

Formal Language Semantics for Triggered Enable Statecharts with a Run-to-Completion Scheduling

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Abstract. The increased complexity of high-integrity digital systems designs with intricate interactions between numerous components has placed a greater need on ensuring that the design satisfies its intended requirements. This digital assurance can only come about through rigorous mathematical analysis of the design. This manuscript provides a detailed description of the formal language semantics used for modeling and verification of systems. Models are developed using Event-B by building on a formalised semantics that enables the constructions of triggered enable statecharts with a run-to-completion scheduling. Rodin has been used to develop and analyse models using this semantics.

Keywords: Run-To-Completion · Statecharts · Formal Semantics · SCXML · Event-B

1 Introduction

Motivation. Statechart notations, with event-triggered, ‘run-to-completion’ semantics [3,4], provide an intuitive and visual modelling interface with which engineers can design and communicate. SCXML [11] is an example of such a statechart notation. Formal refinement, as in Event-B [1], provides a safe way to abstract important properties and introduce detail in a manageable way. Hence we have proposed the introduction of refinement into SCXML in previous work [7,8,9]. In order to define our proposals for refinement in SCXML more precisely, we first need to formally specify the semantics of SCXML itself. In this paper, we focus on the formalization of SCXML semantics using Event-B. SCXML can be viewed as 2 superimposed behaviours: a) the underlying behaviour of how the statechart is structured and how it changes its active state and b) the run-to-completion schedule that dictates the order in which enabled transitions should be fired. We approach the formalisation by addressing these issues as separate Event-B models and then compose them using the CamilleX extension [6] to obtain the complete formalisation of the SCXML semantics.

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Contribution. Our contributions are as follows.

- We provide a formalization of the run-to-completion semantics of SCXML statecharts in Event-B. Although we refer to SCXML as an example, we believe that it is typical of other similar statechart notations and we have not relied on specific SCXML definitions.
- The work illustrates how Event-B with its automatic proof obligation generators and theorem provers can be used to define formal semantics of notations. Here we explain the Event-B models and outline the more interesting proofs that required manual intervention.
- The work also illustrates how the composition feature of CamilleX can be used to structure such semantics definitions into more manageable sub-issues.

As far as we know, this is the first tool-supported semantics model for SCXML statecharts supporting features such as parallel regions.

Structure. In Section 2, we provide a high-level description of statecharts, and the run-to-completion execution model, as well as, the Event-B modeling method used to formalized the semantics of our triggered statechart modeling language. Section 3 introduces a turnstile running example which will be used in subsequent sections to illustrate the construction and analysis of a system using our formalized semantics. We develop the semantics model of the run-to-completion statecharts in three separate steps: (1) first, we model the semantics the un-triggered statecharts (Section 4); and (2) we then specify the run-to-completion triggering mechanism (Section 5); and (3) we compose the two individual semantics models to construct the triggered statechart semantics (Section 6). A summary of our findings and concluding remarks are discussed in Section 7.

2 Background

2.1 Statecharts

Statecharts have been used for visual representation in the specification and design of complex systems for several decades. Different statechart formalizations exist depending on the features supported and specific characteristics of the modeled systems [3,4]. These diagrams provide a compact representation for modeling hierarchy, concurrency and communication in a systems design. Statecharts were originally developed to address unbounded modeling complexity in other state diagrams.

2.2 Run-to-Completion and SCXML

State Chart eXtensible Markup Language (SCXML) [11] is a general-purpose event-based statemachine language that combines concepts from Call Control eXtensible Markup Language (CCXML) and Harel State Tables. Harel State Tables are included in the Unified Modeling Language (UML). The concrete

syntax for SCXML is based on XML. Hence, SCXML is an XML notation for UML style state-machines extended with an action language that is intended for call control features in voice applications. SCXML uses a run-to-completion semantics, also known as macro-step/micro-step semantics. This means that trigger events may be needed to enable transitions. Trigger events are queued when they are raised, and then one is de-queued and consumed by firing all the transitions that it enables, followed by any (un-triggered) transitions that then become enabled due to the change of state caused by the initial transition firing. This is repeated until no transitions are enabled, and then the next trigger is de-queued and consumed. There are two kinds of triggers: internal triggers are raised by transitions and external triggers are raised non-deterministically by the environment. An external trigger may only be consumed when the internal trigger queue has been emptied. This means that an external trigger is only consumed when no transition can be taken without doing so.

2.3 Event-B

Event-B [1] is a formal method used to design and model software systems, of which certain properties must hold, such as safety properties. This method is useful in modelling safety-critical systems, using mathematical proofs to show consistency of models in adhering to its specification. Models consist of constructs known as machines and contexts. A *context* is the static part of a model, such as *carrier sets* (which are conceptually similar to types), *constants*, and *axioms*. Axioms are properties of carrier sets and constants which always hold. A context can *extend* one or more contexts by adding more carrier sets, constants and axioms. *Machines* describe the dynamic part of the model, that is, how the state of the model changes. The state is represented by the current values of the *variables*, which may change values as the state changes. *Invariants* are declared in the machine, stating properties of variables which should always be true, regardless of the state. *Events* in the machine describe state changes. Events can have *parameters* and *guards* (predicates on variables and event parameters); the guard must hold true for event execution. Each event has a set of *actions* which happen simultaneously, changing the values of the variables, and hence the state. Every machine has an initialisation event which sets initial variable values. Contexts can be extended and machines can be refined to introduce details of the formal model gradually. We utilise context extension in this paper, but not event extension. An important set of proof obligations are invariant preservation. They are generated and required to be discharged to show that no event can potentially change the state to one which breaks any invariant, a potentially unsafe state. Event-B is supported by the extensible Rodin platform [2]. Extension of Event-B and Rodin such as CamilleX [6] allows the composition of models.

3 Turnstile Example

A turnstile is used to illustrate the construction of a system model under the developed semantics. Figure 1, shows the statechart diagram for the turnstile.

The system has two sub-components that manage the operations performed by the gate and card reader in the turnstile. These components are represented by the *GATE* and *CARD_READER* parallel regions, respectively.

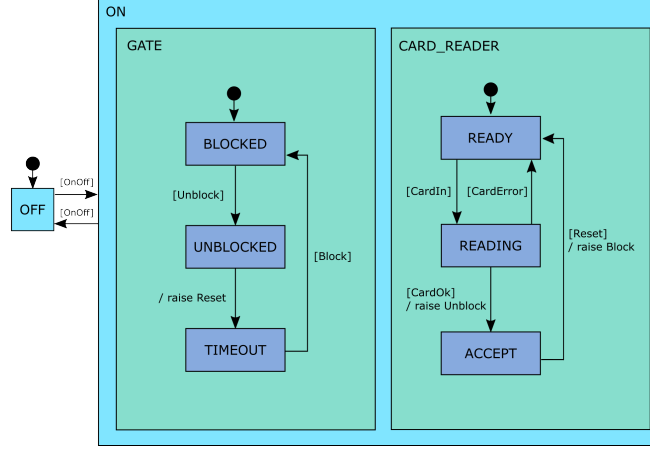


Fig. 1. Design model for a turnstile system

The model has two external triggers (*OnOff*, and *CardIn*), which are signals provided by the environment under which the system operates. In addition, there are internal triggers (*CardOk*, *Unblock*, *Block*, and *Reset*) that are raised by the components in the system. In the current model some of the internal triggers are raised non-deterministically (e.g. *CardOk*, *CardError*), as the details of the actual mechanisms responsible for the generation of the signal are not fully specified. Even in the presence of this non-determinism the system must satisfy certain requirements. Refinement of the abstract model presented in Figure 1 can be used to incorporate implementation details. For example, a nested statechart could be added to the *READING* state to specify the process by which the aforementioned triggers are raised. This manuscript focuses on formalizing the modeling language semantics for triggered statecharts of this form, and leaves the extensions required for refinement proofs to a future publication.

4 Formalization of Untriggered Statecharts

In this section, we formalise the syntactic elements and semantics of untriggered statecharts. This includes finding sufficient conditions for well-defined untriggered statecharts to guarantee consistent semantic behaviour of such models. The main ideas for our formalisation are (1) to use the Event-B *contexts* to capture the *syntactic elements* of the model with axioms ensuring that the model is well-defined (Section 4.1), and (2) to use the Event-B *machines* to capture the *semantics* of the models (Section 4.2).

4.1 Formalisation of the Untriggered Statechart Syntactic Elements

As the syntactic elements of untriggered statecharts are fairly complex, we develop them gradually, together with their well-definedness conditions, using Event-B context extension in the following steps.

1. Model the tree-structure of the states
2. Model the parallel regions
3. Model the transformations between states

Tree-structure States The structure of the states of a statechart is represented by the following constants; **states** (the set of all states), **root** (the implicit root state), **container** (the relationship between a child state and its parent container), **leaves** (the set of leaf states). These constants satisfy an axiom stating that they form a tree-shaped structure (here \mapsto is the notation for specifying tuples).

$$\text{states} \mapsto \text{root} \mapsto \text{container} \mapsto \text{leaves} \in \text{Tree}$$

Where, **Tree** is a constant, defined using transitive closure, that formalises the definition of tree-shaped structures. An important derived property of a tree-shaped structured (proved as a theorem) is needed to allow us to prove properties by induction.

Theorem 1 (Tree induction from root). *Consider a property P . If*

1. *The **root** satisfies P , and*
2. *for every non-root state s , if the container of s , i.e., $\text{container}(s)$, satisfies P , then s also satisfies P ,*

then every state in the tree satisfies P .

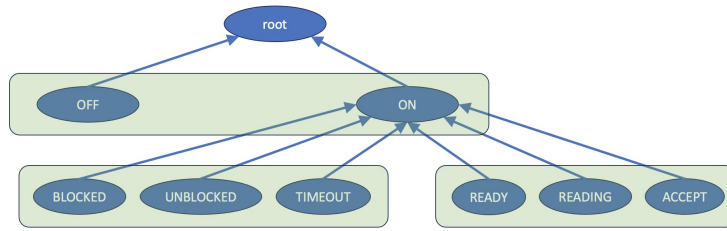


Fig. 2. The states and regions structure for the turnstile example

Example 1 (Turnstile Example. Tree-shaped Structure States). The tree-shaped structure of states in the turnstile example can be seen in Figure 2. Formally, we have the following definitions.

```

1 // All states of the turnstile examples
2 partition(states, {root}, {OFF}, {ON}, {BLOCKED}, {UNBLOCKED}, {TIMEOUT},
3   {READY}, {READING}, {ACCEPT})
4 // The container relationship between states
5 container = {OFF ↦ root, ON ↦ root, BLOCKED ↦ ON, UNBLOCKED ↦ ON,
6   TIMEOUT ↦ ON, READY ↦ ON, READING ↦ ON, ACCEPT ↦ ON}
7 // leaf states
8 leaves = {BLOCKED, UNBLOCKED, TIMEOUT, READY, READING, ACCEPT, OFF}

```

Regions Untriggered statecharts support the parallel composition of two or more nested statechart regions. That is a single state of a statechart may represent several sub components and associate with each component a corresponding region. In the turnstile example, the container state *ON* has two regions *GATE* and *CARD_READER*. We formalise the notion of regions as partitions of the set of non-root states by using the following axioms to constrain the constant regions.

Axiom 1 (Regions are subsets of states) *Each region is a subset of the statechart's states. (Here \mathbb{P} is the notation for powerset.)*

$$@region_type: regions \subseteq \mathbb{P}(states)$$

Axiom 2 (Regions are disjoint) *Every pair of distinct regions does not share any states.*

$$@region_disjoint: \forall r1, r2 \cdot r1 \in regions \wedge r2 \in regions \wedge r1 \neq r2 \Rightarrow r1 \cap r2 = \emptyset$$

Axiom 3 (Regions cover non-root states) *Every non-root state belongs to a region.*

$$@region_complete: union(regions) = states \setminus \{root\}$$

Axiom 4 (Region has a unique container) *Every region has a unique container state. Here $container[region]$ is the image of the relation *container* applying to *region*.*

$$@region_same_parent: \forall region \cdot region \in regions \Rightarrow (\exists parent \cdot container[region] = \{parent\})$$

Example 2 (Turnstile Example. Regions). The parallel regions for the turnstile example can be seen in Figure 2. Formally, we have three regions as follows.

```

1 regions = { {ON, OFF},
2   {BLOCKED, UNBLOCKED, TIMEOUT}, // GATE region
3   {READY, READING, ACCEPT} // CARD_READER region
4 }

```

Note that states *ON* and *OFF* implicitly form a region without any sibling parallel region.

Transformations Unlike the common definitions of transitions, which map a source state to a target state, we define **transformations**, which give an hierarchical view of the set of all simultaneously enabled transitions of the system, from one *enabling* state configuration to the next configuration. There are different types of transformation including *forking* (starting from a state and ending in one or more states in different parallel regions), *joining* (starting from two or more states in different parallel regions and ending in a state), *parallel* (updating parallel regions at the same time), and any combination of these types. To model all transformation types, we formalise each transformation by three sets of states.

- **enabling**: A transformation is enabled (i.e., can be executed) if its (non-empty) set of enabling states are active.
- **exiting**: The non-empty set of states that the transformation will exit upon execution.
- **entering**: The non-empty set of states the transformation will enter upon execution.

Formally, these notions are formalised as constants as follows.

- 1 **enabling** \in **transformations** $\rightarrow \mathbb{P}_1(\text{states})$
- 2 **exiting** \in **transformations** $\rightarrow \mathbb{P}_1(\text{states})$
- 3 **entering** \in **transformations** $\rightarrow \mathbb{P}_1(\text{states})$

Example 3 (Turnstile Example. Transformation). We give the enabling, exiting, and entering states of the following example transformations.

- Consider the transformation from the **BLOCKED** state to the **UNBLOCKED** state, we call this **BLOCKED_2_UNBLOCKED**. We have
 - 1 **enabling**(**BLOCKED_2_UNBLOCKED**) = {**BLOCKED**}
 - 2 **exiting**(**BLOCKED_2_UNBLOCKED**) = {**BLOCKED**}
 - 3 **entering**(**BLOCKED_2_UNBLOCKED**) = {**UNBLOCKED**}
- Consider the transformation from the **OFF** state to the **ON** state, we call this **OFF_2_ON**. Notice that the transformation will take into account also the transition from the initial states within the two sub-statecharts (regions). As a result, we have
 - 1 **enabling**(**OFF_2_ON**) = {**OFF**}
 - 2 **exiting**(**OFF_2_ON**) = {**OFF**}
 - 3 **entering**(**OFF_2_ON**) = {**ON**, **BLOCKED**, **READY**}
- Consider the transformation from the **ON** state to the **OFF** state, we call this **ON_2_OFF**. Notice that the transformation will take into account the non-deterministic exit from the two sub-statecharts (regions). As a result, we have
 - 1 **enabling**(**ON_2_OFF**) = {**ON**}
 - 2 **exiting**(**ON_2_OFF**) = {**ON**, **BLOCKED**, **UNBLOCKED**, **TIMEOUT**, **READY**, **READING**, **ACCEPT**}
 - 3 **entering**(**ON_2_OFF**) = {**OFF**}

There are several additional constraints (well-definedness conditions) relating the enabling, exiting, and entering states for a transformation. We identified some of the constraints directly. For instance, the following axioms related to **exiting** states.

Axiom 5 (Exiting a contained region) *If the container of a region is an exiting state, there must be an exiting state within that region.*

- 1 **@exiting-contained_region:**
- 2 $\forall \text{trf}, s, r. \text{trf} \in \text{transformations} \wedge s \in \text{exiting}(\text{trf}) \wedge r \in \text{regions} \wedge \text{container}[r] = \{s\}$
- 3 $\Rightarrow \text{exiting}(\text{trf}) \cap r \neq \emptyset$

Axiom 6 (Exiting one or all states in a region) *If a region r has an exiting state s then either s is the unique exiting state or all the states in r are exiting states.*

- 1 **@exiting-either_one_or_all_in_a_region:**
- 2 $\forall \text{trf}, s, r. \text{trf} \in \text{transformations} \wedge s \in \text{exiting}(\text{trf}) \wedge r \in \text{regions} \wedge s \in r$
- 3 $\Rightarrow \text{exiting}(\text{trf}) \cap r = \{s\} \vee r \subseteq \text{exiting}(\text{trf})$

The following axioms linking **exiting** and **enabling** states was “discovered” during the proof of the invariant preservation proof obligations (see Theorem 3).

Axiom 7 (Exiting a unique enabling state in a region) *Given a region r and an exiting state s in r , if r has states other than s , s must be the unique enabling state in r .*

- 1 **@exiting-unique_enabling_state_in_a_region:**
- 2 $\forall \text{trf}, s, r. \text{trf} \in \text{transformations} \wedge r \in \text{regions} \wedge \text{exiting}(\text{trf}) \cap r = \{s\} \wedge r \neq \{s\}$
- 3 $\Rightarrow \text{enabling}(\text{trf}) \cap r = \{s\}$

Axiom 8 (An enabling state is the unique exiting state) *Given a region r and an enabling state s in r , s must be the unique exiting state in r .*

- 1 **@enabling-unique_exiting_state_in_a_region:**
- 2 $\forall \text{trf}, s, r. \text{trf} \in \text{transformations} \wedge s \in \text{enabling}(\text{trf}) \wedge r \in \text{regions} \wedge s \in r$
- 3 $\Rightarrow \text{exiting}(\text{trf}) \cap r = \{s\}$

The following axiom relates **entering** with **enabling** and **exiting** states. It is also discovered during the proof of the invariant preservation proof obligations (see Theorem 3).

Axiom 9 (Transformation stays within a state) *If a transformation enters a region r but not the container of r (called c), then c is not an exiting state and there is an enabling state which is a descendant of c . Here, $cl(\text{container})$ denote the transitive closure of **container** relationship, hence the inverse of that (i.e., $cl(\text{container}) \sim$ is the descendant relationship between states.*

- 1 **@entering-stay_within_state:**
- 2 $\forall \text{trf}, r, c. \text{trf} \in \text{transformations} \wedge r \in \text{regions} \wedge \text{container}[r] = \{c\} \wedge$
- 3 $\text{entering}(\text{trf}) \cap r \neq \emptyset \wedge \text{entering}(\text{trf}) \cap \text{container}[r] = \emptyset$
- 4 $\Rightarrow \text{enabling}(\text{trf}) \cap cl(\text{container}) \sim \{\{c\}\} \neq \emptyset \wedge c \notin \text{exiting}(\text{trf})$

4.2 Formalisation of the Untriggered Statechart Semantics

Given an untriggered statechart (characterized by the tree-shape structured states, the regions, and the transformation), the semantics of the statechart is characterized by the set of active states during its execution. For instance, consider the turnstile example in Figure 1, initially, the turnstile has one active state, namely **OFF**, i.e., the set of active states is $\{\text{OFF}\}$

- A transformation **OFF_2_ON** (from **OFF** to **ON**) will change the set of active states to $\{\text{ON}, \text{BLOCKED}, \text{READY}\}$.
- A transformation **BLOCKED_2_UNBLOCKED** will change the set of active states to $\{\text{ON}, \text{UNBLOCKED}, \text{READY}\}$.
- A transformation **ON_2_OFF** changes the set of active states to $\{\text{OFF}\}$.

We can now formalize the semantics of the untriggered statechart in a machine using a single variable **active** satisfying $\text{active} \subseteq \text{states}$. The system's functionality is encoded through one event called **transformation** that captures how the design transitions from one configuration to the next. Essentially variable **active** provides a discrete characterization of the information and event **transformation** represents the operation of the system under analysis.

```

1 event transformation
2 any trf where
3   @typeof—trf: trf ∈ transformations
4   @active—enabling: enabling(trf) ⊆ active
5 then
6   @update—active: active := (active \ exiting(trf)) ∪ entering(trf)
7 end

```

Guard **@active—enabling** ensures that the chosen transformation **trf** is enabled and action **@update—active** first removes the **trf**'s exiting states then adds **trf**'s entering states. Notice that this action also allows a transformation to exit a state and re-enter that state.

An important aspect for the semantics of statecharts is that it can only transform amongst valid configurations. For example, we want to ensure that the turnstile statechart is never in the configuration where both **ON** and **OFF** states are active or in a state where **BLOCKED** is active, but **ON** is not active. The following constraints on **active** specify the valid configuration for a statechart, which we encode as 4 invariants for the machine.

Invariant 1 (Container active) *If a non-root state is active then its container is also active.*

$$\text{@container_active: } \forall s \cdot s \in \text{active} \setminus \text{root} \Rightarrow \text{container}(s) \in \text{active}$$

Invariant 2 (Content active) *If a container state is active then one of its sub-state must be active.*

$$\text{@content_active: } \forall s \cdot s \in \text{ran}(\text{container}) \wedge s \in \text{active} \Rightarrow \text{container} \sim \{\{s\}\} \cap \text{active} \neq \emptyset$$

Invariant 3 (Unique active state within a region) *There can be at most one active state in a region.*

$$\text{@active-region-unique: } \forall r, s \cdot r \in \text{regions} \wedge s \in r \cap \text{active} \Rightarrow r \cap \text{active} \subseteq \{s\}$$

Invariant 4 (Parallel regions are inactive/active at the same time) *All parallel regions are inactive (hence active) at the same time.*

$$\text{@active-region-parallel: } \forall r1, r2 \cdot r1 \in \text{regions} \wedge r2 \in \text{regions} \wedge \text{container}[r1] = \text{container}[r2] \wedge r1 \cap \text{active} = \emptyset \Rightarrow r2 \cap \text{active} = \emptyset$$

As a consequence of Invariant 1, we can prove the following machine theorem.

Theorem 2 (Ancestors active). *If a state s is active then all ancestors of s are also active.*

$$\text{@ancestor_active: } \forall s \cdot s \in \text{active} \Rightarrow \text{cl}(\text{container})[s] \subseteq \text{active}$$

Proof. The proof of the theorem relying on Invariant 1 and the inductive nature of transitive closure. We omit the details here.

We have to prove that event **transformation** maintains the invariants relying on the well-definedness constraints that we have put as axioms.

Preservation of invariant @active_container The proof obligation for ensuring that invariant **@active_container** is maintained by event **@transformation** after simplification can be stated as the following theorem.

Theorem 3 (Event transformation maintains @active_container). *Given a non-root state s such that either (1) s is active but non-exiting state, or (2) s is an entering state, then either (G1) $\text{container}(s)$ is active but non-exiting state, or (G2) $\text{container}(s)$ is an entering state.*

Proof. Since s is a non-root state, there exists a region r containing s (follows from Axiom 3 **@region_complete**). We continue the proof by considering Cases 1 and Case 2.

Case 1 (s is active but non-exiting state).

- According to Axiom 8 (**@enabling-unique_exiting_state_in_a_region**), s cannot be an enabling state since s is a non-exiting state.
- We discharge (G1) by proving that (G1-1) $\text{container}(s)$ is an active state, and (G1-2) $\text{container}(s)$ is a non-exiting state.
 - *Proof of (G1-1).* According to Invariant 1 (**@container_active**), since s is an active state, $\text{container}(s)$ is an active state
 - *Proof of (G1-2).* We prove this by contradiction, i.e., assuming that $\text{container}(s)$ is an exiting state.
 - * According to Axiom 5 (**@exiting-contained_region**), there must be an exiting state in the region r since the state containing r (in this case $\text{container}(s)$) is an exiting state, let us call this state x .

- * According to Axiom 6 (*@exiting—either_one_or_all_in_a_region*), either x is the unique exiting state in r or all states in r are exiting states.
- * Since s is a non-exiting state, x must be r 's unique exiting state.
- * From Axiom 7 (*@exiting—unique_enabling_state_in_a_region*), x is the unique enabling state in r .
- * According to guard *@active—enabling* of *transformation*, x (being an enabling state) must be an active state.
- * We now have two distinct active states x and s in r , which contradicts Invariant 3 (*@active—region—unique*).

Case 2 (s is an entering state). In this case, we proceed with the proof by assuming that (G2) does not hold (i.e., *container(s)* is not a container state) and prove (G1).

- According to Axiom 9, the transformation entering region r but not the container of r (i.e., *container(s)*), hence *container(s)* is not an exiting state and has a enabling descendant state, let us call this x .
- According to guard *@active—enabling* of *transformation*, x (being an enabling state) must be an active state.
- According to Theorem 2, *container(s)* (being an ancestor of x) must be an active state.
- We therefore has *container(s)* is active but non-exiting state (G1).

□

We omit the proof of the preservation of other Invariants 2, 3, and 4 due to limited space. The proofs are done manually utilising several axioms and the tree induction theorem (Theorem 1). Details can be found in [5].

5 Formalization of the Run-to-Completion Schedule

Similar to the previous section, the formalization is done using an Event-B context to capture the syntactic elements (Section 5.1) and an Event-B machine to model the semantics (Section 5.2).

5.1 Formalization of the Run-to-completion Syntactic Elements

To define run to completion execution we first specify the syntactic elements involved. Triggers, are partitioned into either Internal or External triggers. We define the internal and external trigger queues as sequences of internal and external triggers respectively. Sequences and their allowed operations (e.g. append trigger, head of sequence) are constructively defined via a series of theorems and axioms which are omitted here.

Figure 3 shows the state machine for the run-to-completion schedule. From the *Ready to de-queue* state, an internal or (if internal queue empty) external trigger is de-queued from the corresponding queues and the system moves to

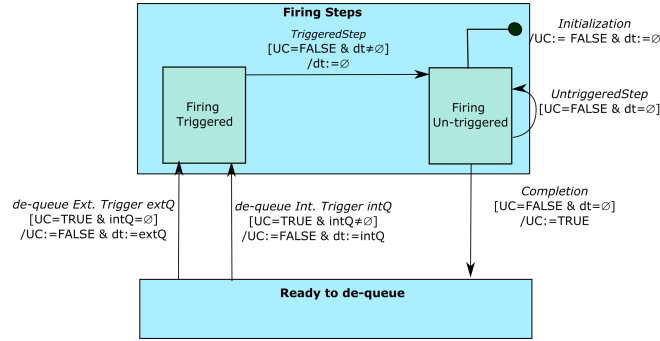


Fig. 3. State diagram for run to completion scheduling

the *Firing Triggered* state. A trigger step that consumes the dequeued trigger is then fired and the system moves to the *Firing Untriggered* state. From this state, untriggered steps can be fired repeatedly or the system moves back to the *Ready to de-queue* state. Both triggered and untriggered steps can raise more internal triggers, which will need to be handled in the future runs.

The context defining the syntactic elements for a run to completion is shown in Listing 1. An important notion for the run to completion schedule are steps. A triggered step is taken when an internal or external trigger is consumed and a non-deterministic number of untriggered steps may be taken after a triggered step. In the next section we will show how these steps relate to triggered and untriggered transitions

```

1 context r2c_ctx
2 extends r2c_c0_2_dequeue
3 sets Steps
4 constants InternalTriggers ExternalTriggers Triggers StepTrigger StepRaised
5 axioms
6 @typeof _IT: InternalTriggers ⊆ InternalTriggerType
7 @typeof _XT: ExternalTriggers ⊆ ExternalTriggerType
8 @def _T: Triggers = InternalTriggers ∪ ExternalTriggers
9 @typeof _StepTrigger: StepTrigger ∈ Steps → Triggers
10 @typeof _StepRaised: StepRaised ∈ Steps → Seq(InternalTriggers)
11 end

```

Listing 1. Context for Run-to-Completion Semantics

Here *StepTrigger* (as a partial function) defines the required trigger for a Step (if any), and *StepRaised* defines the sequence of internal triggers that will be raised for a step (including an empty sequence). Steps that do not have any required trigger will be untriggered steps.

5.2 Formalization of the Run-to-Completion Semantics

Given the syntactic elements defined earlier, the machine defining the semantics for the run-to-completion schedule models the trigger queues in the system

according to Figure 3. The dynamic status of the run-to-completion schedule is represented by the variables `int_q`, `ext_q`, `dt` (dequeue trigger), and `completion` with the following invariants.

```

1  @int_q: int_q ∈ Seq(InternalTriggers)
2  @ext_q: ext_q ∈ Seq(ExternalTriggers)
3  @dequeue_trigger: dt ∈ DeQueueType
4  @dequeue_triggerwd: dt ⊆ InternalTriggers ∪ ExternalTriggers
5  @firingTriggered: dt ≠ ∅ ⇒ completed=FALSE

```

Where `int_q` and `ext_q` represent the sequences of internal and external triggers that need to be handled, `dt` keeps track of the trigger that has been removed from the queues (de-queue) to be consumed by a *TriggeredStep*. Note that `dt` is a singleton set of trigger when the system is in the *Firing Triggered* state and empty otherwise (this is also the definition of *DeQueueType*). Finally, variable `complete` (denoted as `UC` in Figure 3) is `TRUE` indicates that the system is in the *Ready to de-queue* state.

Starting from the *Ready to de-queue* state (where `completed = TRUE`), the system will de-queue an internal/external trigger if there are any. Note that an external trigger is only de-queued when the internal queue is empty. After that the system moves to the *Firing triggered* state (where `completed = FALSE` and `dt` is no longer empty).

<pre> 1 event dequeueInternalTrigger 2 any trigger where 3 @grd1: completed = TRUE 4 @grd2: int_q ≠ ∅ 5 @grd3: trigger = Seq_head(int_q) 6 then 7 @act1: dt := {trigger} 8 @act2: int_q := Seq_tail(int_q) 9 @act3: completed := FALSE 10 end </pre>	<pre> 1 event dequeueExternalTrigger 2 any trigger where 3 @grd1: completed = TRUE 4 @grd2: ext_q ≠ ∅ 5 @grd3: trigger = Seq_head(ext_q) 6 @grd4: int_q = ∅ 7 then 8 @act1: dt := {trigger} 9 @act2: ext_q := Seq_tail(ext_q) 10 @act3: completed := FALSE 11 end </pre>
---	---

The behaviour of the *TriggeredStep* and *UntriggeredStep* are formalised by the corresponding events as follows. Both events can raise a sequence of internal triggers which is concatenated to the the internal queue.

<pre> 1 event triggeredStep 2 any Step where 3 @grd1: Step ∈ dom(StepTrigger) 4 @grd2: StepTrigger(Step) ∈ dt 5 then 6 @act1: dt := dt \ {StepTrigger(Step)} 7 @act2: int_q := Seq_concat(int_q ↦ StepRaised(Step)) 8 end </pre>	<pre> 1 event untriggeredStep 2 any Step where 3 @grd1: Step ∈ Steps \ dom(StepTrigger) 4 @grd2: dt = ∅ 5 then 6 @act1: int_q := Seq_concat(int_q ↦ StepRaised(Step)) 7 end </pre>
--	---

While the internal triggers are raised through steps, external triggers can be raised non-deterministically by event `raiseExternalTrigger` and appended to the external queue. Note that at this stage (without the statemachine), there is a non-determinism between `untriggeredStep` and `completion`. When we combine the untriggered statechart and the run-to-completion schedule in the next section, we will distinguish the two cases.

<pre> 1 event raiseExternalTrigger 2 any trigger where 3 @grd1: trigger ∈ ExternalTriggers 4 then 5 @act1: ext_q := Seq_append(ext_q ↦→ 6 trigger) 7 end </pre>	<pre> 1 event completion 2 where 3 @grd1: dt = ∅ 4 @grd2: completed = FALSE 5 then 6 @act1: completed := TRUE 7 end </pre>
--	--

The model of the run-to-completion schedule maintains its invariants straightforwardly (relying on operations of sequence manipulation).

6 Formalization of Triggered Statecharts

The triggered statecharts is an unified statechart model representation based on SCXML [11]. To formalize the complete semantics we compose the previous two models, of the untriggered statechart (Sections 4) and of the run-to-completion schedule (Section 5). The composition is performed by, using the inclusion mechanism built into the CamilleX extension [6] of the Rodin platform.

6.1 Triggered Statechart Syntactic Elements

Since the a triggered statechart is a combination of its untriggered statechart and the run-to-completion schedule, the syntactic elements of triggered statechart extend the syntactic elements of the untriggered statechart and the run-to-completion (Listing 2). We introduce some syntactic elements *gluing* the sub-context together, namely, the link between untriggered statechart's transformations and run-to-completion's steps, to form `transitions`.

At the same time, when we combine the two sub-models, we need to ensure an important aspect of triggered statecharts which is their responsiveness. In particular, if a trigger (internal or external) is de-queued, but no enabled transformation that can consume the trigger, we need to ensure that the system can still progress. More precisely, the system will need to *discard* the problematic trigger in order to continue. Constants `discardSteps` are introduced to capture the special steps that discarding triggers.

Axiom `@typeof-transitions` specifies that `transitions` is a one-to-one correspondence between transformations and non-discarding steps (\mapsto is the symbol for bijective functions). Axioms `@discardSteps-Triggers` and `@discardSteps-Raised` ensure that there is a discard step for every trigger and the discard steps do not raise any trigger.

```

1 context tstc_ctx
2 extends r2c_ctx utstc_ctx
3 constants transitions discardSteps
4 axioms
5   @typeof—transitions: transitions  $\in$  transformations  $\mapsto$  Steps  $\setminus$  discardSteps
6   @discardSteps—Triggers: discardSteps  $\triangleleft$  StepTrigger  $\in$  discardSteps  $\mapsto$  Triggers
7   @discardSteps—Raised: StepRaised[discardSteps] =  $\{ \emptyset \}$ 
8 end

```

Listing 2. Context for Triggered Statecart

6.2 Triggered Statechart Semantics

The semantics of a triggered statechart is capture by a machine that includes both the untriggered statechart and the run-to-completion schedule. We also use prefixing mechanism, e.g., **as r2c**, so that all modelling elements of the included machine are prefixed accordingly.

```

1 machine tstc sees tstc_ctx
2 includes r2c as r2c
3 includes utstc as utstc

```

The following events are *lifted* from the run-to-completion machine (unchanged): **raiseExternalTrigger**, **dequeueExternalTrigger**, **dequeueInternalTrigger**. Essentially, they only concern the management of trigger queues and do not relate to the statechart's status.

Events to model the transitions (**triggeredTransition** and **untriggeredTransition**) *synchronises* with the events from the untriggered statechart and the run-to-complete schedule. The additional guard of the events ensure that the chosen transformation (from the untriggered statechart) and the step (from the run-to-completion model) corresponds with each other.

<pre> 1 event triggeredTransition 2 synchronises r2c.triggeredStep 3 synchronises utstc.transformation 4 where 5 @grd1: transitions(utstc_trf) = r2c_Step 6 end </pre>	<pre> 1 event untriggeredTransition 2 synchronises r2c.untriggeredStep 3 synchronises utstc.transformation 4 where 5 @grd1: transitions(utstc_trf) = r2c_Step 6 end </pre>
--	--

As discussed before, we need to introduce the events to discard triggers **discardTrigger** (in the case where **triggeredTransition** is not available). This condition is formalised as **discardTrigger**'s **grd3**. We also strengthen the guard of **completion** to ensure that the system will complete a run only when **untriggeredTransition** is not available (see **completion**'s **grd1**).

<pre> 1 event discardTriggered 2 any trigger 3 synchronises r2c.triggeredStep 4 where 5 @grd1: r2c_Step ∈ discardSteps 6 @grd2: trigger = StepTrigger(r2c_Step) 7 @grd3: ∀trf · transitions(trf) ∈ StepTrigger~[{trigger}] ⇒ ¬enabling (trf) ⊆ utstc_active 8 end </pre>	<pre> 1 event completion 2 synchronises r2c.completion 3 where 4 @grd1: 5 ∀ r2c_Step · r2c_Step ∈ Steps \ dom(StepTrigger) 6 ⇒ 7 ¬(enabling(transitions~(r2c_Step)) ⊆ utstc_active) 8 end </pre>
---	--

Using composition via inclusion mechanism means that the triggered state-chart semantics inherit the invariants from the sub-machines without the need to prove them (correct-by-construction).

7 Conclusions

In this paper, we formalize the semantics of SCXML run-to-completion state-charts using Event-B. We formalize the syntactic elements using Event-B contexts and using Event-B machines to model the dynamic semantics. Moreover the semantics model is built in a compositional fashion: the semantics of (un-triggered) statechart and run-to-completion schedule is developed independently and composed to create the SCXML statechart's semantics model. This approach allows us to reduce the complexity on the consistency reasoning by focusing on the different parts of the models. The combined model inherits the consistency of the sub-models by construction.

In order to ensure the consistency of the semantics, several well-definedness conditions on the syntactic elements have been identified. They are encoded as axioms in the formal models. These well-definedness conditions can be used as the specification of a validation tool to ensure the consistency of SCXML models. Given the semantic model are consistent, any instantiation will inherit this consistency without the need for re-proving. For instance, the model of the turnstile example can have consistency about the active states and the triggering mechanism. Often, these consistency checks make up the majority of the proof obligations, and only a small number are related to the specific properties of the model.

We plan to extend this work to accommodate for the semantics of refinement as described in [7,8]. In particular, we will formalise the syntactic constraints which allow the consistent refinement of SCXML run-to-completion statecharts, proving the consistency of the refinement rules, e.g., in [9]. The consistency of the semantic models focuses on safety properties, expressed as invariants. Furthermore, we model some of the syntactic elements in our formal models at a fairly abstract level, e.g., the notion of **enabling**, **exiting**, **entering** states for transformation. This means that the model can be applied to different statechart notations, e.g. UML-B [10].

All data supporting this study are openly available at <https://shorturl.at/agtHN>.

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