P o C S What's The Story?

Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 6701, 6713, & a pretend number
University of Vermont, Fall 2023
Solutions to Assignment 15
Dangerous Beans

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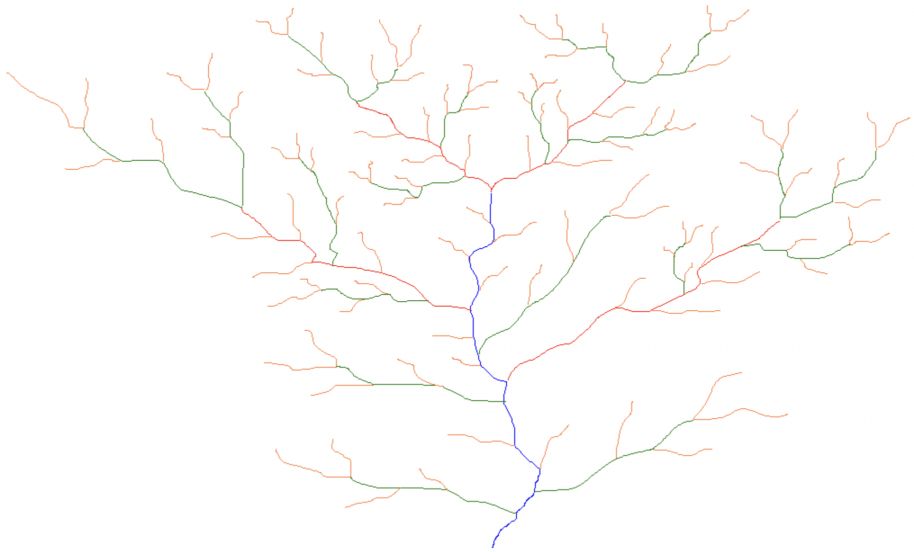
- Please use Overleaf for writing up your project.
- Build your paper using:
  https://github.com/petersheridandodds/universal-paper-template
- Please use Github and Gitlab to share the code and data things you make.
- For this first assignment, just getting the paper template up is enough.

1. Tokunaga's law is statistical but we can consider a rigid version. Take $T_1 = 2$ and $R_T = 2$ and draw an example network of order $\Omega = 4$ with these parameters.

   Please take some effort to make your network look somewhat like a river network.

   Solution:

$$T_{2,1} = 2(2)^{2-1-1} = 2$$
$$T_{3,1} = 2(2)^{3-1-1} = 4$$
$$T_{3,2} = 2(2)^{3-2-1} = 2$$
$$T_{4,1} = 2(2)^{4-1-1} = 8$$
$$T_{4,2} = 2(2)^{4-2-1} = 4$$
$$T_{4,3} = 2(2)^{4-3-1} = 2$$



$\square$

2. Show $R_s = R_\ell$. In other words show that Horton's law of stream segments matches that of main stream lengths, and do this by showing they imply each other.

   Solution:

Showing $R_s \implies R_l$

$$\begin{aligned}
\overline{s}_{\omega+1} &= \overline{s}_\omega R_s \\
&= \overline{s}_{\omega-1} R_s^2 \\
&= \overline{s}_{\omega-2} R_s^3 \\
&\vdots \\
&= \overline{s}_1 R_s^\omega \\
s_\omega &= \overline{s}_1 R_s^{\omega-1} \\
l_\omega &= \sum_{i=1}^{\omega} \overline{s}_i = \overline{s}_1 + \overline{s}_2 + \overline{s}_3 + \cdots + \overline{s}_\omega \\
&= \sum_{i=1}^{\omega} R_s^{i-1} \overline{s}_i = \overline{s}_1 (1 + R_s + R_s^2 + \cdots + R_s^{\omega-1})
\end{aligned}$$

Note*: $\sum_{k=1}^{n} ar^{k-1} = a\left(\frac{1-r^n}{1-r}\right)$

$$l_\omega = \overline{s}_1 \left(\frac{1 - R_s^\omega}{1 - R_s} \right) \Rightarrow R_s \neq 1$$

$$R_l = \frac{\overline{l}_{\omega+1}}{\overline{l}_\omega} = \frac{\overline{s}_1 \frac{1 - R_s^{\omega+1}}{1 - R_s}}{\overline{s}_1 \frac{1 - R_s^\omega}{1 - R_s}} = \frac{1 - R_s^{\omega+1}}{1 - R_s} = \frac{1 - R_s}{1 - R_s^\omega}$$

For large R_s ...

$$\frac{1 - R_s}{1 - R_s^\omega} \simeq R_s$$

so...

$$R_l = R_s$$

Showing $R_l \implies R_s$

$$R_l = \frac{\overline{l}_{\omega+1}}{\overline{l}_\omega}$$
$$\overline{l}_\omega = \overline{l}_1 R_l^{\omega-1}$$
$$\overline{s}_\omega = \overline{l}_\omega - \overline{l}_{\omega-1} \quad \text{and} \quad \overline{s}_{\omega+1} = \overline{l}_{\omega+1} - \overline{l}_\omega$$
$$R_s = \frac{\overline{s}_{\omega+1}}{\overline{s}_\omega} = \frac{\overline{l}_{\omega+1} - \overline{l}_\omega}{\overline{l}_\omega - \overline{l}_{\omega-1}} = \frac{\overline{l}_1 R_l^\omega - \overline{l}_1 R_l^{\omega-1}}{\overline{l}_1 R_l^{\omega-1} - \overline{l}_1 R_l^{\omega-2}} = \frac{R_l^{\omega-1}(R_l - 1)}{R_l^{\omega-2}(R_l - 1)} = \frac{R_l^{\omega-1}}{R_l^{\omega-2}} = R_l$$

so...

$$R_s = R_l$$

$\square$