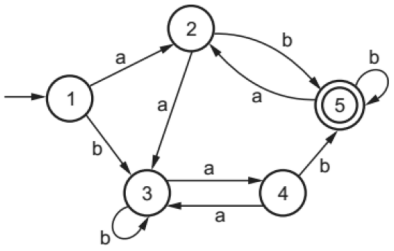
	a	b
→ q_0	$\{q_1, q_2\}$	$\emptyset$
q_1	$\{q_2, q_3, q_4\}$	$\{q_1, q_2, q_3, q_4\}$
q_2	$\{q_2, q_3, q_4\}$	$\{q_3, q_4\}$
q_3	$\{q_3, q_4\}$	$\{q_1, q_2\}$
★ q_4	$\emptyset$	$\emptyset$

DFA

	a	b
→ q_0	q_{12}	$\emptyset$
q_{12}	q_{234}	q_{1234}
★ q_{234}	q_{234}	q_{1234}
★ q_{1234}	q_{234}	q_{1234}
~~q_1~~	~~q_{234}~~	~~q_{1234}~~
~~q_2~~	~~q_{234}~~	~~q_{34}~~
~~q_{34}~~	~~q_{34}~~	~~q_{12}~~
~~q_3~~	~~q_{34}~~	~~q_{12}~~
~~q_4~~	~~$\emptyset$~~	~~$\emptyset$~~
$\emptyset$	$\emptyset$	$\emptyset$



2. [10 points] Minimize the following DFA using Myhill Nerode theorem and verify using state elimination method.



	1	2	3	4	5
1	-	-	-	-	-
2	X	-	-	-	-
3	·	X	-	-	-
4	X	·	X	-	-
5	X	X	X	X	-

$\delta(1,2) = \delta(1,a) = 2 \quad \delta(1,b) = 3$
$\delta(2,a) = 3 \quad \delta(2,b) = 5$

$\delta(1,4) = \delta(1,a) = 2 \quad \delta(1,b) = 3$
$\delta(4,a) = 3 \quad \delta(4,b) = 5$

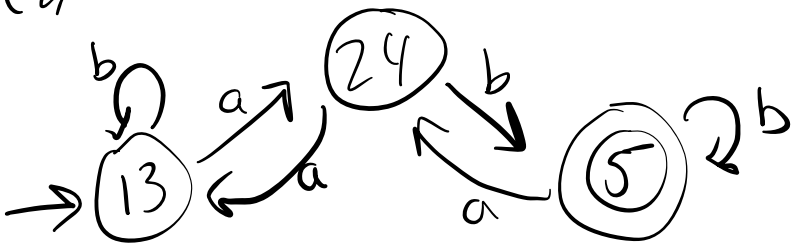
$\delta(2,4) = \delta(2,a) = 3 \quad \delta(2,b) = 5$
$\delta(4,a) = 3 \quad \delta(4,b) = 5$

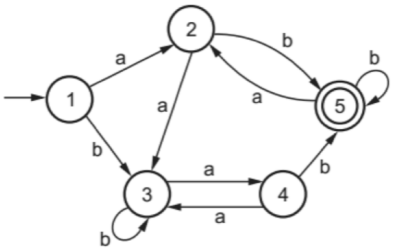
$\delta(1,3) = \delta(1,a) = 2 \quad \delta(1,b) = 3$
$\delta(3,a) = 4 \quad \delta(3,b) = 3$

$\delta(2,3) = \delta(2,a) = 3 \quad \delta(2,b) = 5$
$\delta(3,a) = 4 \quad \delta(3,b) = 3$

$\delta(3,4) = \delta(3,a) = 4 \quad \delta(3,5) = 3$
$\delta(4,a) = 3 \quad \delta(4,b) = 5$

(2,4) (1,3)

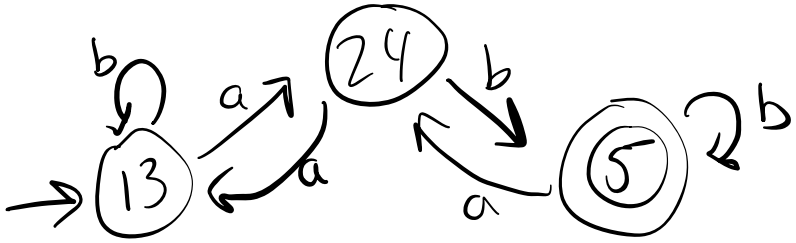




0 eqv.
{1,2,3,4} {5}

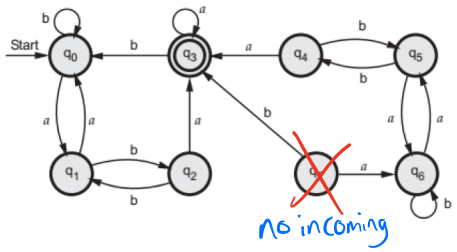
1 eqv.
{1,3} {5}
{2,4}

2 eqv.
{1,3} {5}
{2,4}



3. [5 points] Construct a minimum state automaton for the following transition diagram,

	a	b
→q_6	q_1	q_0
q_1	q_0	q_2
q_2	q_3	q_1
★q_3	q_3	q_0
q_4	q_3	q_5
q_5	q_6	q_4
q_6	q_5	q_6
q_7	q_6	q_3



0 eqv.

$\{q_0, q_1, q_2, q_4, q_5, q_6, q_7\}$ $\{q_3\}$

1 eqv.

$\{q_0, q_1, q_5, q_6\}$

$\{q_2, q_4\}$

$\{q_3\}$

2 eqv.

$\{q_0, q_6\}$

$\{q_2, q_4\}$

$\{q_3\}$

$\{q_1, q_5\}$

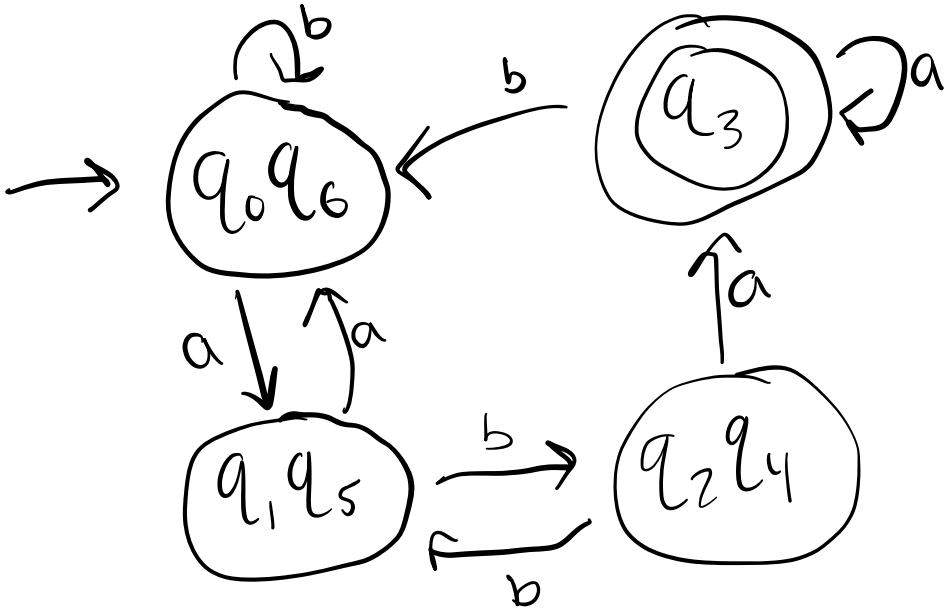
3 eqv.

$\{q_0, q_6\}$

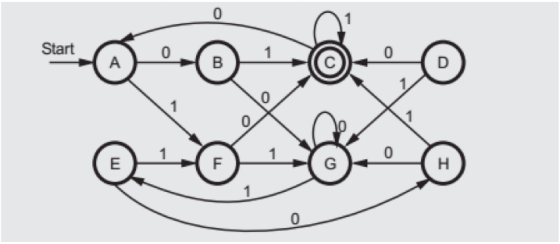
$\{q_2, q_4\}$

$\{q_3\}$

$\{q_1, q_5\}$


b
b
a
q_0q_6
q_3
a
a
a
b
q_1q_5
q_2q_4
b

4. [10 points] Minimize the following DFA using Myhill Nerode theorem,



$\delta(A,B)$: $\delta(A,0)=B$ $\delta(A,1)=F$
$\delta(B,0)=G$ $\delta(B,1)=C$

	A	B	C	D	E	F	G	H
A	-	-	-	-	-	-	-	-
B	X	-	-	-	-	-	-	-
C	X	X	-	-	-	-	-	-
D	X	X	X	-	-	-	-	-
E		X	X	X	-	-	-	-
F	X	X	X		X	-	-	-
G	X	X	X	X	X	X	-	-
H	X		X	X	X	X	X	-

$\delta(A,D)=\delta(A,0)=B$ $\delta(A,1)=F$
$\delta(D,0)=C$ $\delta(D,1)=G$

$\delta(A,E)=\delta(A,0)=B$ $\delta(A,1)=F$
$\delta(E,0)=H$ $\delta(E,1)=F$

$\delta(A,F)=\delta(A,0)=B$ $\delta(A,1)=F$
$\delta(F,0)=C$ $\delta(F,1)=G$

$\delta(A,G) = \delta(A,0) = B \quad \delta(A,1) = F$

$\delta(G,0) = G \quad \delta(G,1) = E$

$\delta(A,H) = \delta(A,0) = B \quad \delta(A,1) = F$

$\delta(H,0) = G \quad \delta(H,1) = C$

$\delta(B,D) = \delta(B,0) = G \quad \delta(B,1) = C$

$\delta(D,0) = C \quad \delta(D,1) = G$

$\delta(B,E) = \delta(B,0) = G \quad \delta(B,1) = C$

$\delta(E,0) = H \quad \delta(E,1) = F$

$\delta(B,F) = \delta(B,0) = G \quad \delta(B,1) = C$

$\delta(F,0) = C \quad \delta(F,1) = G$

$\delta(B,G) = \delta(B,0) = G \quad \delta(B,1) = C$

$\delta(G,0) = G \quad \delta(G,1) = E$

$\delta(B,H) = \delta(B,0) = G \quad \delta(B,1) = C$

$\delta(H,0) = G \quad \delta(H,1) = C$

$\delta(D,E) = \delta(D,0) = C \quad \delta(D,1) = G$

$\delta(E,0) = H \quad \delta(E,1) = F$

$\delta(D,F) = \delta(D,0) = C \quad \delta(D,1) = G$

$\delta(F,0) = C \quad \delta(E,1) = G$

$\delta(D,G) = \delta(D,0) = C \quad \delta(D,1) = G$

$\delta(G,0) = G \quad \delta(G,1) = E$

$\delta(D,H) = \delta(D,0) = C \quad \delta(D,1) = G$

$\delta(H,0) = G \quad \delta(H,1) = C$

$\delta(E,F) = \delta(E,0) = H \quad \delta(E,1) = F$

$\delta(F,0) = C \quad \delta(F,1) = G$

$\delta(E,G) = \delta(E,0) = H \quad \delta(E,1) = F$

$\delta(G,0) = G \quad \delta(G,1) = E$

$\delta(E,H) = \delta(E,0) = H \quad \delta(E,1) = F$

$\delta(H,0) = G \quad \delta(H,1) = C$

$\delta(F,G) = \delta(F,0) = C \quad \delta(F,1) = G$

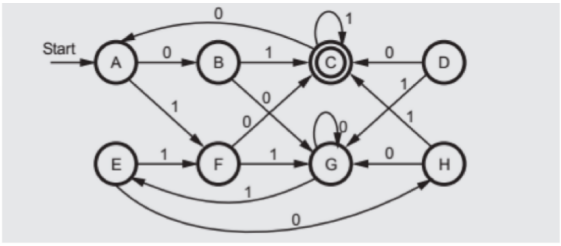
$\delta(G,0) = G \quad \delta(G,1) = E$

$\delta(G,H) = \delta(G,0) = G \quad \delta(G,1) = E$

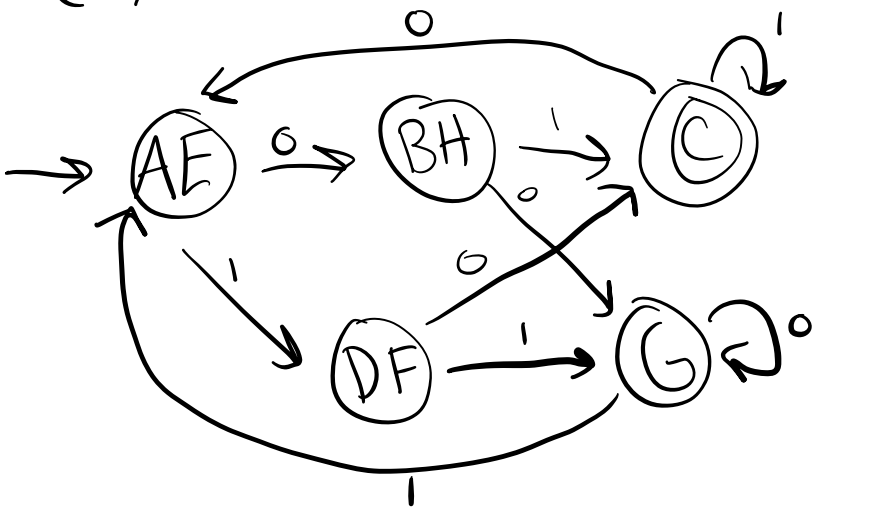
$\delta(H,0) = G \quad \delta(H,1) = C$

$\delta(F,H) = \delta(F,0) = C \quad \delta(F,1) = G$

$\delta(H,0) = G \quad \delta(H,1) = C$

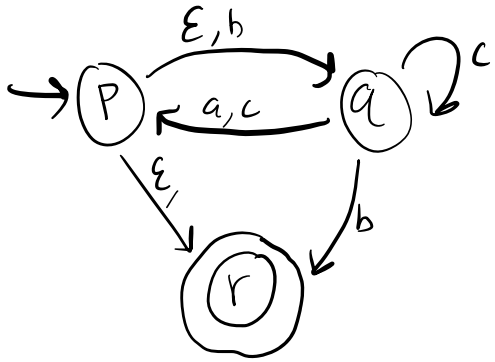


(A,E) (B,H) (D,F)



5. [5 points] Convert the given ε-NFA to DFA.

STATES	INPUTS			
	ε	a	b	c
=>p	{q,r}	{Φ}	{q}	{r}
q	{Φ}	{p}	{r}	{p,q}
*r	Φ	Φ	Φ	Φ



closure

p: {p,q,r}

q: {q}

r: {r}

	ε*	a	ε*
p	p, q, r	∅, p, ∅	∅, {p,q,r}, ∅
q	q	p	{p,q,r}
r	r	∅	∅

	ε*	b	ε*
p	p, q, r	q, r, ∅	q, r, ∅
q	q	r	r
r	r	∅	∅

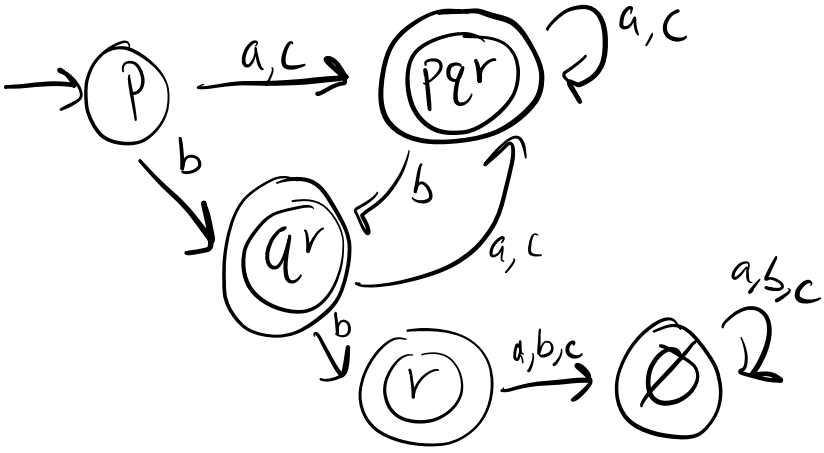
	ε*	c	ε*
p	p, q, r	r, p, q, ∅	r, {p,q,r}, q, ∅
q	q	p, q	{p,q,r}, q
r	r	∅	∅

NFA

	a	b	c
p	{p,q,r}	{q,r}	{p,q,r}
q	{p,q,r}	r	{p,q,r}
r	∅	∅	∅

DFA

	a	b	c
→ p	pqr	qr	pqr
★ pqr	pqr	qr	pqr
★ qr	pqr	r	pqr
~~q~~	~~pqr~~	~~r~~	~~pqr~~
★ r	∅	∅	∅
∅	∅	∅	∅



6. [5 points] Convert the given Λ-NFA to DFA.

	Q	δ (q, Λ)	δ (q, 0)	δ (q, 1)
→	A	{B, D}	{A}	ϕ
	B	ϕ	{C}	{E}
	C	ϕ	ϕ	{B}
	D	ϕ	{E}	{D}
★	E	ϕ	ϕ	ϕ

closure:
A: {A, B, D}
B: {B}
C: {C}
D: {D}
E: {E}

	ε*	0	ε*
A	A B D	A C E	{A, B, D} C E
B	B	C	C
C	C	∅	∅
D	D	E	E
E	E	∅	∅

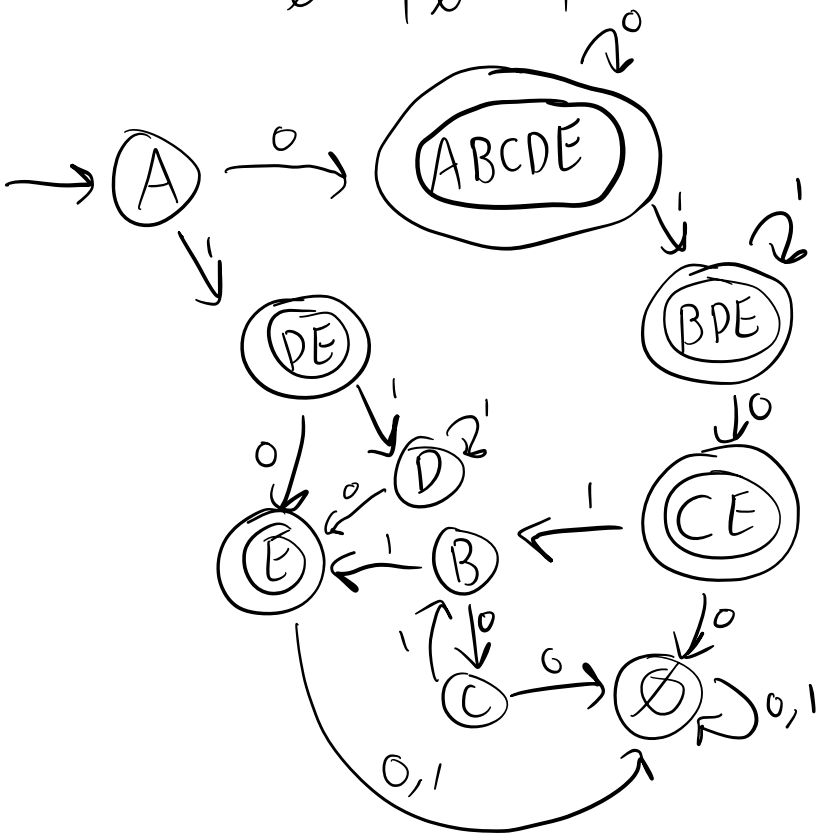
	ε*	1	ε*
A	A B D	E D	∅ E D
B	B	E	E
C	C	B	B
D	D	D	D
E	E	∅	∅

NFA

	0	1
A	{A, B, C, D, E}	{D, E}
B	C	E
C	∅	B
D	E	D
★ E	∅	∅

DFA

	0	1
A	ABCDE	DE
★ ABCDE	ABCDE	BDE
★ DE	E	D
★ BDE	CE	BDE
★ CE	∅	B
B	C	E
C	∅	B
D	E	D
E	∅	∅
∅	∅	∅



7. [5 points] Convert the given NFA to DFA.

State \ Input	0	1
→ p	{p, q}	p
q	r	r
r	s	–
★ s	s	s

DFA

	0	1
→ p	pq	p
pq	pqr	pr
pqr	pqrs	pr
pr	pqs	p
★ pqrs	pqrs	prs
★ pqs	pqrs	prs
★ prs	pqs	ps
★ ps	pqs	ps
~~q~~	~~r~~	~~r~~
~~r~~	~~s~~	~~∅~~
~~s~~	~~s~~	~~s~~
~~∅~~	~~∅~~	~~∅~~