

# A-B-Test---Kristy Morris Project 7-- Udacity

## Metric Choice--Gross Conversion/Retention/Net Conversion

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The metrics I chose are listed below and why--

Gross conversion: That is, The numbers of user-ids who decide to start the free trial and are expected to depend on how the "start free trial" page is carried out --- whether a "5 or more hours per week" is suggested --- this is one question we would like to understand through this A/B test. Therefore, this is a good evaluation metric.

Retention: Along the same lines, as the above, we would like to understand whether carrying out a "5 or more hours per week" suggestion is helpful to increase ratio of user-ids who will make payments over those who finish the free trial, and therefore this metric is good for evaluation.

Net conversion: That is, The result of the previous two evaluation metrics: gross conversion and retention, and it can be considered as a more general goal of the A/B test --- whether carrying out a "5 or more hours per week" suggestion helps increase the ratio of users who make payment over those who see the start free trial page. Therefore, again a good evaluation metric.

## Measuring Variability--

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The analytical standard deviation is computed as  $\text{std} = \sqrt{p * (1-p)/N}$

Gross conversion:  $\text{std} = \sqrt{0.20625*(1-0.20625)/3200} = 0.00715$  (correspond to 3200 clicks & 40000 pageviews). For 5000 pageviews, we have new  $\text{std} = 0.00715 * \sqrt{40000/5000} = 0.0202$

Retention:  $\text{std} = \sqrt{(0.53*(1-0.53)/660) * \sqrt{40000/5000}} = 0.549$

Net conversion:  $\text{std} = \sqrt{0.1093125*(1-0.1093125)/3200} = 0.0055159$  (correspond to 3200 clicks & 40000 pageviews). For 5000 pageviews, we have new  $\text{std} = 0.00715 * \sqrt{40000/5000} = 0.0156$

Gross conversion: 0.0202 Retention: 0.0549 Net conversion: 0.0156

For each metric you selected as a evaluation metrics, indicate whether you think the analytic estimate would be comparable to the the empirical estimate of the variability, if you had time? Expained below--

For both Gross Conversion and Net Conversion using number of cookies as denominator, which is also unit of diversion. Here, the unit of diversion is equal to unit of analysis, which indicate the analytical estimate would be comparable to the emperical variability.

For Retention, the denominator is "Number of users enrolled (complete checkout)" which is not similar as Unit of Diversion. The unit of analysis and the unit of diversion are not the same therefore the analytical and the empirical estimates are different.

## Sizing--

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### Number of Samples vs. Power

I decided not to use Bonferroni correction, because the metrics in the test has high correlation and the Bonferroni correction will be too conservative to use it.

I calculated the number of samples needed for each metric using the online calculator, with  $\alpha = 0.05$ ,  $1 - \beta = 0.2$ . The baseline conversion rate and minimum detectable effect ( $d_{min}$ ) are listed individually below. Also note that the number produced by the online calculator is per branch, and in order to have both control and experiment, we need to double the number of required page views.

Gross conversion = The baseline conversion rate is 0.20625, and  $d_{min}$  is 0.01. The required number of samples calculated from the online calculator is 25,835. Note that this is the number of clicks on "start free trial", and in order to get that number, we need  $25,835 / 0.08 * 2 = 645,875$  page views.

Retention = The baseline retention rate is 0.53, and  $d_{min}$  is 0.01. The required number of samples calculated from the online calculator is 39,115. Note that this is the number of users who finished the 14 day free trial, and in order to get that number, we need  $39,115 / 0.08 / 0.20625 * 2 = 4,741,212$  page views.

Net conversion = The baseline conversion rate is 0.1093125, and  $d_{min}$  is 0.0075. The required number of samples calculated from the online calculator is 27,413. Note that this is the number of clicks on "start free trial", and in order to get that number, we need  $27,413 / 0.08 * 2 = 685,325$  page views.

If we keep the retention rate as a evaluation metric, the number of required pages will be too large (in order to get 4.7 million page views, it takes 117 days of 100% or full site traffic, which is not realistic). Therefore, we decide to drop the retention rate evaluation metric, and use gross conversion and net conversion as evaluation metrics, and the required number of page views (taking the larger one) is 685,325.

## Duration vs. Exposure

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I selected to redirect 50% of the traffic to our experiment, and the length of the experiment is therefore  $685,325 / (40,000 * 0.5) = 34.3$  days (where 40,000 is the baseline number of visitors per day).

The 50% traffic is being redirected to the experiment means that 25% will go to control group and 25% to experiment group, and therefore we risk about a quarter of users seeing an not yet evaluated feature. This is large risk, because those 25% users will see a different view of "start free trial" page

which potentially discourages them to start the free trial (although the intention is to increase the overall net conversion). But this large risk is a uncertain choice in order to keep the length of the experiment in a reasonable amount of time. If we reduce the risk by half (sending 12.5% users to see not-yet-evaluated feature), the length will be doubled, taking more than 2 months, which is a little too long to run in most cases.

## Analysis--

## Sanity Checks

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For counts ("number of cookies" and "number of clicks"), we model the assignment to control and experiment group as a Bernoulli distribution with probability 0.5. Therefore the standard deviation is  $\text{std} = \sqrt{0.5 * 0.5 / (N_1 + N_2)}$ , and the margin of error is  $\text{me} = 1.96 * \text{std}$ . The lower bound will be  $0.5 - \text{me}$  and the higher bound will be  $0.5 + \text{me}$ . The actual observed value is number of assignments to control group divide by the number of total assignments.

Number of cookies

control group total = 345543 experiment group total = 344660 standard deviation =  $\sqrt{0.5 * 0.5 / (345543 + 344660)}$  = 0.0006018 margin of error =  $1.96 * 0.0006018$  = 0.0011796 lower bound =  $0.5 - 0.0011797$  = 0.4988 upper bound =  $0.5 + 0.0011797$  = 0.5012 observed =  $345543 / (345543 + 344660)$  = 0.5006 The observed value is within the bounds, and therefore this invariant metric passed the sanity check.

Number of clicks on "start free trial"

control group total = 28378 experiment group total = 28325 standard deviation =  $\sqrt{0.5 * 0.5 / (28378 + 28325)}$  = 0.0021 margin of error =  $1.96 * 0.0021$  = 0.0041 lower bound =  $0.5 - 0.0041$  = 0.4959 upper bound =  $0.5 + 0.0041$  = 0.5041 observed =  $28378 / (28378 + 28325)$  = 0.5005 The observed value is within the bounds, and therefore this invariant metric passed the sanity check.

Click-through-probability on "start free trial"

For click through probability, we first compute the control value  $p_{\text{cnt}}$ , and then estimate the standard deviation using this value with experiment group's sample size, i.e.  $\text{std} = \sqrt{p_{\text{cnt}} * (1 - p_{\text{cnt}}) / N_{\text{exp}}}$ . The margin of error is 1.96 times of standard deviation.

control value = 0.0821258 standard deviation =  $\sqrt{0.0821258 * (1 - 0.0821258) / 344660}$  = 0.000468 margin of error =  $1.96 * 0.000468$  = 0.00092 lower bound =  $0.0821258 - 0.00092$  = 0.0812 upper bound =  $0.0821258 + 0.00092$  = 0.0830 experiment value = 0.0821824 The observed value (experiment value) is within the bounds, and therefore this invariant metric passed the sanity check.

## Effect Size Tests

Let N denote the number of total samples (denominator) and X denote the number of target samples (numerator), and \_cnt denote controlled group and \_exp the experiment group.

We first computed pooled probability and pooled standard error as

$p_{\text{pooled}} = (X_{\text{cnt}} + X_{\text{exp}}) / (N_{\text{cnt}} + N_{\text{exp}})$   $se_{\text{pooled}} = \sqrt{p_{\text{pooled}} * (1 - p_{\text{pooled}}) * (1/N_{\text{cnt}} + 1/N_{\text{exp}})}$  The probability difference is computed as

$d = X_{\text{exp}} / N_{\text{exp}} - X_{\text{cnt}} / N_{\text{cnt}}$  With these values in hand, the lower bound and upper bound are

$lower = d - se_{\text{pooled}}$   $upper = d + se_{\text{pooled}}$

### Gross conversion

For gross conversion, the total samples (denominator) are the clicks of "start free trial", and the target samples (numerator) are enrolled users.

The calculation is shown below.

$N_{\text{cnt}} = \text{clicks\_controlled} = 17293$ .  $X_{\text{cnt}} = \text{enroll\_controlled} = 3785$ .  $N_{\text{exp}} = \text{clicks\_experiment} = 17260$ .  $X_{\text{exp}} = \text{enroll\_experiment} = 3423$ .

$p_{\text{pooled}} = (X_{\text{cnt}} + X_{\text{exp}}) / (N_{\text{cnt}} + N_{\text{exp}}) = 0.2086$   $se_{\text{pooled}} = \sqrt{p_{\text{pooled}} * (1 - p_{\text{pooled}}) * (1/N_{\text{cnt}} + 1/N_{\text{exp}})} = 0.00437$

$d = X_{\text{exp}} / N_{\text{exp}} - X_{\text{cnt}} / N_{\text{cnt}} = -0.02055$

$lower = d - se_{\text{pooled}} = -0.0291$   $upper = d + se_{\text{pooled}} = -0.0120$

Since the interval does not contain 0, the metric is statistical significant. It does not include  $d_{\text{min}} = 0.01$  or  $-d_{\text{min}} = -0.01$  either, and therefore it is also practical significant.

### Net conversion

For net conversion, the total samples (denominator) are the clicks of "start free trial", and the target samples (numerator) are paid users.

The calculation is shown below.

$N_{\text{cnt}} = \text{clicks\_controlled} = 17293$ .  $X_{\text{cnt}} = \text{pay\_controlled} = 2033$ .  $N_{\text{exp}} = \text{enroll\_experiment} = 17260$ .  $X_{\text{exp}} = \text{pay\_experiment} = 1945$ .

$p_{\text{pooled}} = (X_{\text{cnt}} + X_{\text{exp}}) / (N_{\text{cnt}} + N_{\text{exp}}) = 0.1151$   $se_{\text{pooled}} = \sqrt{p_{\text{pooled}} * (1 - p_{\text{pooled}}) * (1/N_{\text{cnt}} + 1/N_{\text{exp}})} = 0.00343$

$d = X_{\text{exp}} / N_{\text{exp}} - X_{\text{cnt}} / N_{\text{cnt}} = -0.0048$

$lower = d - se_{\text{pooled}} = -0.0116$   $upper = d + se_{\text{pooled}} = 0.0019$

Since the interval contains 0, it is not statistically significant, and consequently not practically significant either.

## Sign Tests

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I used the online calculator to perform the sign tests. For gross conversion, the number of days we see an improvement in experiment group is 4, out of total 23 days of experiment. With probability 0.5 (for sign test), the online calculator calculates a p-value 0.0026, which is smaller than  $\alpha = 0.05$ . Therefore the change is statistically significant.

For net conversion, the number of days we see an improvement in experiment group is 10, out of total 23 days of experiment. With probability 0.5 (for sign test), the online calculator calculates a p-value 0.6776, which is larger than  $\alpha = 0.05$ . Therefore the change is not statistically significant.

## Summary--

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I decided not to use Bonferroni correction, because the metrics in the test have high correlation and the Bonferroni correction will be too conservative to use it.

Both the effective size hypothesis tests and sign tests state that the change will practically significantly reduce the gross conversion, however not affect the net conversion rate in a practically significant way.

## Recommendation--

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Based on the analysis above, I recommend not to adopt the changes of adding "5 or more hour" recommendation to "start free trial" date. The reason is that the A/B test shows that this will not practically significantly increase the net conversion rate. Meaning, it does not increase the number of paid users, which fails the original goal of our experiment/test.

## Follow-Up Experiment--

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I would recommend adding a "accelerated completion program discount option" button on the home page. This would be in addition to the "start free trial" button, not replacing it. This feature will allow users to skip the "free trial" phase, if they desire so, and in exchange they get a tuition discount, if they complete the program within a shortened time period, such as 6 months. This feature will be potentially engage the users who are already determined to take the course, and want to jump right in.

The hypothesis is that by providing this additional option, the number of enrollees will increase, because those who decide to take the course will directly enroll rather than experiencing the free trial, during which they might decide not to enroll for certain reasons. Another hypothesis is that this

feature will bring more revenue to Udacity --- even though the users who enroll directly pay less than others, the increasing number of users will be more significant.

Corresponding to each hypothesis, there are two evaluation metrics: 1. the conversion rate from home page viewers to enrolled users. This will test whether the additional option helps to boost the enrollment. 2. the ratio of revenue over number of home page viewers. This will test for the same unit number of users who viewed the home page, whether the additional option helps to increase the overall revenue.

The initial unit of diversion will be a cookie, because the home page viewers are not necessarily signed in. When users are signed in, user id will be used instead of cookies.

## References--

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Udacity A/B Course Lesson Videos Online Calculator-- <http://www.evanmiller.org/ab-testing/sample-size.html> Bernoulli Distribution [https://en.wikipedia.org/wiki/Bernoulli\\_distribution](https://en.wikipedia.org/wiki/Bernoulli_distribution)