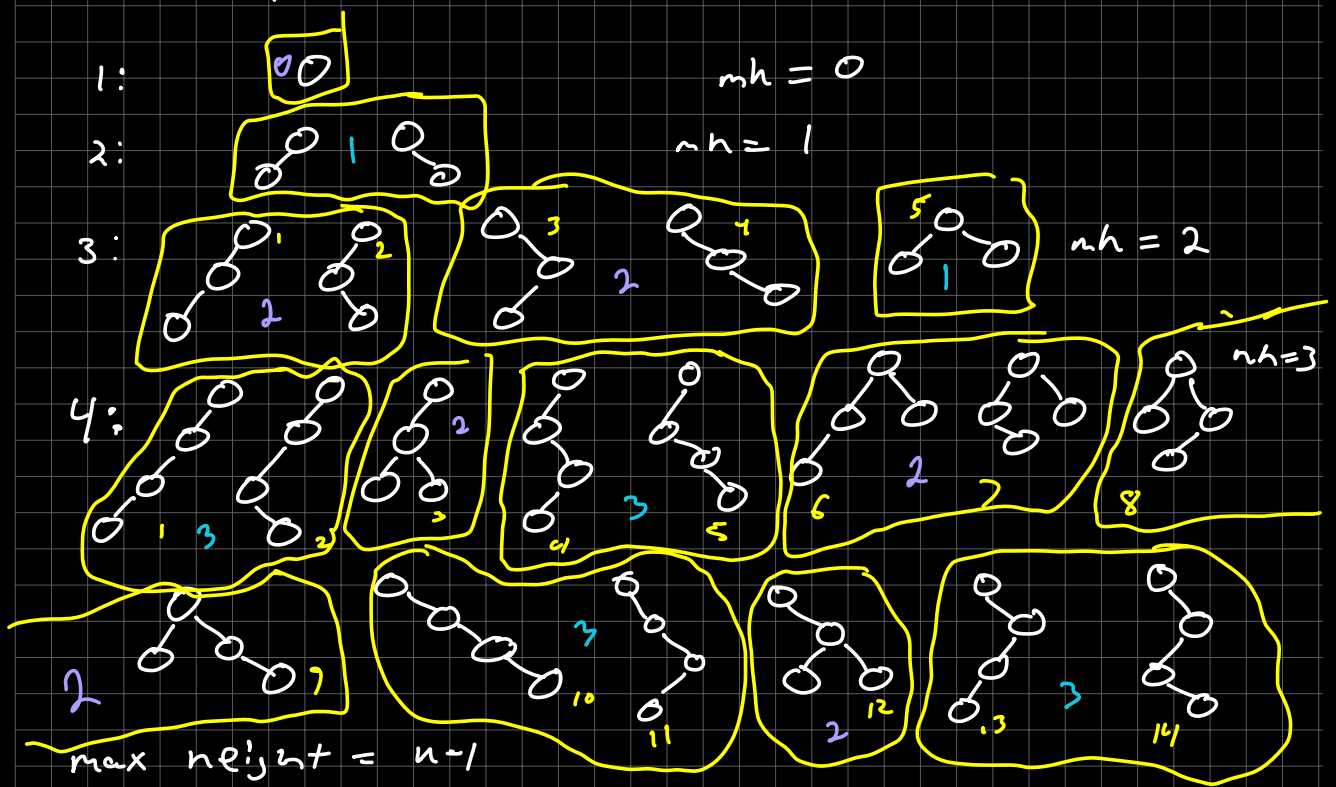
1) Method that takes n & finds BST tree combinations with $1 \rightarrow n$ nodes

$n = 4$



$2^{h+1} - 1$ = # of nodes AVL

$n = 0$

$2^1 - 1 = 1$

$n = 1$

$2^2 - 1 = 3$

$n = 2$ $2^3 - 1 = 7$

$2^{3-1} + 1 = 2^2 + 1 = 5$

$2^{4-1} + 1 = 2^3 + 1 = 9$

$(2 \cdot 3) + 2^3$

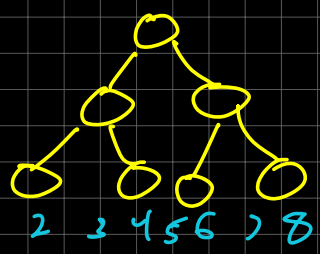
$(2(n-1)) + 2^{n-1}$

$2^h + 2^h + 2^h$

$2^0 = 1 \quad 2^1 = 2 \quad 2^2 = 4 \quad 2^3 = 8 \quad 2^4 = 16$

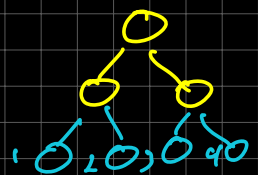
31

2^3 +



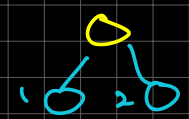
3 possible for node on level 4

2^2 +



4 possible for node on level 3

2^1 +



2 possible for node on level 2

2^0 + 0

1 possible for node on level 1

= total possibilities for n nodes

$$2^n + 2^{n-1} + 2^{n-2} \ldots + 2^0$$

$$2^{\wedge}(k) + 2^{n-1}(k+1) + 2^{n-2}(k+2) \ldots + 2^0(k+n)$$

$n=4$
$k=1$ should be 22

$$2^3(1) + 2^2(2) + 2^1(3) = 8 + 8 + 6$$

22

$$2(n + (n-1) + (n-2) + (n-3) \ldots + 0)$$

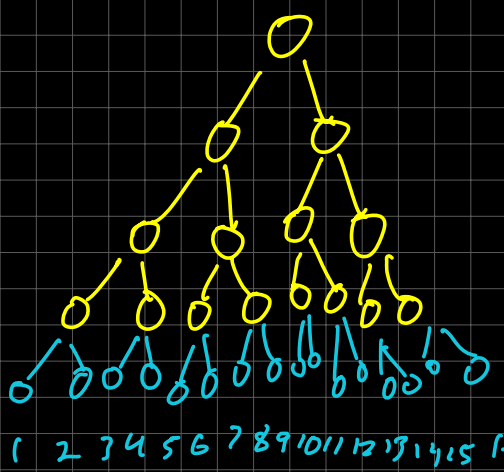
$n=3$

$k=1$

$2^2(1) + 2^1(2) \quad = \quad 4+4=8$

$2n(+2(n-1)+(2(n-2)))$

$2(3+2(2+2(1+2(0))))$



$$\frac{(2n)!}{(n+1)!\,n!} = \frac{(2\cdot 4)!}{(4+1)!\,4!} = \frac{8!}{5!\,4!}$$

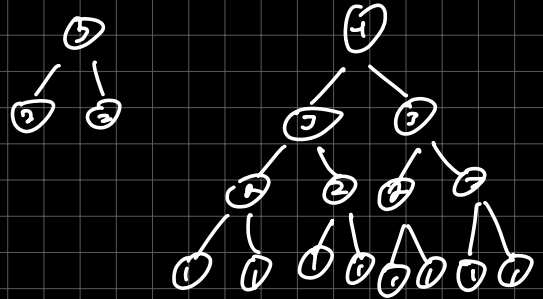
$=$

$$= \sum_{i=1}^{n} (i-1)(n-i)$$

$n=4$

$$\sum_{i=1}^{4} (i-1)(4-i)$$

$(1-1)(4-1) + (2-1)($



$4 \cdot 2 \quad 8$

$3 \cdot 2 \quad 6$

$2 \cdot 2 \quad 4$

$$t(3) = t(0)\,t(2) + t(1)\,t(1) + t(2)\,t(0)$$

$$t(0)\,t(1) + t(1)\,t(0) \qquad t(0)\,t(1) + t(1)\,t(0)$$

$$\sum_{i=1}^{n-1} \sum_{i=1}^{n} (i-1)(n-i)$$

$$= \sum_{i=1}^{n-1} \left(\left(\frac{n(n-1)}{2} \right) - 1 \right) \left(n - \left(\frac{n(n-1)}{2} \right) \right)$$

$-$ <

- 4

- if you can construct 2^{n-1} BST's of height n-1

$$2^{n-1} + 2^{n-2} + 2^{n-3} + 2^{n-4}$$

say 4 nodes
height = 3

$$2^{4-1} + 2^{4-2} + 2^{4-3} + 2^{4-4}$$

$$2^3 + 2^2 + 2^1 + 2^0$$

$$= 8 + 4 + 2 + 1 = 15$$

say 5 nodes
height = 4

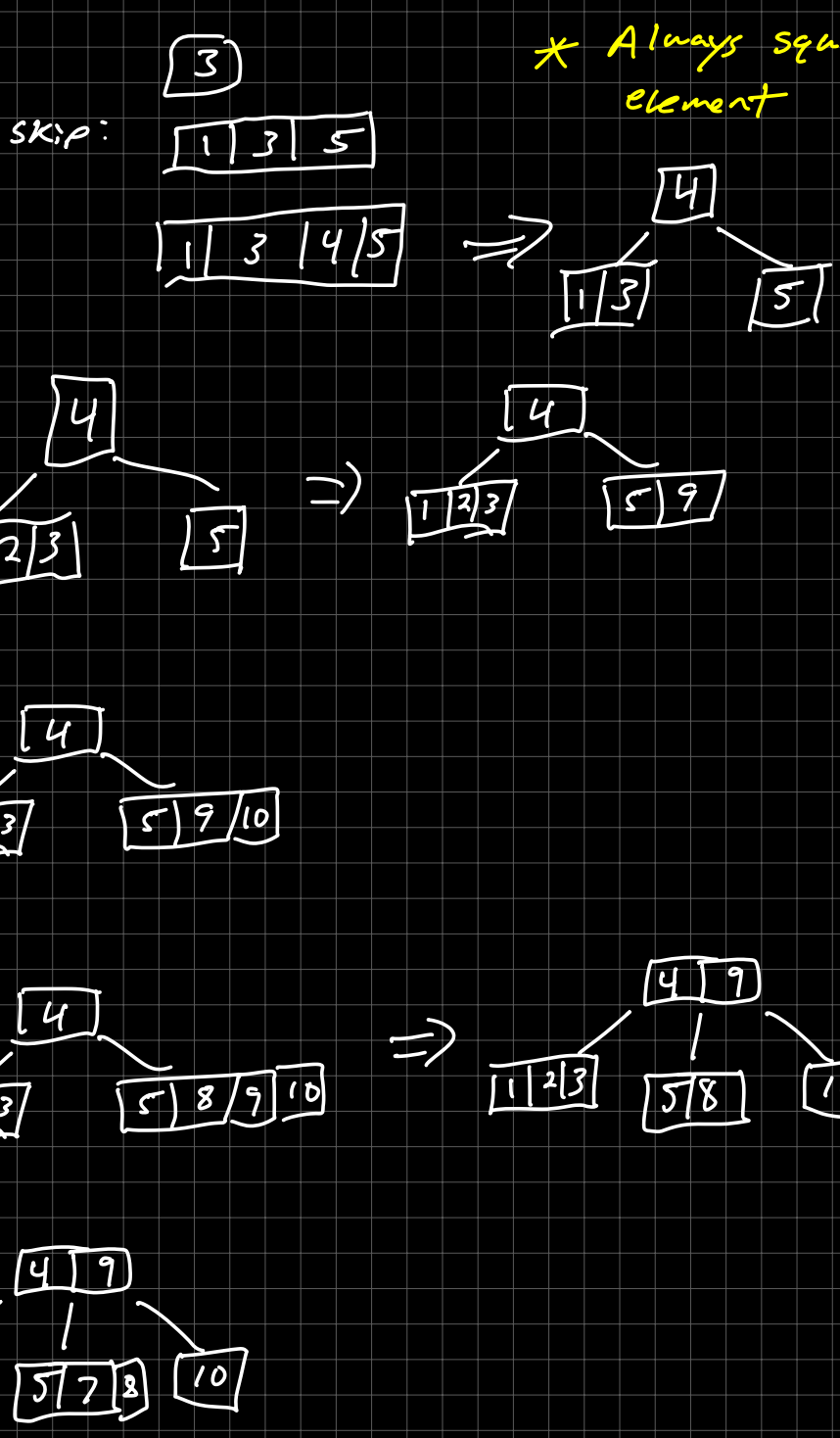
$$2^4 + 2^3 + 2^2 + 2^1 + 2^0$$

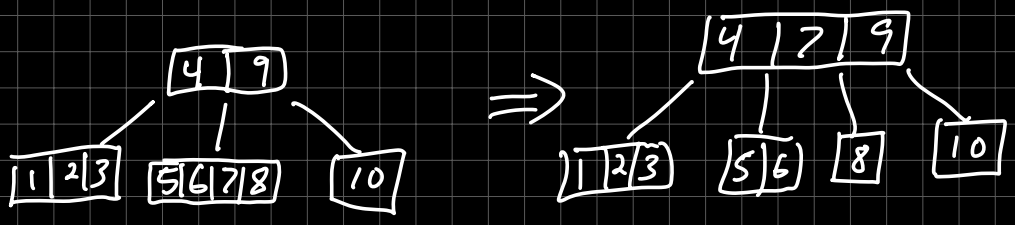
$$= 16 + 8 + 4 + 2 + 1$$

$$= 24 + 4 + 2 + 1 = 31$$

Actual 2-4 problems / other practice

- 3 1 5 4 2 9 10 8 7 6




4 9
1 2 3
5 6 7 8
10
4 7 9
1 2 3
5 6
8
10

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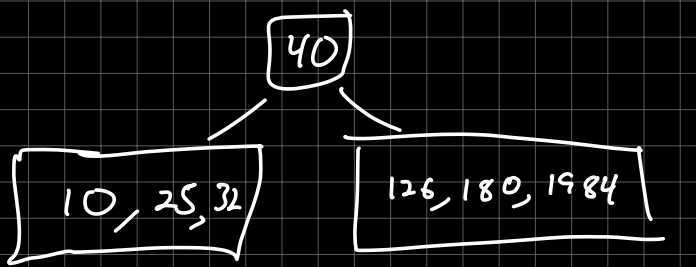
insert 86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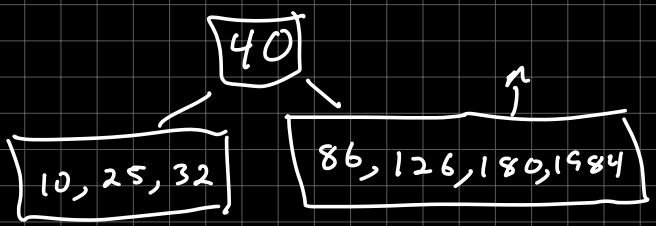
10, 25, 32
126, 180, 1984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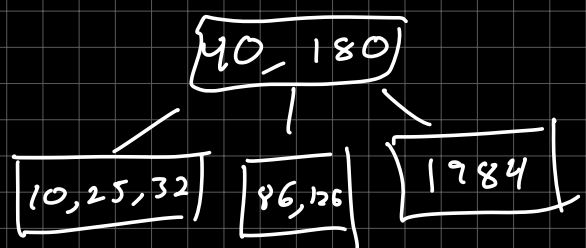
86, 126, 180, 1984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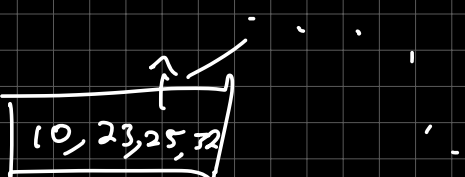
10, 25, 32
86, 126
1984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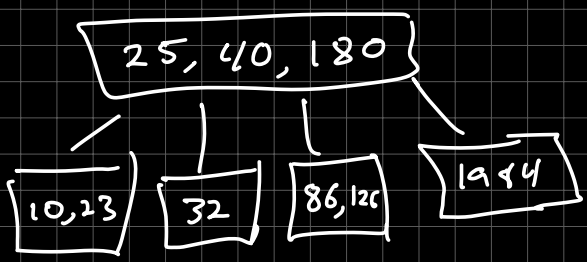
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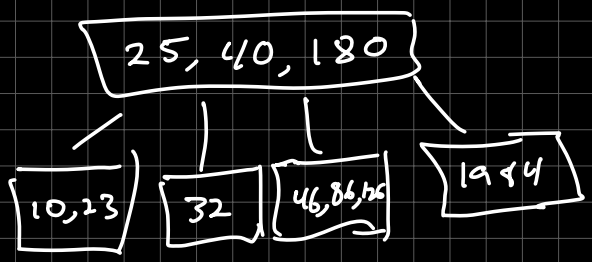
10, 23
32
86, 126
1984

46:



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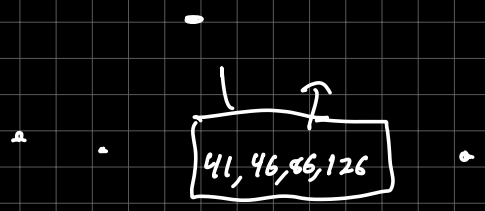

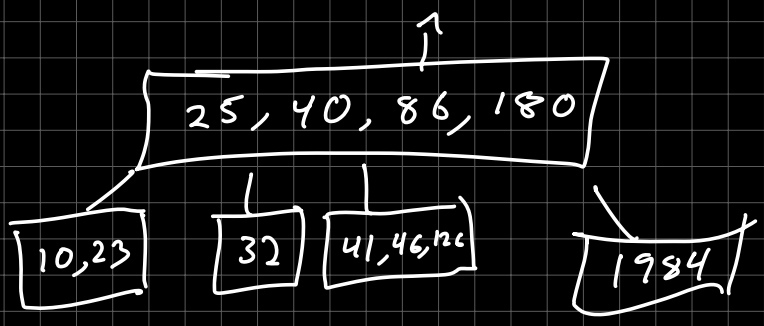

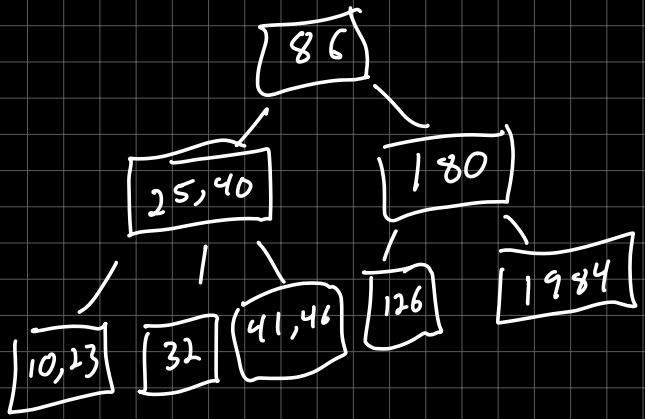

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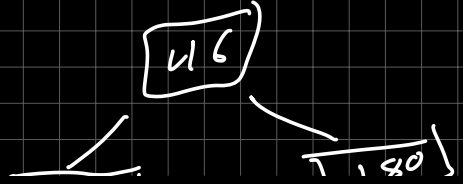

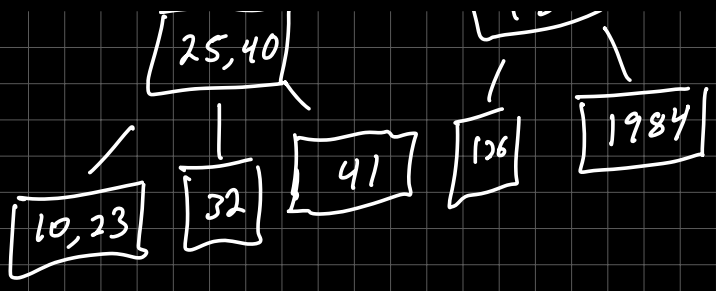
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10,23
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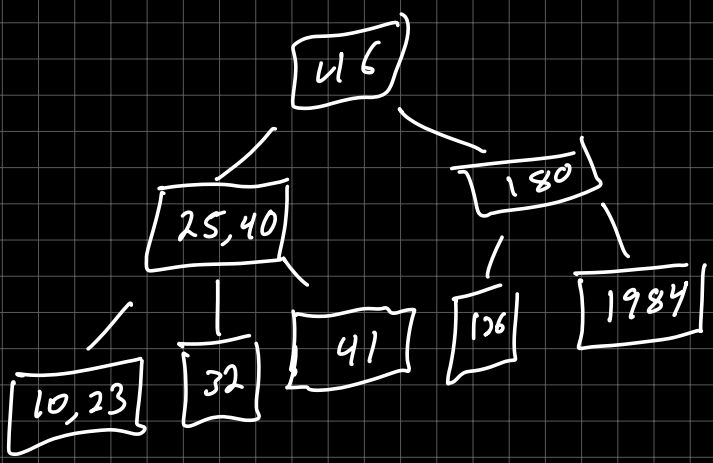
41
126
1984

delete 32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10,23
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126
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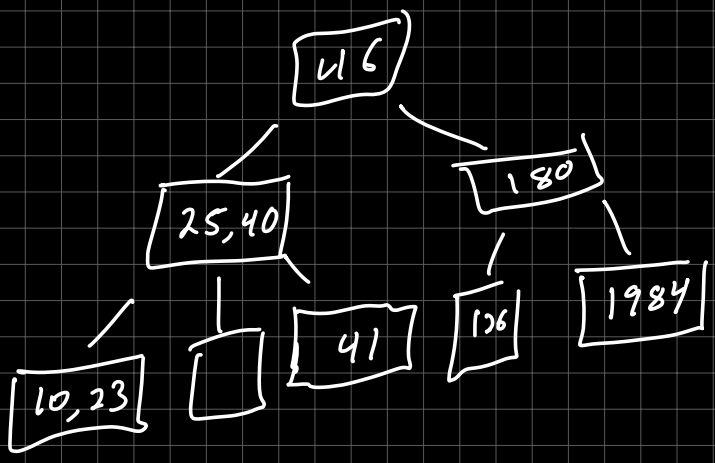
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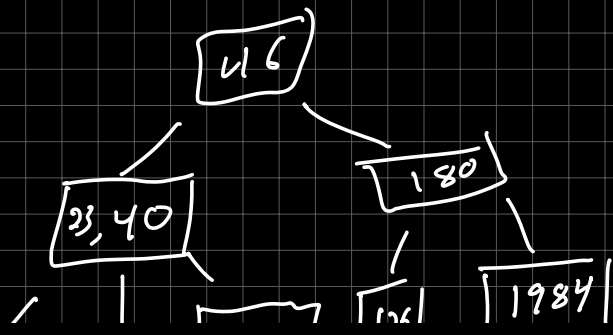
116
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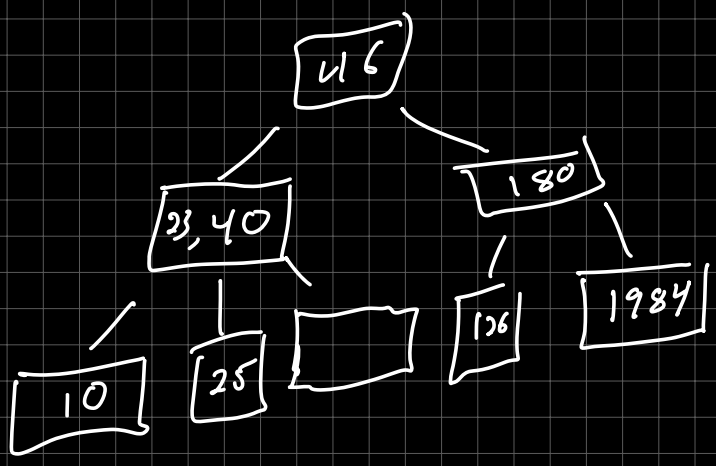
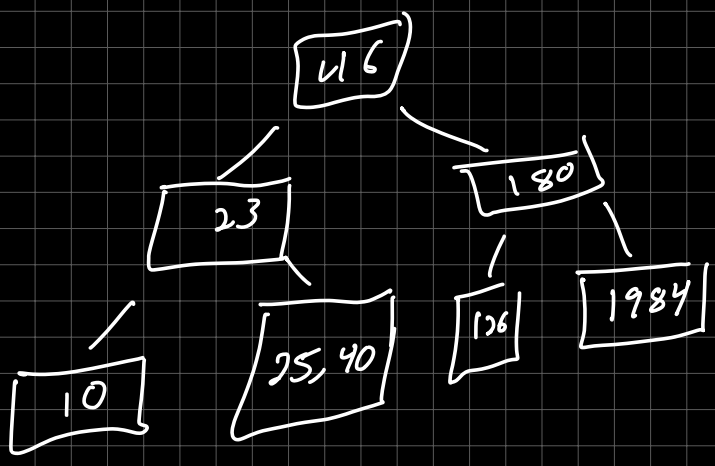
33,40
1984

delete 41:

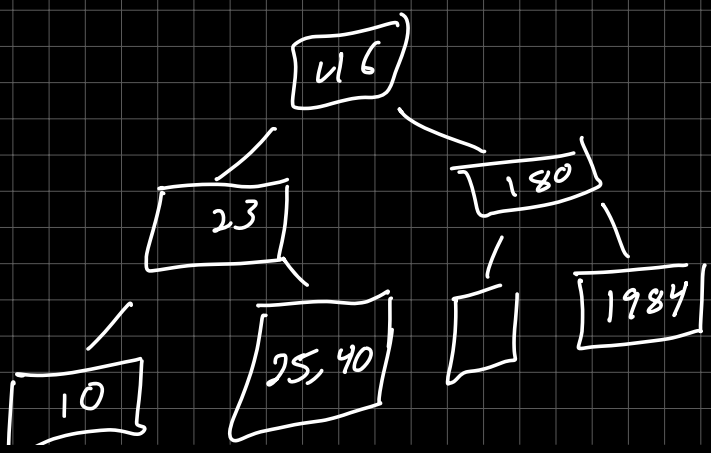
can't borrow

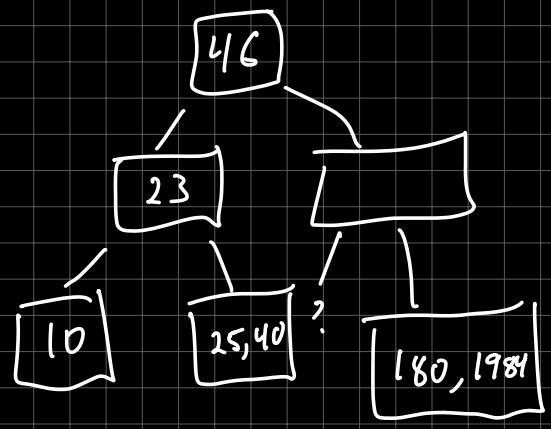


merge + move down



delete 126

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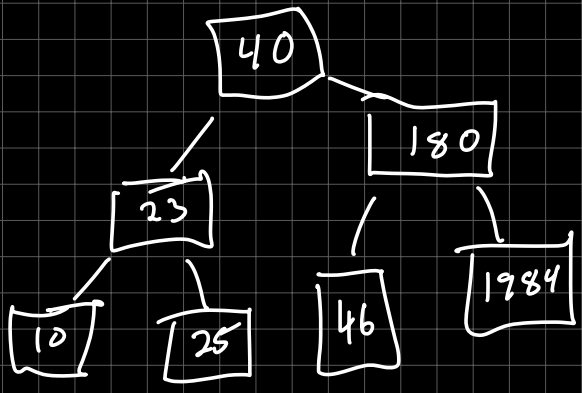
10
25,40
180, 1984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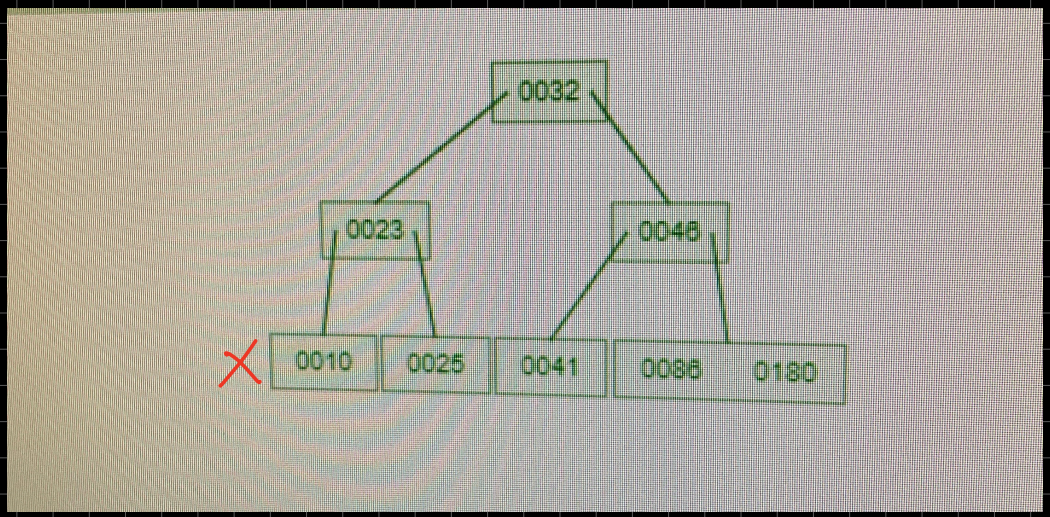
1984

delete 10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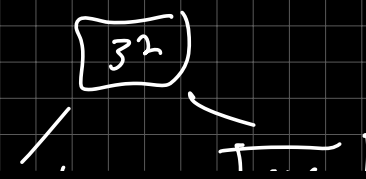
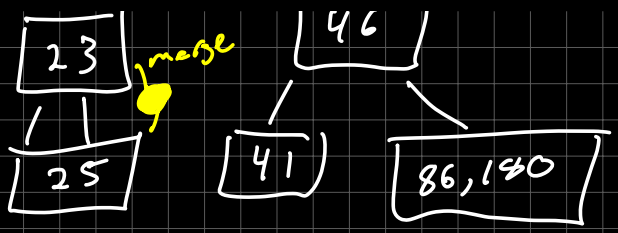
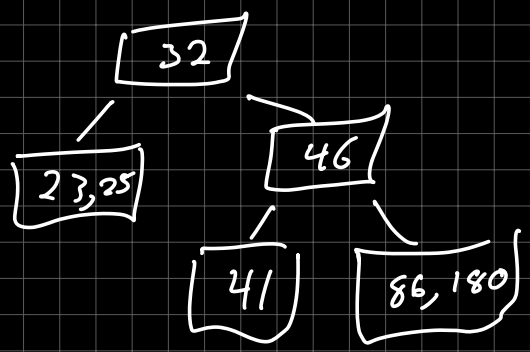
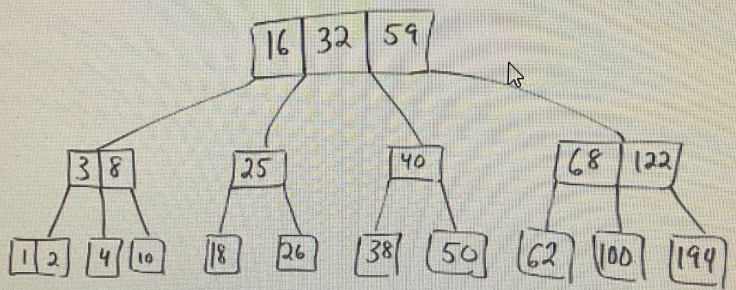
0180


32

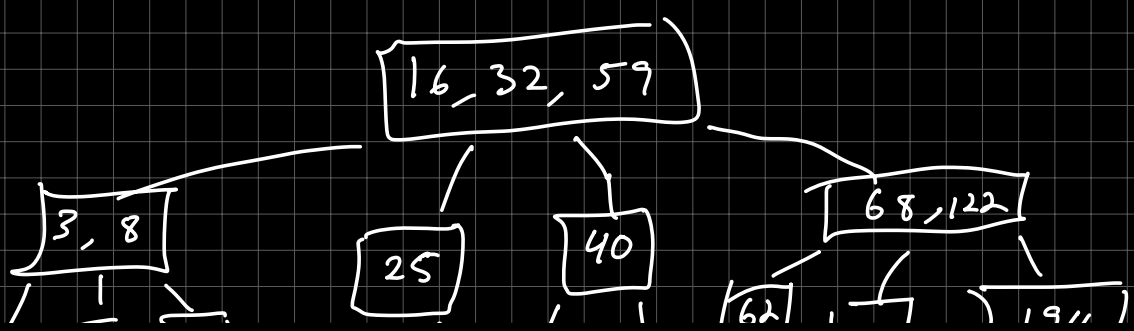


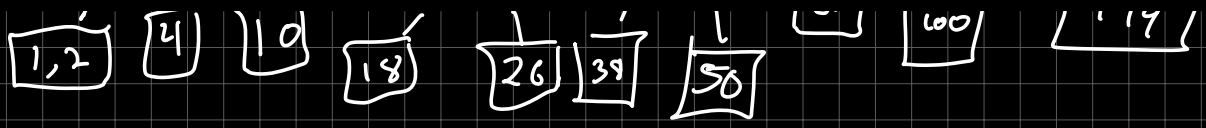


5. Show the following 2-4 tree after performing each of the following operat

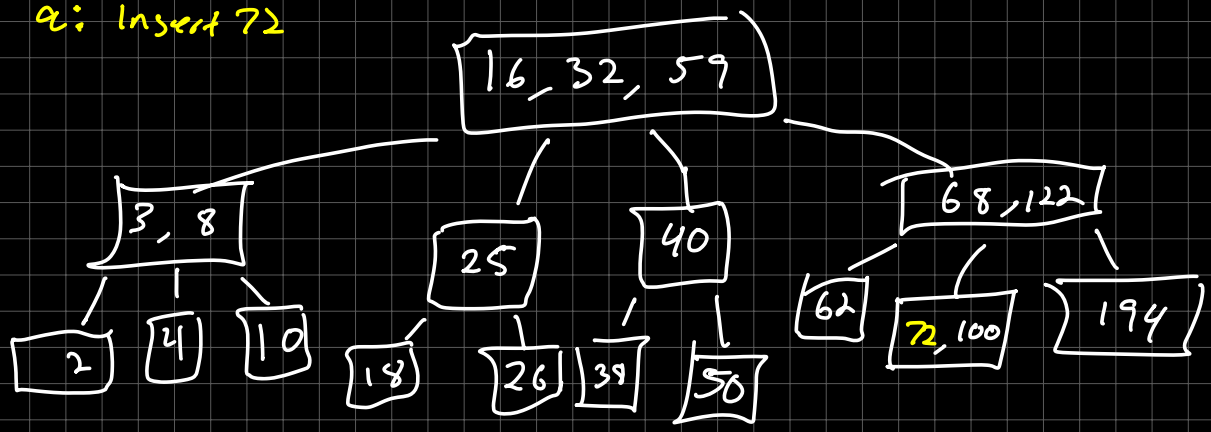


a. Insert 72
b. Insert 106
c. Insert 74
d. Delete 25
e. Insert 78
f. Insert 79
g. Delete 4
h. Delete 16

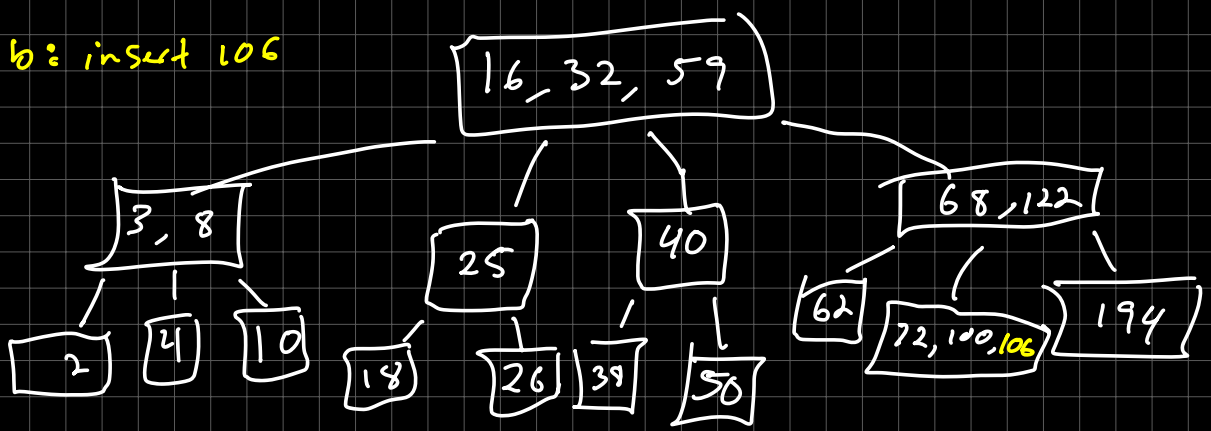




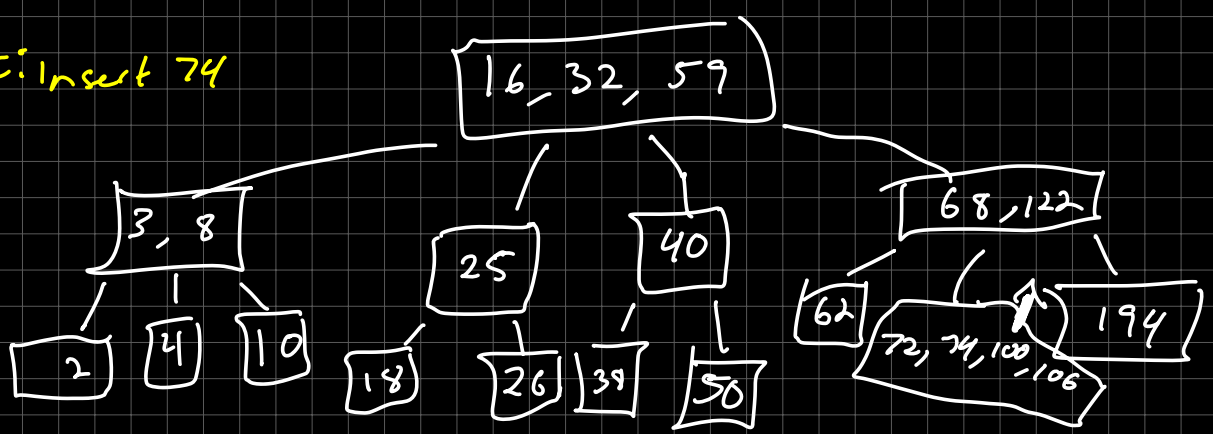
a: Insert 72

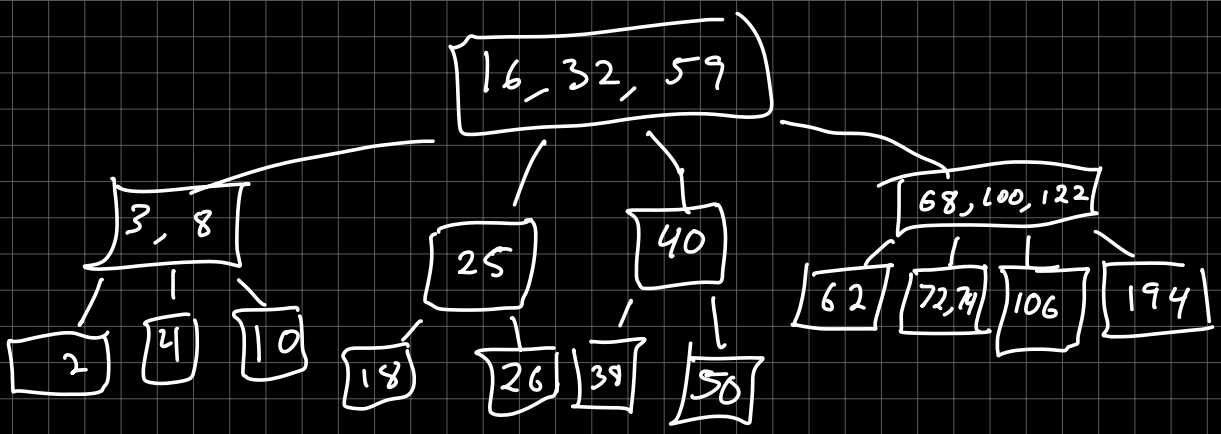


b: insert 106

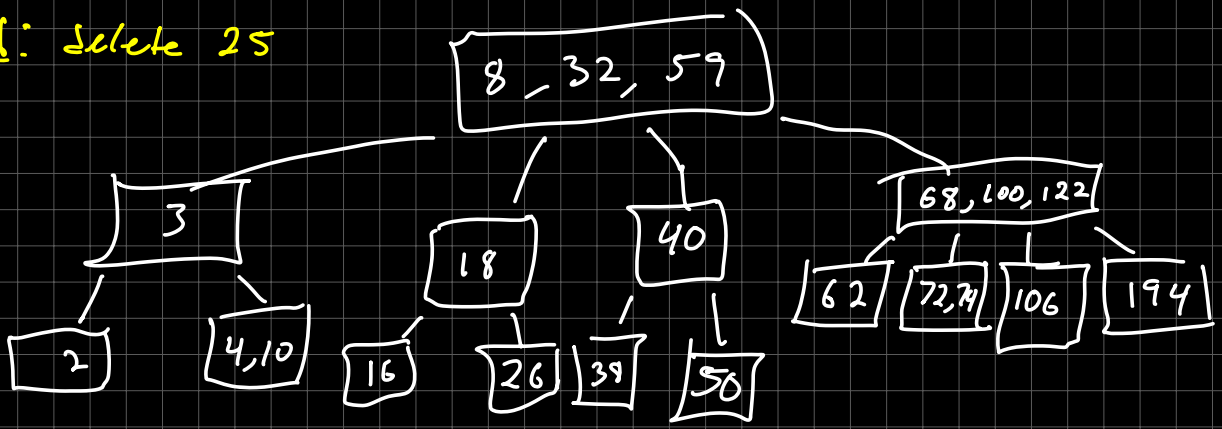


c: Insert 74

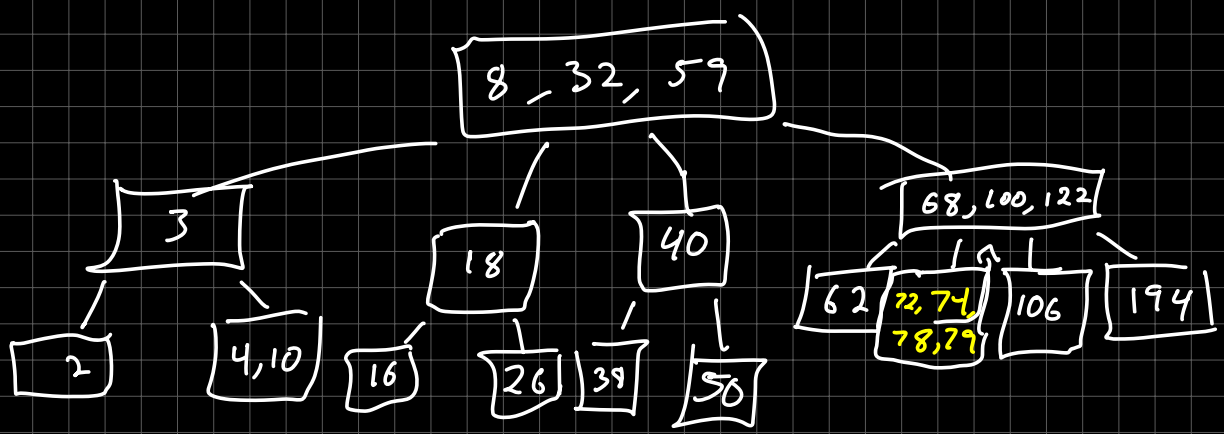


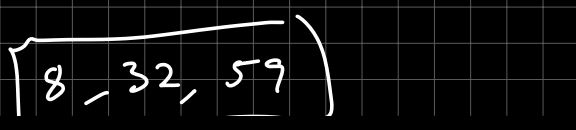


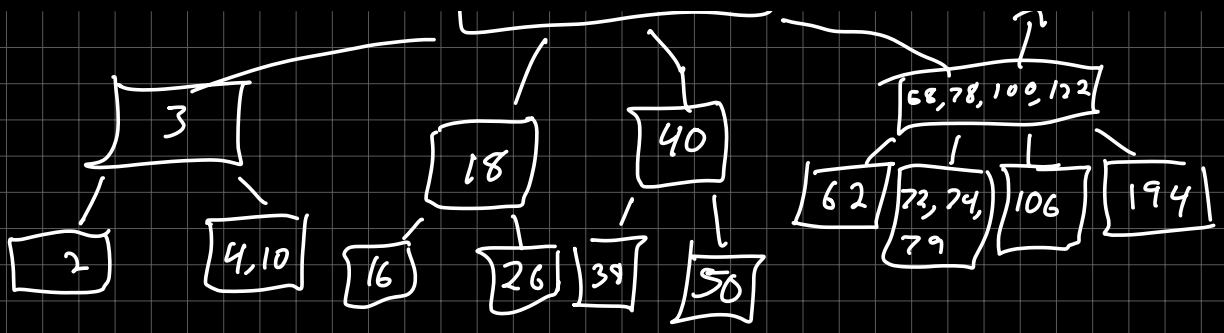
d: delete 25



e & f: insert 78, 79



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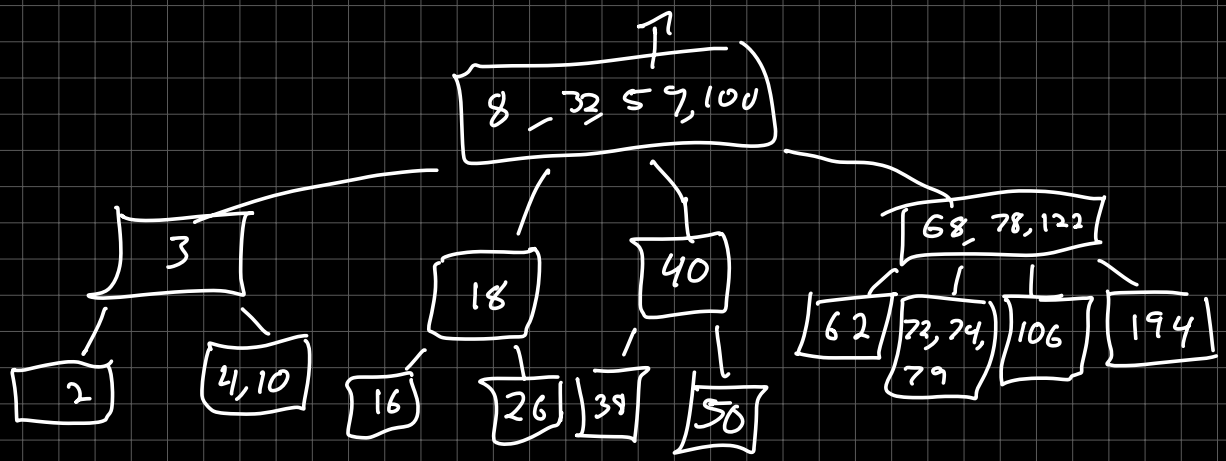
68,78,100,122
2
4,10
16
26
39
50
62
72,74,
79
106
194


8,32,57,100
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68,78,122
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4,10
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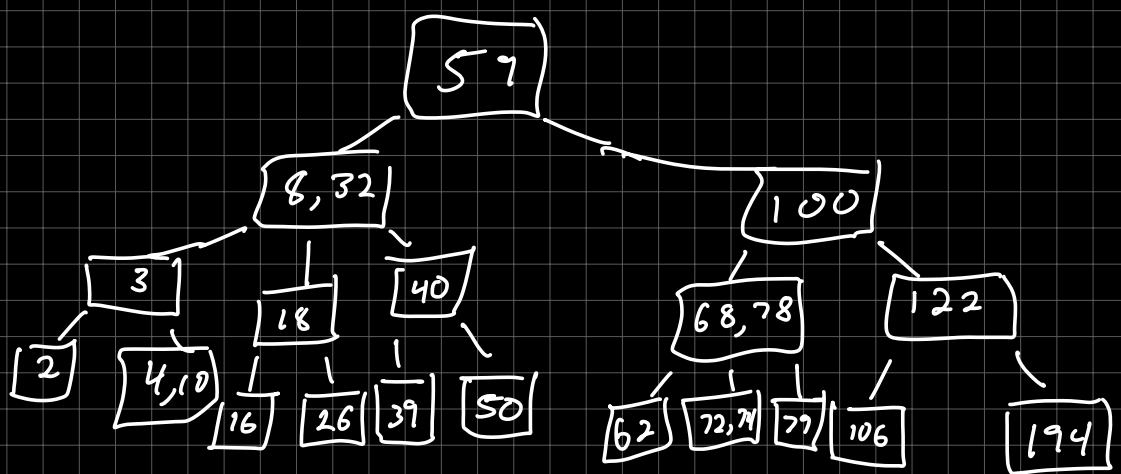
62
72,74,
79
106
194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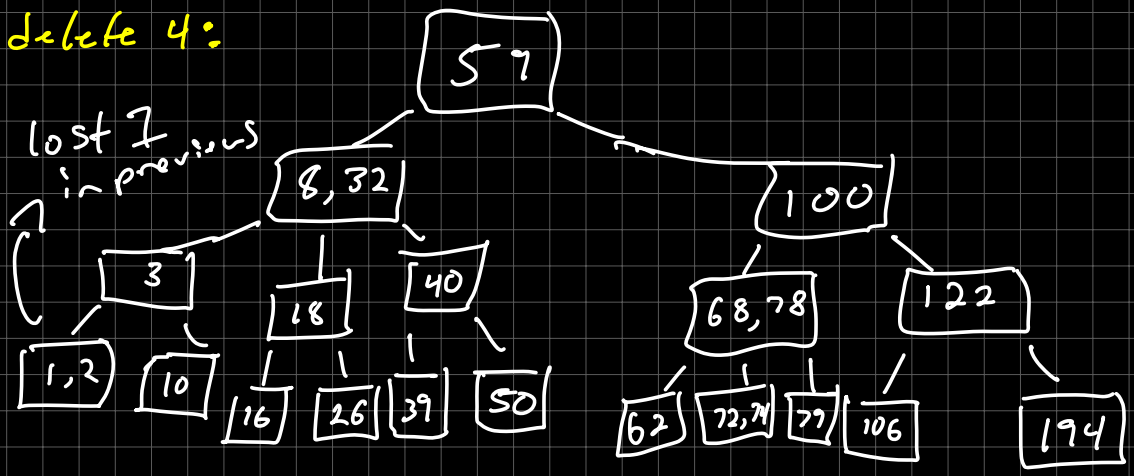
106
194

delete 4:



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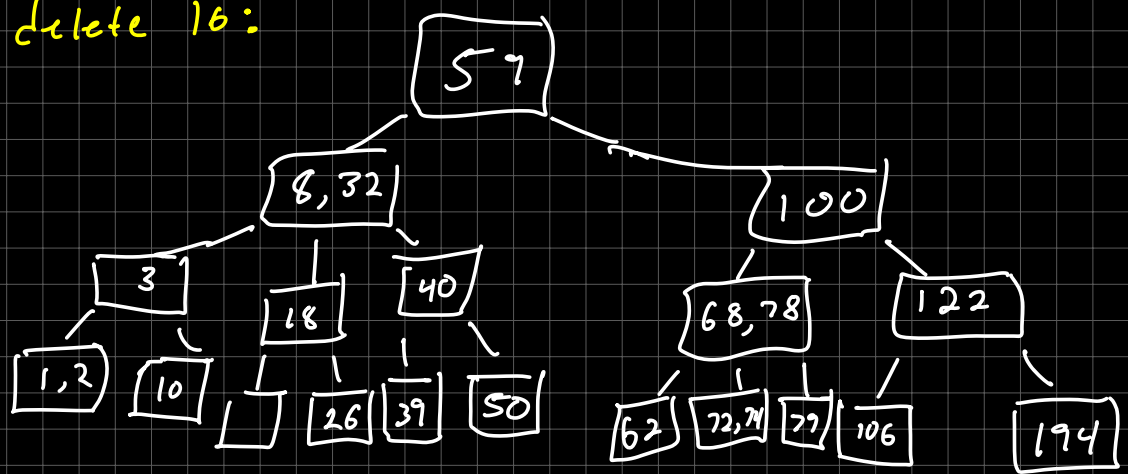
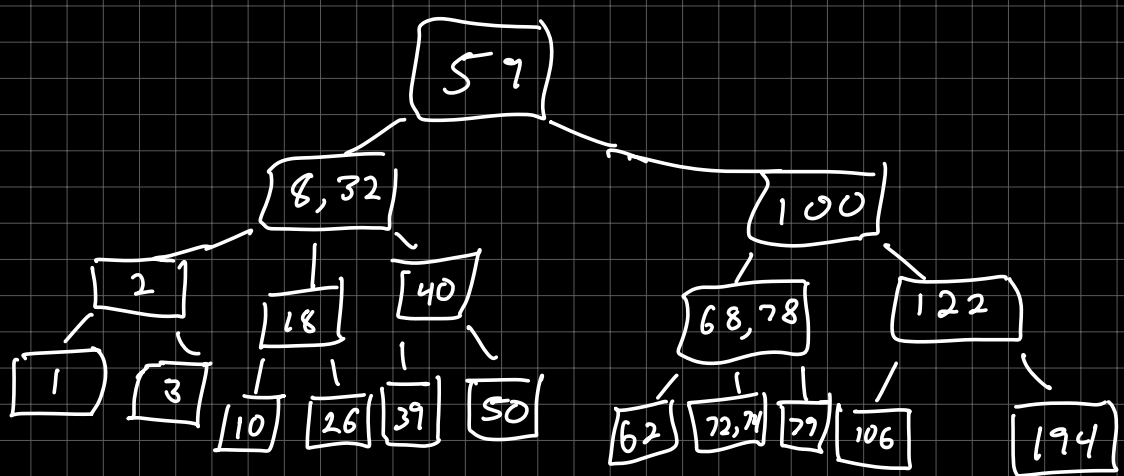




1 2 3 4 inserts

3

1 2 4

5 total

$\log_2 4 = 2$

$2^2 = 4$

$\log_2(n+1) \geq \log_2(n)$

$\log_2(n+1) = \log_2\left(n\left(1+\frac{1}{n}\right)\right)$

$\log_2 n + \log_2\left(1+\frac{1}{n}\right)$

assume n is large

$* \frac{1}{\infty} = 0$

$\lim_{n\to\infty} \log_2 n + \lim_{n\to\infty} \log_2\left(1+\frac{1}{n}\right)$

$= \lim_{n\to\infty} \log_2 n + \log_2(1)$

$= \lim_{n\to\infty} \log_2 n + [2^x = 1\ ?] = 0$

$= \lim_{n\to\infty} \log_2 n + 0 = \lim_{n\to\infty} \log_2 n \to \infty$

this grows properly

their way

$\log_2 n \leq \log_2 2n$ for all n

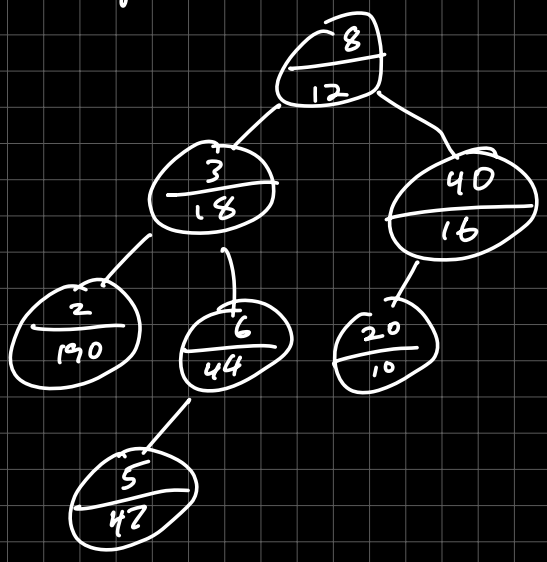
Build a int up from 1 to 99

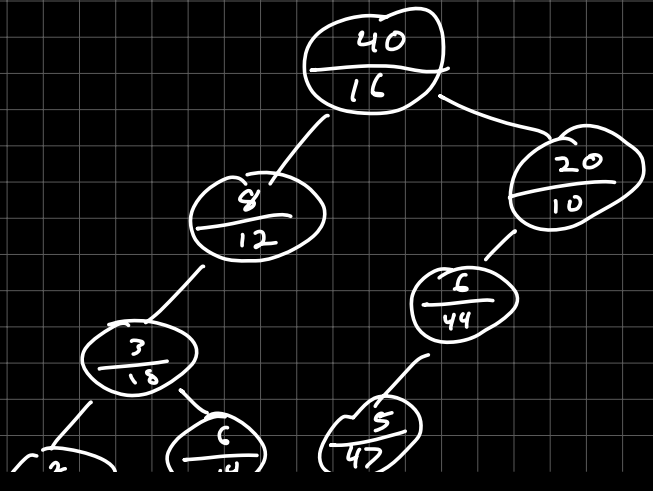
$\frac{1}{2}$ are 1 $\frac{1}{4}$ are 2 $\frac{1}{8}$ are 3

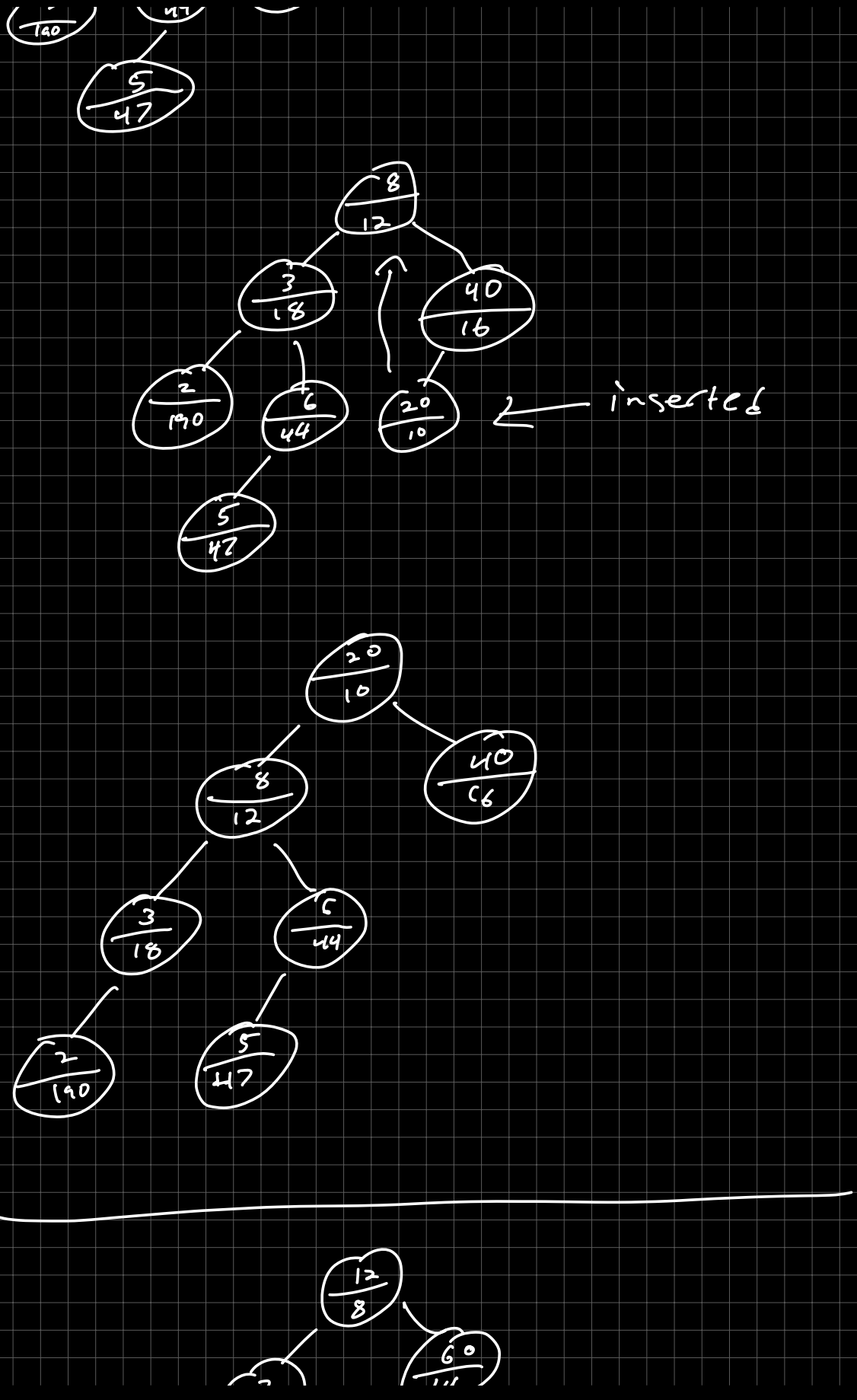
$\frac{1}{2^1}$ $\frac{1}{2^2}$ $\frac{1}{2^3}$

$\frac{1}{2^{99}}$

Treaps:



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190
6
44
20
10
inserted
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8
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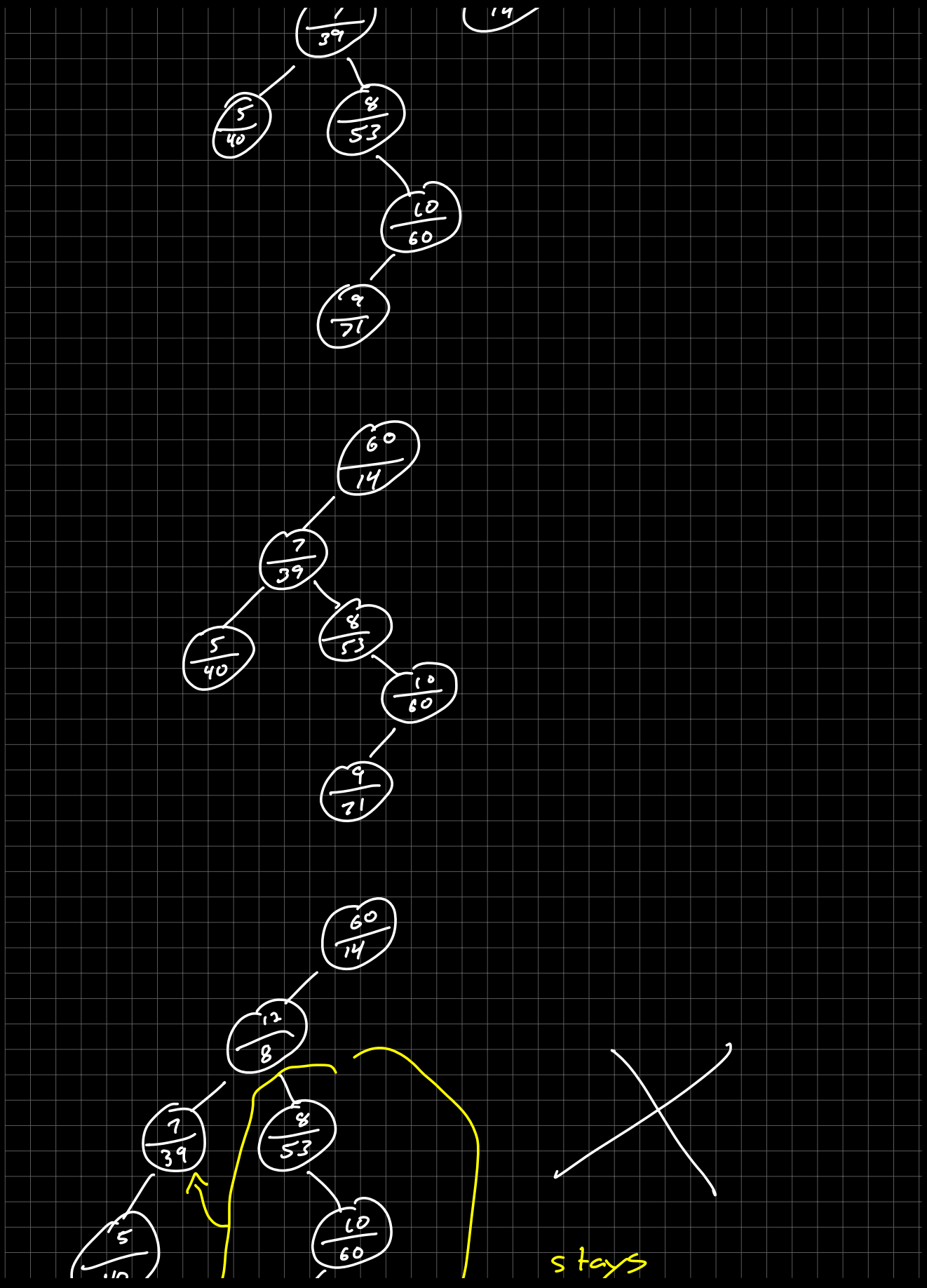
C6
3
18
C
44
2
190
5
47
12
8
60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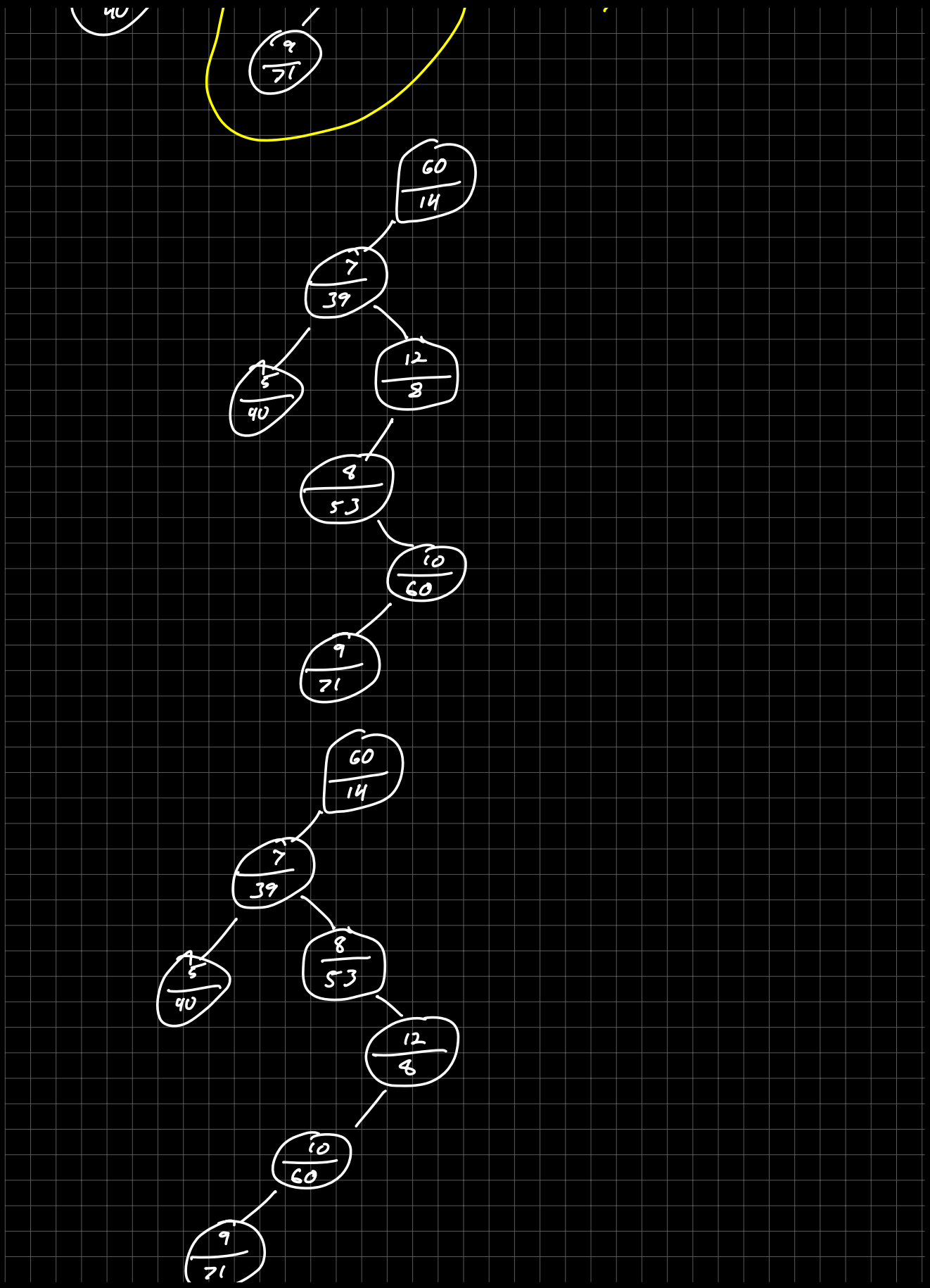
stays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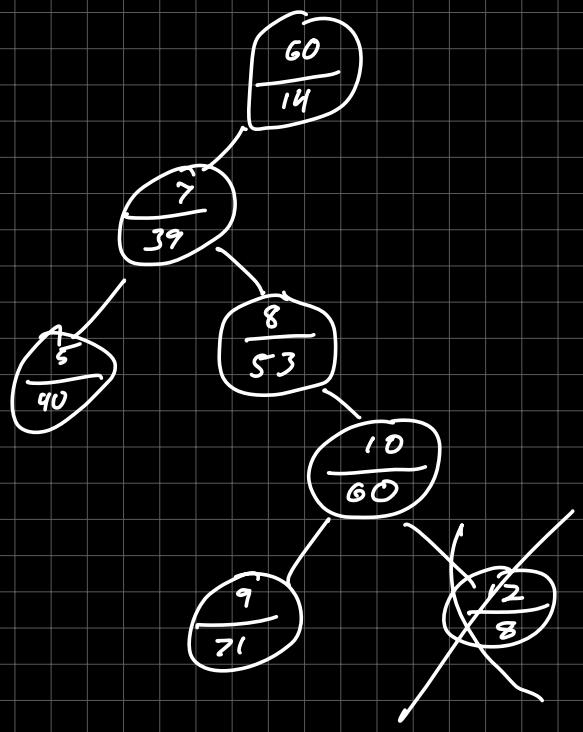
9
71



$T(n) = C n^2$ if $O(n^2)$

Something takes 4 secs to run with 1000 n then

$4 = C(1000)^2$

therefore $C = \frac{4}{1000^2}$

$f(n) = n!$ prove $O(n^n)$

$n! \leq n^n + n$

$$\frac{n!}{n^1+n} = \frac{n(n-1)!}{n(n^{n-1}+1)} = \frac{(n-1)!}{n^{n-1}+1}$$

$$= \frac{(n-1)(n-2)!}{n\left(n^{n-2}+\frac{1}{n}\right)}$$

⟵ this will reduce every factor

⟵ this is approaching infinity quicker

for $n \geq 1$

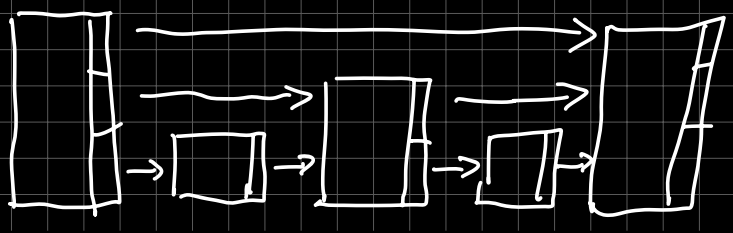
$$O(n^{n+1}) = O(n^n)$$

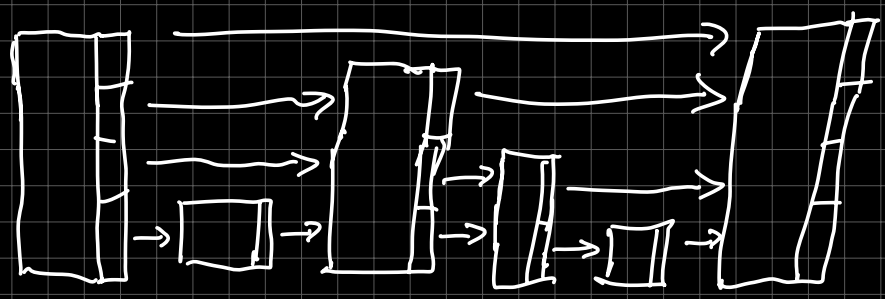
$$n^{n+1} \leq n^n$$

$$n^n \leq n^n$$

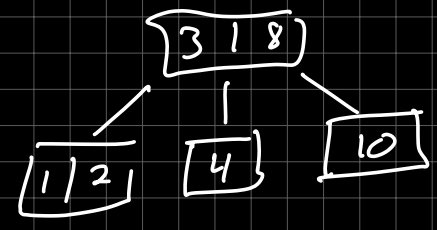
can't multiply by constant n since n is not a constant

$$\log_a b = \frac{\log_c b}{\log_c a}$$

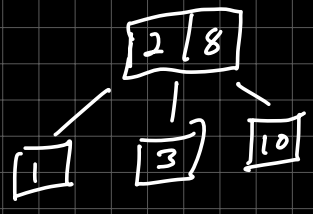




probablistic Skip lists are prefered due to benefit of better chance of run times vs deterministic. The random nature would allow for a larger "skip" within the list to get to desired value. Probablistic is also easy to "re-distribute"/"re assign" heights since you raise the height of the first node + last + coin flip the remaining old max height nodes



delete 4



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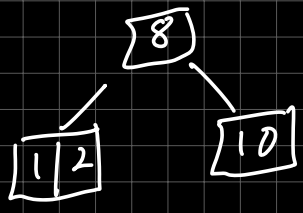

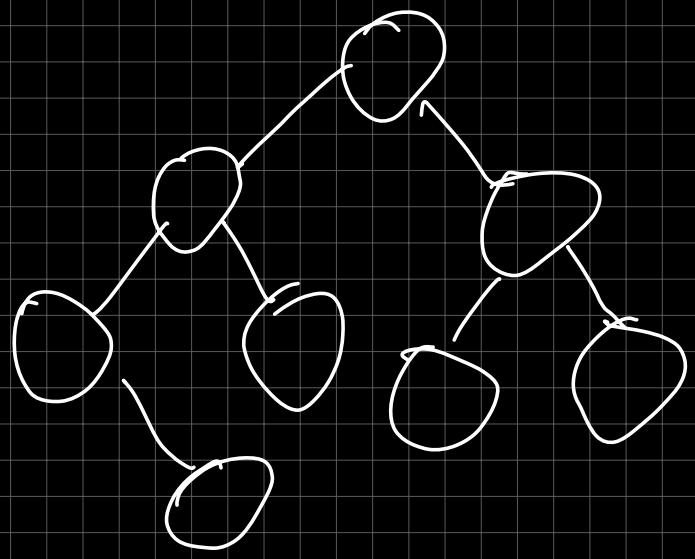

$$\log_{13} n = \frac{\log_2 n}{\log_2 13}$$

$$\sum_{i=1}^{n} 2i = \frac{2(n)(n+1)}{2} = n^2 + n$$ final value

runtime = $\log(n)$

$n = 4$

$\sum_{i=1}^{n}$

$\sum_{i=1}^{n} 2i$

i	x
1	0
2	1
4	2

$\log n$

$n = 8$

i	x
1	0
2	1
4	2
8	3

$$\sum_{i=0}^{n-1} 1 = \sum_{i=1}^{n} (1) = n \text{ times}$$

$$\sum_{i=}$$