1.

a) $O(n^3)$

b) $O(n^3)$

c) $O(n \log n)$

d) $O(n)$

e) $O(n)$

f) $O(1)$

2. $O(1)$

3. $O(100(\log n + n)) \Rightarrow O(100n) \Rightarrow O(n)$

4. $O(100(\log(18) + 13))$

$\Rightarrow O(100 \cdot 13) \Rightarrow O(1)$

constant runtime

5. $\sum_{i=0}^{99} \cancel{(\log(18) + 13)} (18+2) = \sum_{i=1}^{100} (20) = 2000$

$\cancel{= 0(\log 18 + 13) + \sum_{i=1}^{100} (\log(18) + 13) = 100(\log(18) + 13)}$

$\cancel{= (100 \log(18) + 1300}$

6. $\sum_{i=2}^{87} (2n+5)$

$= \sum_{i=1}^{86} (2n+5)$

$= 86(2n+5) = 172n + 430$

* you're still adding "2n+5" up the same amount of times + n doesn't change ever ⇒ its a constant

7. $\sum_{i=10}^{K+1} (3i)$

$= \sum_{i=1}^{K+1} (3i) - \sum_{i=1}^{\cancel{10}\ 9} (3i)$

$= \dfrac{(3(K+1)(K+2))}{2} - \dfrac{3(\cancel{10}\ 9)(\cancel{11}\ 10)}{2}$

$3(90) = 270$

$= \dfrac{(3K+3)(K+2)}{2} - \dfrac{\cancel{330}}{2}$

$= \dfrac{3K^2 + 9K + 6}{2} - \cancel{165}\ \dfrac{270}{2}$

$= \dfrac{3K^2 + 9K - \cancel{324}\ 264}{2}$

$$\frac{n(n+1)}{2}$$

8. $$\sum_{i=2}^{n}(101i+3)$$

$$= \sum_{i=1}^{n}(101i+3) - \sum_{i=1}^{1}(101i+3)$$

$$= \frac{101n(n+1)}{2} + 3n - (104)$$

$$= \frac{101n^2 + 101n + 6n}{2} - 104$$

$$= \frac{101n^2 + 107n - 208}{2}$$

$$\frac{n(n+1)}{2}$$

9. $$\sum_{i=2}^{n-1}(101i+3)$$

$$= \sum_{i=1}^{n-1}(101i+3) - \sum_{i=1}^{1}(101i+3)$$

$$= \frac{101(n-1)(n)}{2} + 3(n-1) - 104$$

$$= \frac{101n^2 - 101n}{2} + 3n - 3 - 104$$

$$= \frac{101n^2 - 101n + 6n}{2} - 107$$

$$= \frac{101n^2 - 95n - 214}{2}$$

10.

$$\sum_{i=0}^{n} \sum_{j=0}^{i} \sum_{k=0}^{n-1} (1)$$

$$\frac{n(n+1)}{2}$$

$$= \sum_{i=0}^{n} \sum_{j=0}^{i} \left[(n-1)(x+1)\right]$$

$$= \sum_{i=0}^{n} \left[i(n-1)(x+1)\right]$$

$$= \left[\frac{n(n+1)}{2}(n-1)(x+1)\right]$$

* when $i=0$ the entire result will add 0 so no need to shift & add

$$\sum_{i=0}^{n} \sum_{j=0}^{i} \sum_{k=0}^{n-1} (1) = \sum_{i=0}^{n} \sum_{j=0}^{i} \sum_{k=1}^{n} (1)$$

$$= \sum_{i=0}^{n} \sum_{j=0}^{i} n \quad = \sum_{i=0}^{n} \sum_{j=1}^{i+1} n$$

$$\boxed{\frac{n(n+1)}{2}}$$

$$= \sum_{i=0}^{n} (i+1)(n) \quad = n \sum_{i=0}^{n} (i+1)$$

$$= n \sum_{i=1}^{n+1} (i-1+1) \quad = n \sum_{i=1}^{n+1} i$$

$$= n \left[\frac{(n+1)(n+1+1)}{2} \right] \quad = \frac{n(n+1)(n+2)}{2}$$

$$= \frac{(n^2+n)(n+2)}{2} = \frac{n^3+2n^2+n^2+2n}{2}$$

$$= \frac{n^3+3n^2+2n}{2}$$

11. $O(n^3)$

$$299 = 500 - 201$$

12. $\sum_{i=0}^{n} \sum_{j=0}^{i} \sum_{K=201}^{500} (1)$

$$= \sum_{i=0}^{n} \sum_{j=0}^{i} \sum_{K=1}^{300} (1) = \sum_{i=0}^{n} \sum_{j=0}^{i} (300)$$

$$= \sum_{i=0}^{n} \sum_{j=1}^{i+1} (300) = \sum_{i=0}^{n} 300(i+1)$$

$$= 300 \sum_{i=1}^{n+1} (i-1)+1 = 300 \sum_{i=1}^{n+1} i$$

$$= \frac{300(n+1)(n+2)}{2} = \frac{300(n^2+3n+2)}{2}$$

$$= 150(n^2+3n+2) = 150n^2+450n+300$$

13. line 6 execution number:

$$\sum_{i=0}^{n-1} (1) = \sum_{i=1}^{n-1+1} (1) = \sum_{i=1}^{n} (1) = \boxed{n \text{ times}}$$

value of x at end:

// note $x = x + (2*i) + (3*n)$

$\Rightarrow x += (2*i) + (3*n)$

$$\sum_{i=0}^{n-1} [2i+3n] = \sum_{i=1}^{n} [2(i-1)+3n]$$

$$= \sum_{i=1}^{n} (2i-2+3n)$$

$$= \frac{2(n)(n+1)}{2} - 2n + 3n^2$$

$$= n(n+1) - 2n + 3n^2 = n^2 + n - 2n + 3n^2$$

$$\boxed{= 4n^2 - n}$$

14. j doesn't get reset

$O(n) \rightarrow$ best $O(2n)$ worst $\Rightarrow O(n)$

15. $O(\log n)$

```
int enigmatic_foo(int n)
{
   int i = 1, x = 0;

   while (i <= n)
   {
      i *= 2;  // same as writing i = i * 2
      x++;
   }
   return x;
}
```

$n = 6$

1st: $i = 2$ $x = 1$

2nd: $i = 4$ $x = 2$

3rd: $i = 8$ $x = 3$

$2^3 = 8$

$n = 10$

1st: $i = 2$ $x = 1$

2nd: $i = 4$ $x = 2$

3rd: $i = 8$ $x = 3$

4th: $i = 16$ $x = 4$

$\log n$

$2^4 = 16$

16. $\log_2 13 = \log_3 x$

$\cancel{x = 3^{\log_2 13}}$

$$\log_a b = \frac{\log_c b}{\log_c a}$$

so

$$\log_2 13 = \frac{\log_3 13}{\log_3 2}$$

17. $T(n) = \log_{13} n$

$$\log_{13} n = \frac{\log_2 n}{\log_2 13} = \log_2\left(\frac{n}{13}\right)$$

$$O\left(\log_2\left(\frac{n}{13}\right)\right) = O(\log_2(n))$$

Redo

3)

$O(n)$

4) $O(1)$

5) $\sum_{i=0}^{100} 18+2 = \sum_{i=0}^{100} 20 = 200$

6) $\sum_{i=2}^{87}(2n+5)$

$\sum_{i=1}^{87}(2n+5) - \sum_{i=1}^{1}(2n+5)$

$$\begin{array}{r} {}^{+1} \\ 87 \\ \times\ 2 \\ \hline 174 \end{array}$$

$87(2n+5) - (2n+5)$

$174n + 435 - 2n - 5$

$= 172n + 430$

* could just do $\sum_{i=1}^{86} 2n+5$

7) $\sum_{i=10}^{k+1} 3i = \sum_{i=1}^{k+1} 3i - \sum_{i=1}^{9} 3i$

$3\left[\frac{(k+1)(k+2)}{2} - \frac{9(10)}{2}\right]$

$= 3\left[\frac{k^2+3k+2}{2} - \frac{90}{2}\right]$

$= 3\left[\frac{k^2+3k-88}{2}\right]$

8) $O(n)$

$$\sum_{i=2}^{n}(101i+3)$$

$$= \sum_{i=1}^{n}(101i+3) - \sum_{i=1}^{1}(101i+3)$$

$$= \sum_{i=1}^{n}101i + \sum_{i=1}^{n}3 - \sum_{i=1}^{1}101i - \sum_{i=1}^{1}3$$

$$= \sum_{i=1}^{n}101i - \sum_{i=1}^{1}101i + \sum_{i=1}^{n}3 - \sum_{i=1}^{1}3$$

$$= 101\left[\sum_{i=1}^{n}i - \sum_{i=1}^{1}i\right] + 3\left[\sum_{i=1}^{n}1 - \sum_{i=1}^{1}1\right]$$

$$101\left[\frac{(n)(n+1)}{2} - 1\right] + 3\left[n-1\right]$$

$$= \frac{101n^2 + 101n - 202}{2} + 3n - 3$$

$$= \frac{101n^2 + 101n + 6n - 202 - 6}{2}$$

$$= \frac{101n^2 + 107n - 208}{2}$$

$$\sum_{i=2}^{n} (101i + 3)$$

$$= \sum_{i=1}^{n} (101i+3) - (101 + 3)$$

$$\frac{101(n)(n+1)}{2} + 3n - 104$$

$$\frac{101n^2 + 101n}{2} + 3n - 104$$

$$= \frac{101n^2 + 101n + 6n - 208}{2}$$

$$= \frac{101n^2 + 107n - 208}{2}$$

8) original method

$$\sum_{i=2}^{n} (101i + 3)$$

$$= \sum_{i=1}^{n-1} (101(i+1) + 3)$$

$$= \sum_{i=1}^{n-1} (101i + 101 + 3)$$

$$= \sum_{i=1}^{n-1} (101i + 104) = \frac{101(n-1)(n)}{2} + 104(n-1)$$

$$= \frac{101(n^2-n)}{2} + 104n - 104$$

$$= \frac{101n^2 - 101n}{2} + 104n - 104$$

$$= \frac{101n^2 - 101n + 208n - 208}{2}$$

$$= \frac{101n^2 - 107n - 208}{2}$$

9) $\sum_{i=2}^{n-1} 101i + 3 = \sum_{i=1}^{n-2} 101(i+1) + 3$

10) $\sum_{i=0}^{n} \sum_{j=0}^{i} \sum_{k=0}^{n-1} (1)$

$$= \sum_{i=0}^{n} \sum_{j=0}^{i} \sum_{K=1}^{n} (1) = \sum_{i=0}^{n} \sum_{j=0}^{i} n = \sum_{i=0}^{n} \sum_{j=1}^{i+1} n$$

$$= \sum_{i=0}^{n} n(i+1) = \sum_{i=0}^{n} ni + n$$

$$= \sum_{i=1}^{n+1} (n(i-1) + n) = \sum_{i=1}^{n+1} (ni - n + n)$$

$$= n \sum_{i=1}^{n+1} i = \frac{n(n+1)(n+2)}{2} = \frac{(n^2+n)(n+2)}{2}$$

$$= \frac{n^3 + 3n^2 + 2n}{2}$$

$500 - 200 = 300$

11) n^3

12) $\sum_{i=0}^{n} \sum_{j=0}^{i} \sum_{K=201}^{500} (1) = \sum_{i=0}^{n} \sum_{j=0}^{i} \sum_{K=1}^{300} 1 = \sum_{i=0}^{n} \sum_{j=0}^{i} 300$

$$= \sum_{i=0}^{n} \sum_{j=1}^{i+1} 300 = \sum_{i=0}^{n} 300i + 300 = \sum_{i=1}^{n+1} 300(i-1) + 300$$

$$= \sum_{i=1}^{n+1} 300i - 300 + 300 = \sum_{i=1}^{n+1} 300i = \frac{300(n+1)(n+2)}{2}$$

$$= \frac{300(n^2+3n+2)}{2} = 150(n^2+3n+2)$$

13) $\sum_{i=0}^{n-1} 1 = \sum_{i=1}^{n} 1 = n$ times

$$x = \sum_{i=0}^{n-1} 2i + 3n = \sum_{i=1}^{n} 2(i-1) + 3n = \sum_{i=1}^{n} 2i - 2 + 3n$$

$$= \frac{2(n)(n+1)}{2} - 2n + 3n^2$$

$$= n^2 + n - 2n + 3n^2 = 4n^2 - n$$

14) worst case

$$\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (1)$$

$$\sum_{i=0}^{n-1} n = n^2$$

j & i dont set reset to 0

so O(n) is worst case

& O(n) is best