Comparing World Quality of Life Measures: Parametric vs. Nonparametric Approaches

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Abstract

BACKGROUND Increasing globalization has generated interest in comparing countries by key quality of life (QoL) measures such as life expectancy, gender equality, and happiness, among others. When considering how countries compare by QoL, it is vital to understand how the measures are related, if at all, and in what ways the statistical tests chosen affect the results. METHODS A dataset containing country names and selected QoL measures for 2016 was explored through descriptive statistics and sensible univariate and bivariate visualizations. Six sets of hypotheses pertaining to relationships within the data were generated and tested with both nonparametric methods and their parametric equivalents, and results of these paired analyses were compared. RESULTS Text goes here CONCLUSION text goes here

Background

With increasing economic globalization, a natural topic of interest is how the world's nations compare with respect to quality of life (QoL). Several organizations monitor global QoL indicators and report single-dimension or aggregate values for indicators of interest. For example, the World Bank reports Gross Domestic Product (GDP), which is a single-dimension indicator often strongly predictive of QoL in a given country (The World Bank (2018)). Additionally, the World Health Organization reports infant mortality rate, life expectancy at birth, and life expectancy at 60 years of age (World Health Organization (2018b), World Health Organization (2018a)).

Other quality of life measures represent compound scores or indices based on several inputs. For example, the United Nations calculates an annual Human Development Index (HDI), representing the developmental level of each country on a scale of zero to one based on several factors, including life expectancy at birth, years of schooling, and per-capita income (The United Nations Development Programme (2018b)). The HDI also categorizes countries into four levels of development (low, medium, high, and very high). Similarly, the Social Progress Imperative publishes the Social Progress Index (SPI), ranging from 0 to 100, and comprising over 50 dimensions in three broad categories: basic human needs (e.g., nutrition, safety), foundations of wellbeing (e.g., basic knowledge, environmental quality), and opportunity (e.g., personal rights, freedoms) (Social Progress Imperative (2018b)). The World Economic Forum's Global Gender Gap Index reports a gender equality index, scaled from 0-1, based on measurements of gender-related gaps in such dimensions as economic participation, level of education, health and survival, and political offices held (World Economic Forum (2016b)). Finally, the World Happiness Report calculates a score from 0-10 by considering per-capita GDP, healthy life expectancy, social support, freedoms, and perception of corruption, among others (Helliwell, Layard, and Sachs (2018)).

The objective of this analysis is to explore the distributions of and relationships between key QoL indicators using both nonparametric and parametric methods, and to assess the appropriateness of each method used.

Methods

The dataset used in this analysis, titled `alldata`, was generated for the MAT 8790 course (Prioli (2018b)). It consists of country-level variables for calendar year 2016 as described in Table 1.

Table 1. `alldata` dataframe contents.

Source	Variable Name	Description
`countrycode` package	`country`	Country names
Social Progress Imperative (2018a)	`SPI`	Social Progress Index value (scale of 0:100)
The World Bank (2018)	`GDP_USD_2018`	2016 Gross Domestic Product (valued in $US 2018)
The United Nations Development Programme (2018a)	`HDIrank`	Human Development Index ranking
The United Nations Development Programme (2018a)	`HDIindex`	HDI index value (scale of 0:1)
The United Nations Development Programme (2018a)	`HDI_cat`	HDI index category (5 levels)

Source	Variable Name	Description
Helliwell, Layard, and Sachs (2018)	`happiness`	World Happiness Score (scale of 0:10)
World Economic Forum (2016a)	`genderequality_index`	Gender Equality Index (scale of 0:1)
World Health Organization (2018b)	`infantmort`	Infant mortality rate
World Health Organization (2018a)	`birth_MF`	Life expectancy at birth, males & females
World Health Organization (2018a)	`sixty_MF`	Life expectancy at 60 years, males & females

All variables pertain to the calendar year 2016. Missing values were omitted from the dataset to ensure that the tests of interest could be performed.

For each variable except `country`, descriptive statistics were run and a sensible visualization was generated, following which a correlation matrix was produced to examine pairwise relationships between continuous variables.

Analyses

Based on the data exploration results, six sets of formal hypotheses were generated about the data (Table 2), and sensible nonparametric tests and their parametric equivalents were chosen to test these hypotheses.

Table 2. Analyses Performed

Analysis	Variable(s)	Null Hypothesis	Alternative Hypothesis	Nonparametric Test	Parametric Test
1	HDIindex	The sample median is equal to its mean	The sample median differs from its mean	One-Sample Sign Test	One-Sample t-Test
2	HDIindex, SPI	Human development and social progress are not associated	Human development and social progress are correlated	Kendall's Tau	Pearson's Correlation Test
3	logGDP, infantmort	There is no relationship between log(GDP) and infant mortality	There is a relationship between log(GDP) and infant mortality	Hoeffding's Test	Pearson's Correlation Test
4	happiness	Happiness is normally distributed	Happiness is not normally distributed	One-Sample Kolmogorov-Smirnov Test	Shapiro-Wilk Test
5	genderequality_index, SPI	Gender equality and adjusted social progress index have the same median	Gender equality index and adjusted social progress index differ in median	Wilcoxon Rank-Sum Test	Two-Sample t-Test
6	HDI_cat, infantmort	Infant mortality rate is the same across levels of human development	Infant mortality rate differs by level of human development	Permutation F-Test	ANOVA

A two-sided one-sample test was chosen for Analysis #1 because the `HDIindex` distribution is very non-normal, yet its median and mean appear quite similar and the standard deviation is small. The one-sample sign test assumes that the sample is random with independent draws, and the data are continuous. Its parametric equivalent, the one-sample *t*-test, shares these assumptions and also requires normality of the sampling distribution.

Analysis #2 was motivated by the correlation matrix analysis, which indicates a linear relationship is likely between `SPI` and `HDIindx`. Kendall's Tau was chosen as the nonparametric test, with the usual Pearson's test as the parametric alternative. Both tests assume continuous data and are equipped to detect linear dependence, and employed two-sided alternatives.

Correlation tests were chosen for Analysis #3 to determine whether increasing GDP is correlated with decreasing infant mortality. Hoeffding's Test was selected because it is sensitive to any departure from independence. Because the relationship appears to be roughly nonlinear, Pearson's test was chosen as the parametric comparator. Both tests employed two-sided alternatives and assume continuous data, and Pearson's test additionally assumes a linear relationship.

Analysis #4 was chosen because `happiness` appears to have a symmetric, possibly normal distribution on univariate analysis. Both the one-sample Kolmogorov-Smirnov test and the Shapiro-Wilk test assume a continuous sample distribution.

Analysis #5 sought to determine whether there is a difference in median between $\frac{1}{100}\texttt{SPI}$ and `genderequality_index`. Dividing `SPI` by 100 ensures it shares the same 0:1 scale as `genderequality_index`. After this rescaling, the two variables appear to have similar medians on descriptive analysis. The Wilcoxon Rank-Sum test assumes all observations are independent and come from the same distribution under H_0, whereas the two-sample t-test assumes independent samples and normality of the sampling distribution. A two-sided alternative was used for each test.

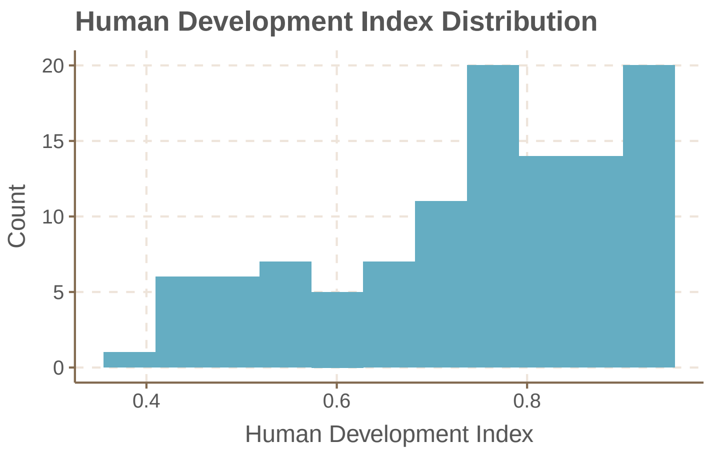
To test for a difference in infant mortality rate by human development category, Analysis #6 employed the permutation F-test and ANOVA as the nonparametric and parametric tests respectively. Both tests assume samples are independent and identically distributed, and ANOVA additionally assumes a normal distribution.

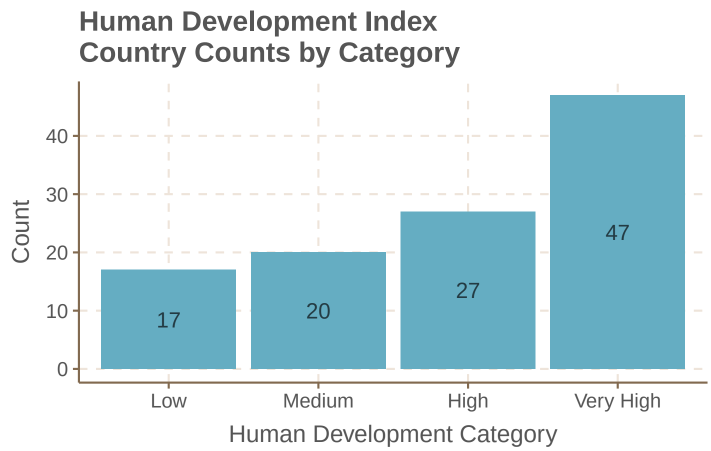
Results of these six paired analyses were compared in the context of the data and assumptions needed. All tests were performed at level $\alpha = 0.05$ in R. The dataset, full code, and this report are available in an online repository (Prioli (2018a)).

Results

Descriptive Statistics and Visualizations

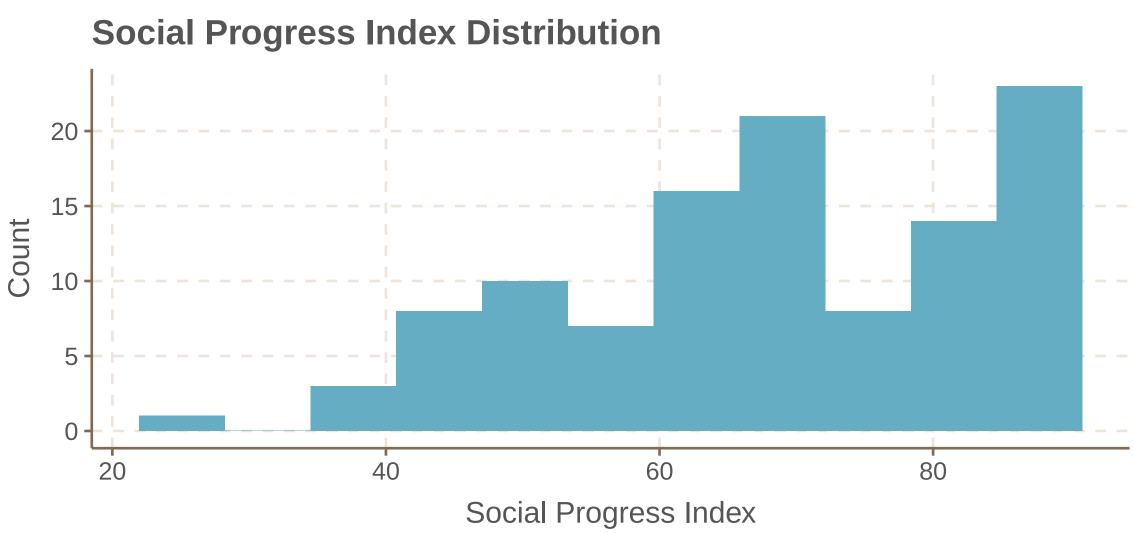
First, exploring the Human Development Index variables:





Min	Q1	Median	Mean	Q3	Max	SD
0.405	0.653	0.772	0.745	0.867	0.951	0.15

Exploring the Social Progress Index data:

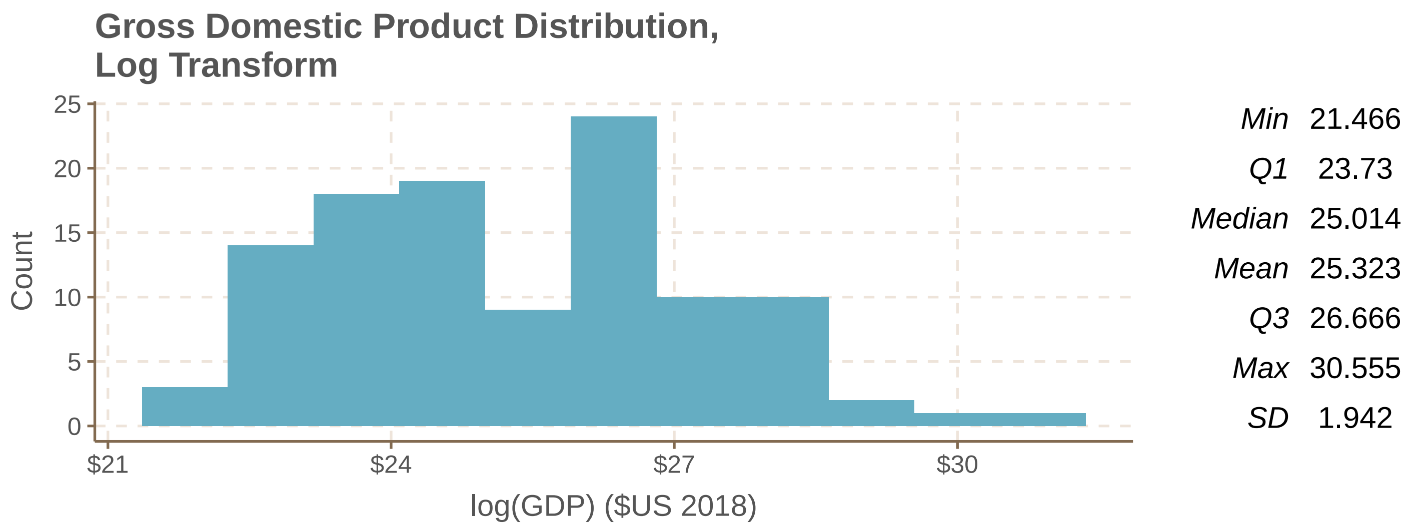


Min	26.92
Q1	58.42
Median	68.94
Mean	68.624
Q3	82.37
Max	89.62
SD	15.381

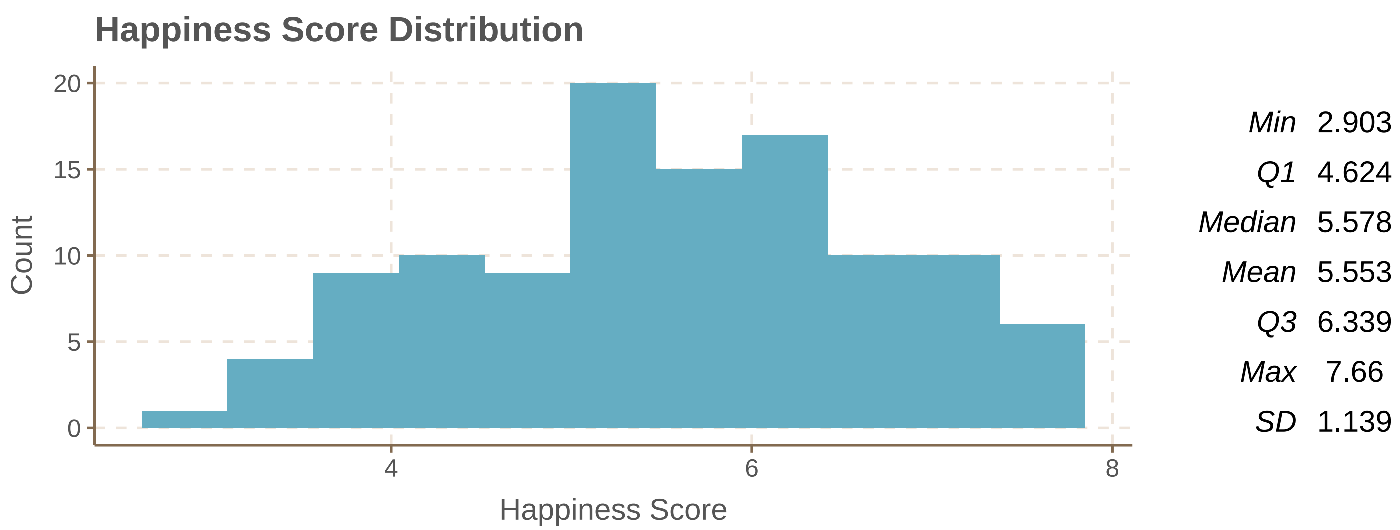
Next, exploring GDP by summary statistics:

Min	Q1	Median	Mean	Q3	Max	SD
2101	20228.99	73000.98	650859.6	380937.5	18624500	2138824

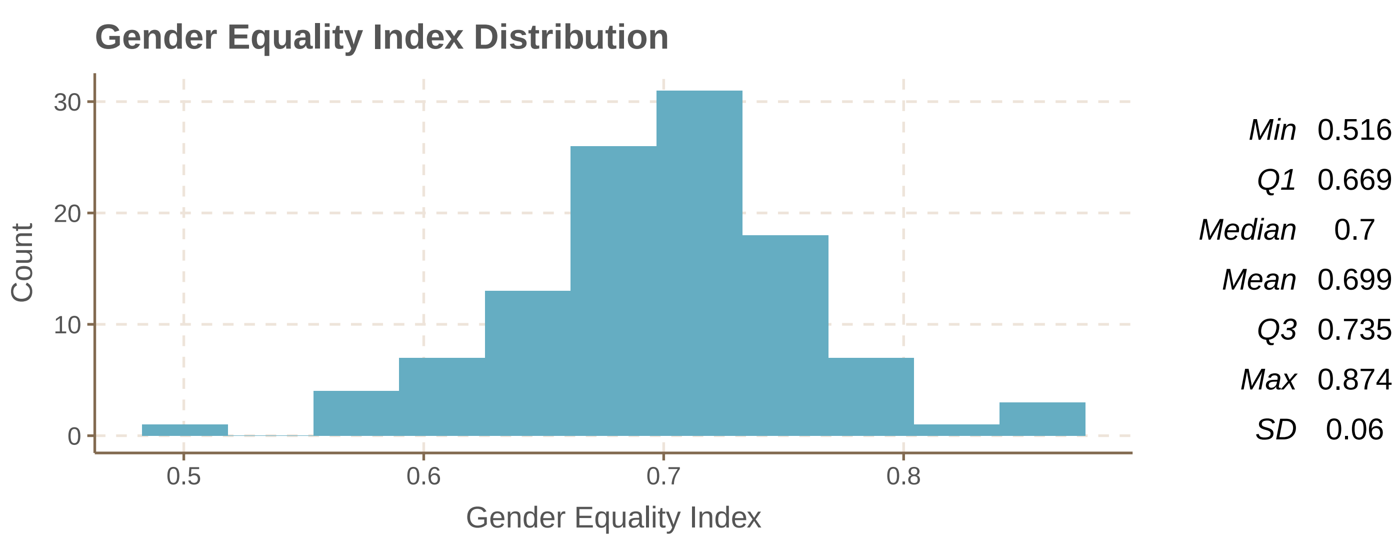
Taking the log transform and plotting:



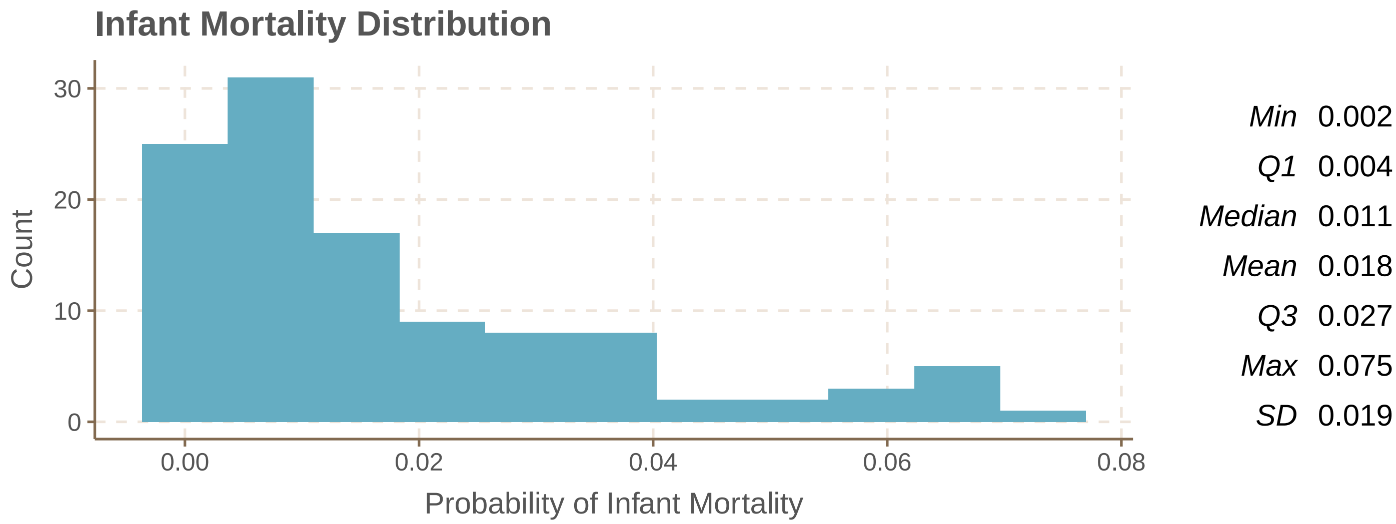
Exploring the World Happiness Report data:



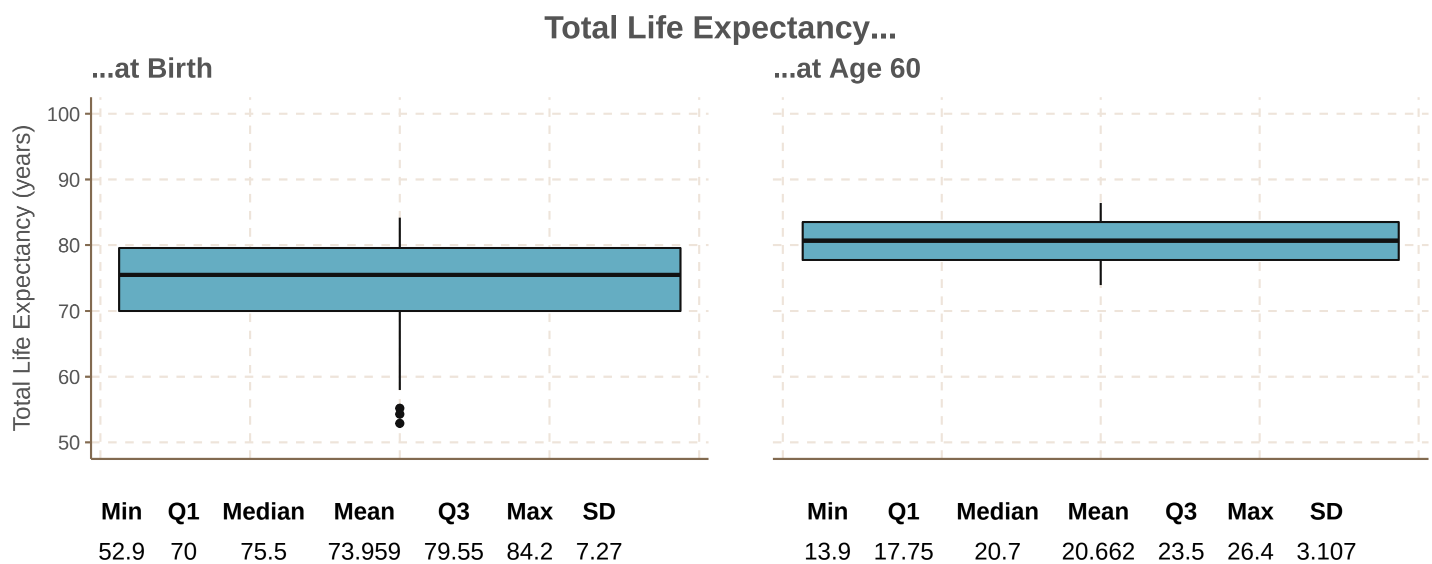
Exploring the gender equality index data:



Exploring the WHO infant mortality rate data:

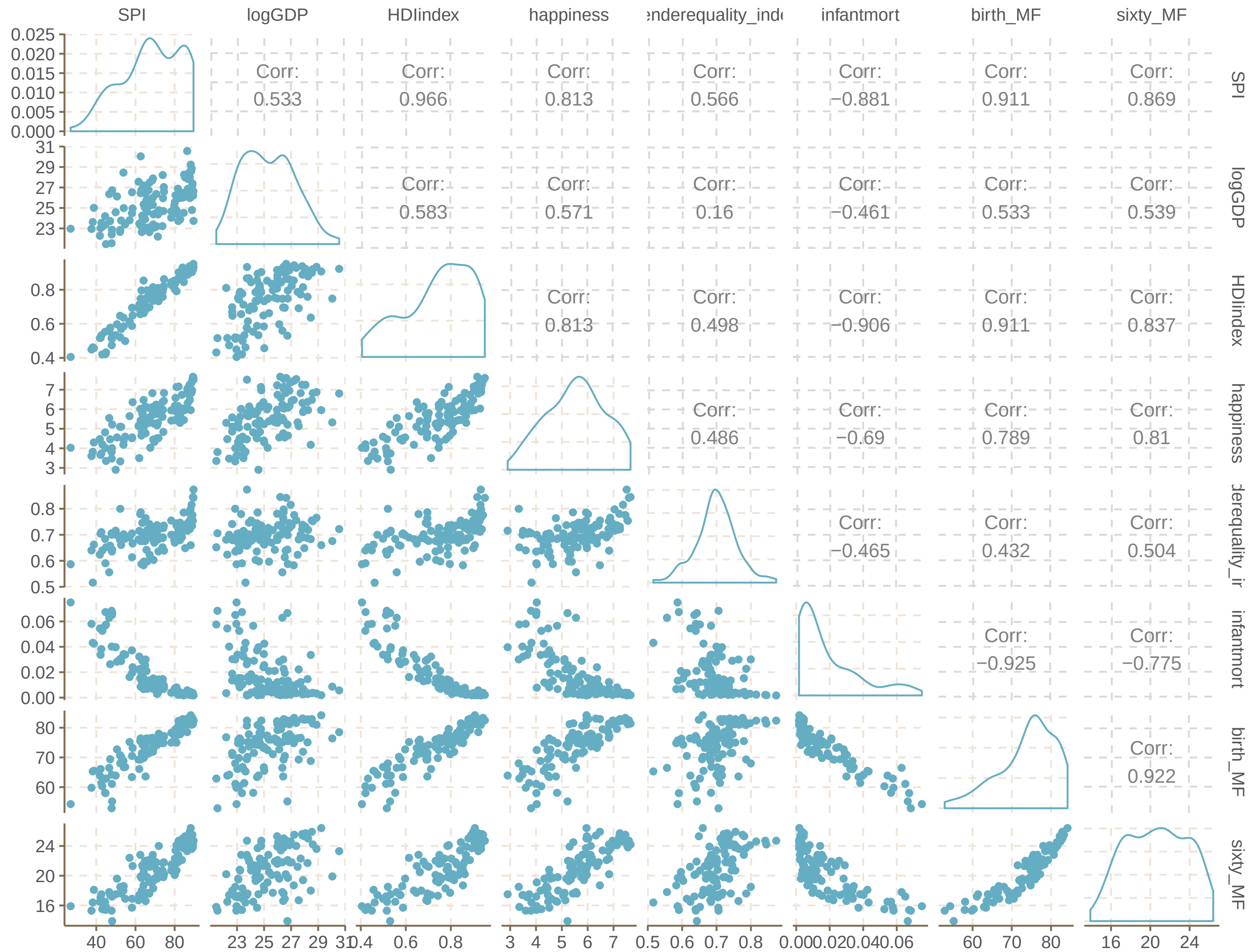


Exploring the WHO life expectancy data:



Investigating pairwise relationships between continuous variables:

Correlation Matrix, Continuous Variables



Strong positive linear relationships are seen between `HDIindex` and `SPI`, `happiness`, and `birth_MF`; between `SPI` and `happiness`, `birth_MF`, and `sixty_MF`; and between `happiness` and `sixty_MF`. Additionally, strong positive relationships that are possibly nonlinear are seen between `HDI_index` and `sixty_MF`, and between `birth_MF` and `sixty_MF`.

Strong negative relationships are seen between `infantmort` and `birth_MF`, between `HDIindex` and `infantmort`, and between `SPI` and `infantmort`, though the latter two of these may not necessarily be linear. A strong negative nonlinear relationship is seen between `infantmort` and `sixty_MF`.

Analysis #1

```r
analysis1_nonpara <- SIGN.test(x = alldata$HDIindex,
                               md = mean(alldata$HDIindex),
                               alternative = "two.sided",
                               conf.level = 0.95)
analysis1_nonpara
```

```
##
##  One-sample Sign-Test
##
## data:  alldata$HDIindex
## s = 67, p-value = 0.03631
## alternative hypothesis: true median is not equal to 0.7445518
## 95 percent confidence interval:
```

```
##  0.7464645 0.8031086
## sample estimates:
## median of x
##   0.7722166
##
## Achieved and Interpolated Confidence Intervals:
##
##                   Conf.Level L.E.pt U.E.pt
## Lower Achieved CI     0.9428 0.7467 0.8021
## Interpolated CI       0.9500 0.7465 0.8031
## Upper Achieved CI     0.9637 0.7461 0.8050
```

```r
analysis1_para <- t.test(alldata$HDIindex,
                         mu = median(alldata$HDIindex),
                         alternative = "two.sided",
                         conf.level = 0.95)
analysis1_para
```

```
##
##  One Sample t-test
##
## data:  alldata$HDIindex
## t = -1.9434, df = 110, p-value = 0.05452
## alternative hypothesis: true mean is not equal to 0.7722166
## 95 percent confidence interval:
##  0.7163408 0.7727629
## sample estimates:
## mean of x
## 0.7445518
```

The one-sample sign test for $H_0 : m =$ `mean(alldata$HDIindex)` versus $H_A : m \neq$ `mean(alldata$HDIindex)` yielded a p-value of $p_{sign} = 0.036$. Since $p_{sign} < \alpha$, the null hypothesis was rejected, leading to the conclusion that the median and mean HDI index values are not equal. However, the parametric test assessed a complementary set of hypotheses $-$ that is, $H_0 : \mu =$ `median(alldata$HDIindex)` versus $H_A : \mu \neq$ `median(alldata$HDIindex)` $-$ and resulted in $p_t = 0.055$. Since $p_t > \alpha$, the null hypothesis was retained with the conclusion that, at the $\alpha = 0.05$ level, there is insufficient evidence to assert that the mean and median are different in the parametric case.

The sign test is appropriate for this data; however, due to the clear nonnormality of the `HDIindex` data, the parametric test is inappropriate because the assumption of normality is violated. Thus the sign test results were retained.

Analysis #2

Analysis #3

Analysis #4

Analysis #5

Analysis #6

Discussion

Limitations

Conclusion

References

Helliwell, John F., Richard Layard, and Jeffrey D. Sachs. 2018. "World Happiness Report." http://worldhappiness.report/ed/2018/.

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