DYNAMICS OF MACHINERY
ME314
COURSE PROJECT

TITLE: DYNAMIC ANALYSIS OF MULTI-STORY BUILDING DUE TO EARTHQUAKE EXCITATION

GROUP MEMBERS:

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CAD MODEL

Matlab Code Defining the model-

```
%Input
N=5; %Number of stories
Ks=1.44e8; %Stiffness
Ms=2.0e8; %Mass of each floor
floor_height = 3;
```

Dimensions:

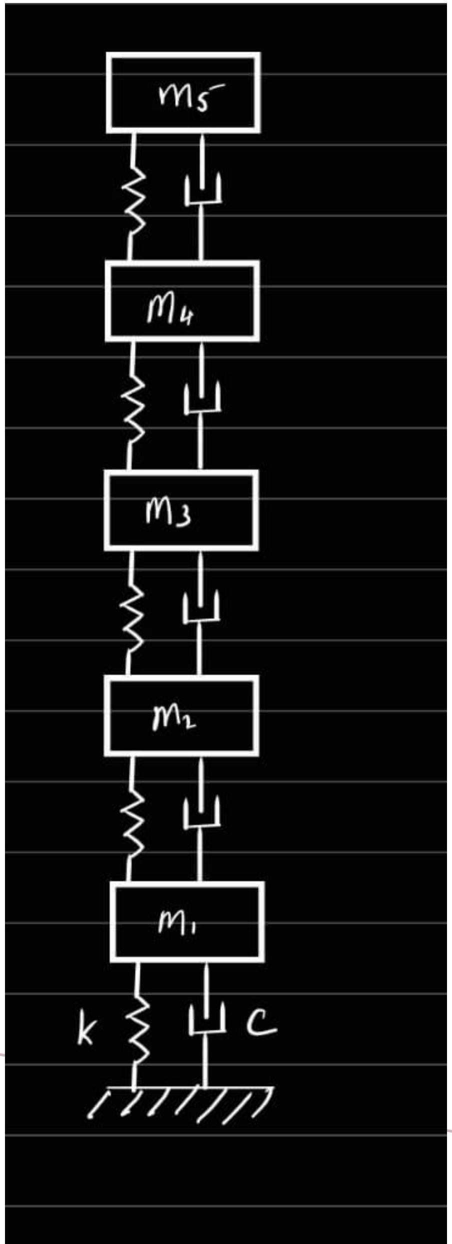
Thickness of each Storey 0.2m;

Height = 0.3m;

Length* breadth = 12m*10m

Reference-https://www.researchgate.net/publication/312274411_Estimation_of_Storey_Stiffness_in_Multi-storey_Buildings

SIMPLIFIED MODEL



Key Features

•Floors as Lumped Masses:

- Each floor's mass is concentrated at a point (center of the slab), ignoring distributed mass.

•Vertical Members as Springs:

- The columns or walls between floors are modeled as springs that only resist *horizontal* motion (shear deformation), not bending.

•Movement Type:

- Only allows each floor to move back and forth (side to side), not rotate or bend—each floor acts as a rigid slab sliding horizontally.

•Equations of Motion:

- The system is described by a set of coupled differential equations, one for each mass/floor, capturing how each interacts with the rest through the springs

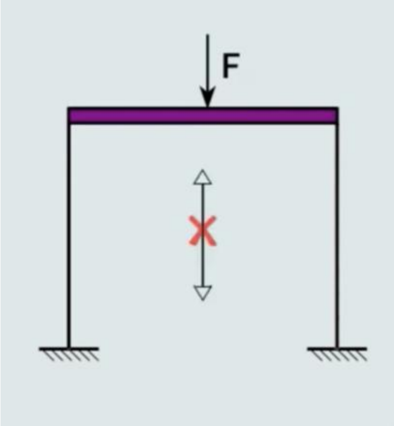
Lumped-mass shear building model

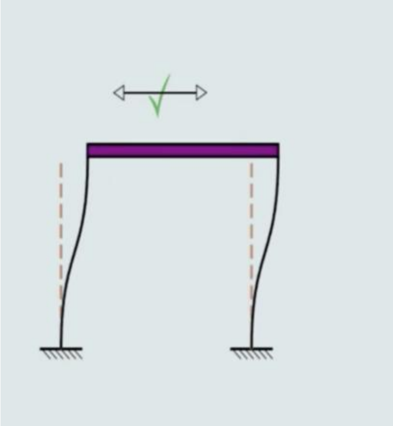
Reference-**Seismic Analysis of Multistorey Reinforced Concrete Structures**

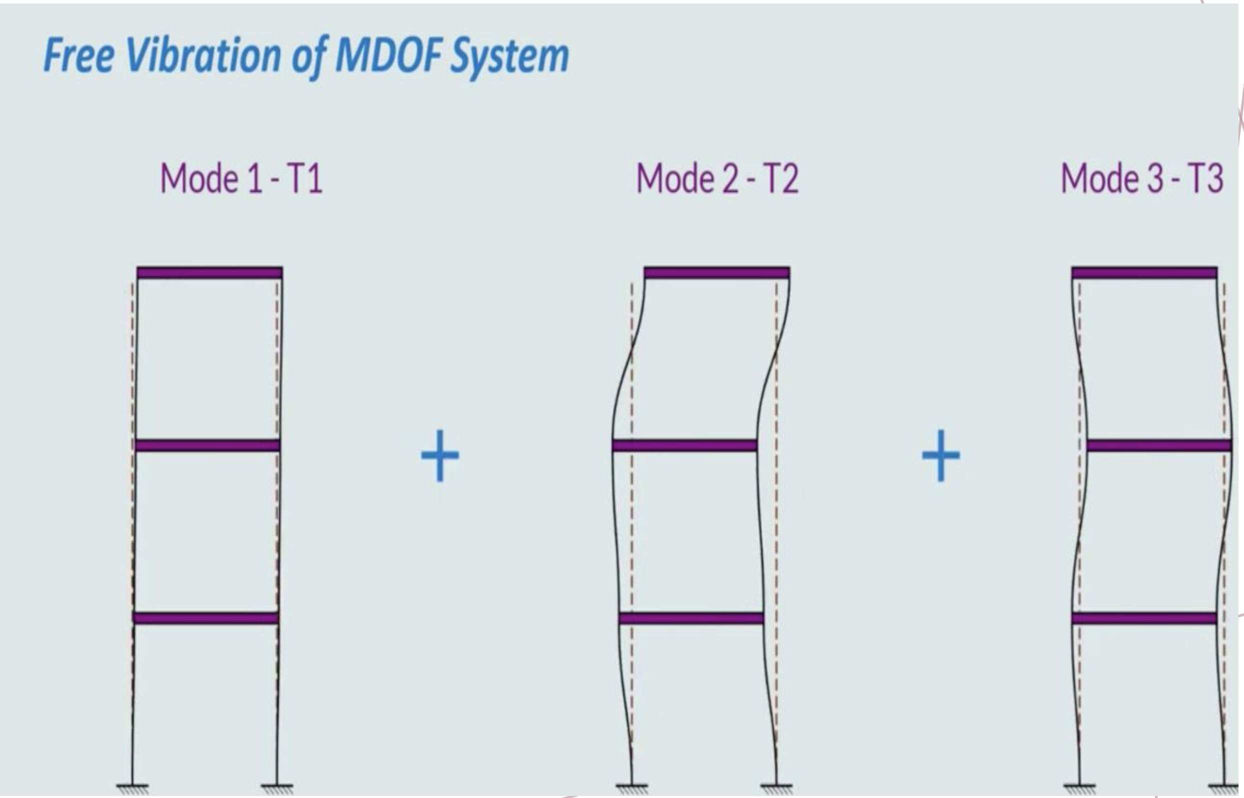
https://imanagerpublications.com/assets/htmlfiles/JCE9%281%29December-February201914592.html

SOME IMPORTANT TERMS

- Shear Frame- Special type of frame used to simplify the dynamic behavior of Single Story and multi story building.
- Doesn’t allow vertical deflections or span rotations, but allows horizontal sway.
- Mode Shape- Each Natural Frequency is associated with a specific shape of vibration known as the mode shape

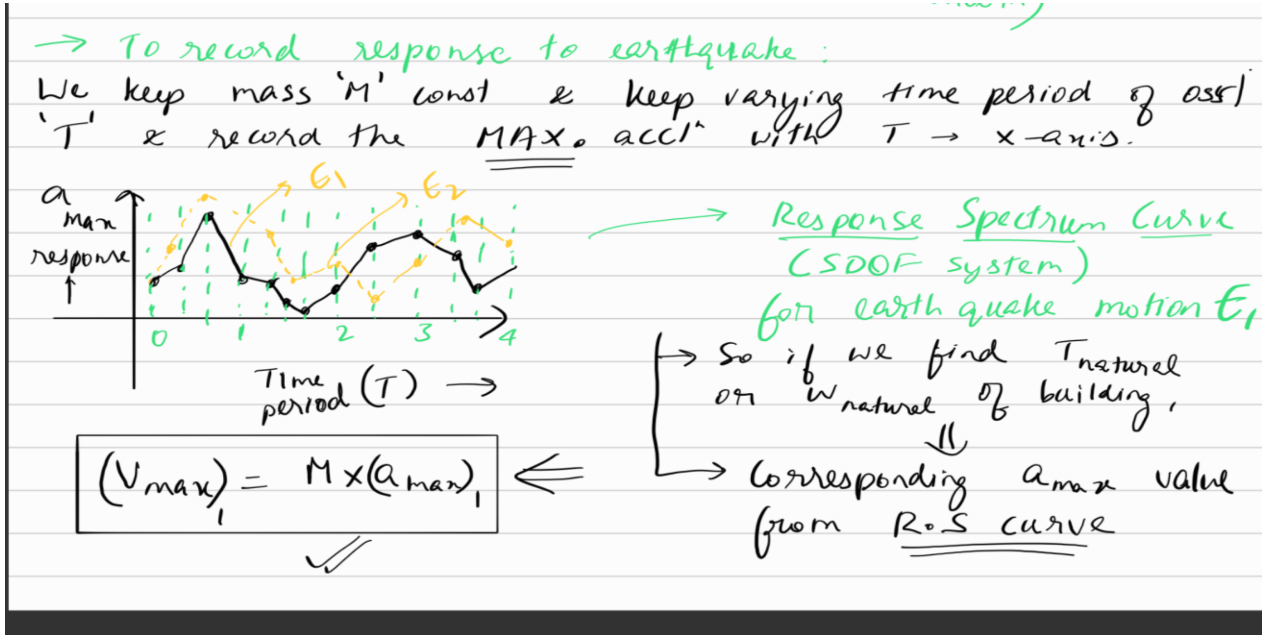


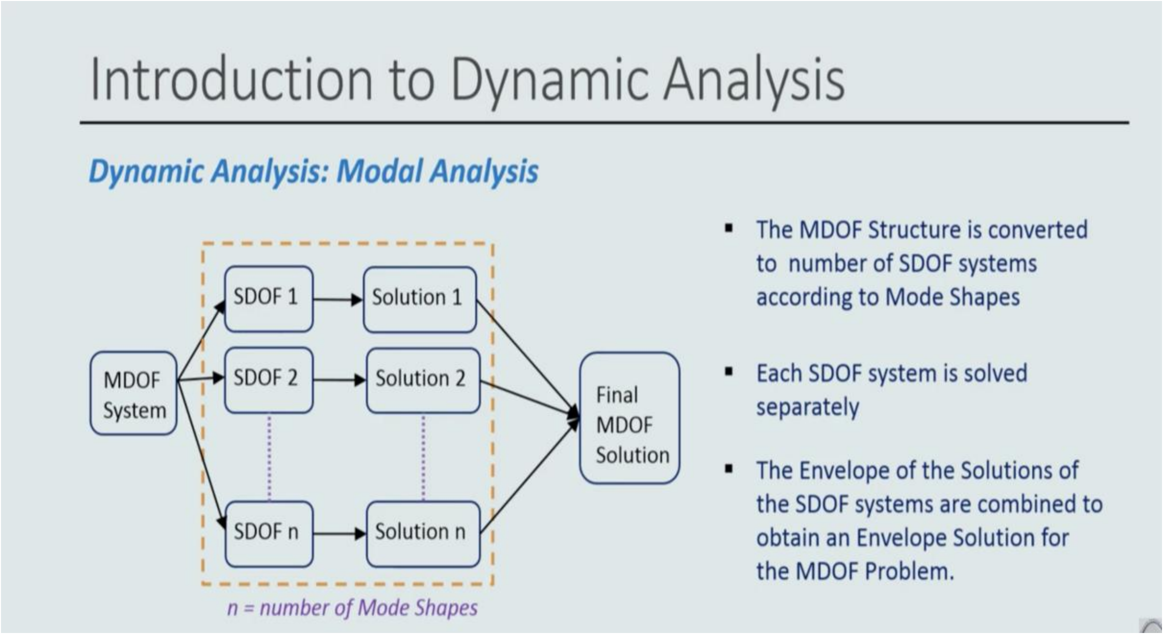




References:-
https://ijret.org/volumes/2018v07/i03/IJRET20180703009.pdf
https://www.iitk.ac.in/nicee/wcee/article/14_K001.pdf
https://ijettjournal.org/archive/ijett-v71i2p247

THEORETICAL BACKGROUND





We have to solve the problem for Multi Degree of freedom system

We will be using the method of Modal Analysis because –

- The solution is fast compared to Time history method
- Note- We will be getting the Approx. solution with max. and min. values , and our solution is limited to Linear problems.

Reference-Chopra, A. K. (2017). Dynamics of Structures: Theory and Applications to Earthquake Engineering (5th Edition). Pearson.

MODAL ANALYSIS – OBTAINING UNCOUPLING MATRICES

MDOF → SDOF 1, SDOF 2, SDOF 3, ⋮ N

SDOF 1 → $S_{a_1} \times \Gamma_1 \times \phi_1$ → mode shape

↳ spectral response (spectral accl^n max.) ; ↳ particip^n factor

Final response : $\sum_{i=1}^{N} S_{a_i} \times \Gamma_i \times \phi_i$ ☆

Modal analysis : Easy to use SDOF methods for MDOF systems also, but based on superpos^n ⇒ only LINEAR analysis

→ Max response of each mode may not occur @ the same time

→ How to combine a_{max} from different mode shapes ??

↳ Sum of Absolutes (Very consv.) ✗

↳ Sq. Root of sum of squares. (SRSS) ✓

⇒ for a mode i : Final Resp (accl^n) = $\sqrt{\sum_{i=1}^{N} (S_{a_i} \Gamma_i \phi_i)^2}$ ☆

→ $\boxed{\Gamma_i = \dfrac{\sum_{j=1}^{N} M_j \times \phi_{ji}}{\sum_{j=1}^{N} M_j \times \phi_{ji}^2}}$: Particip^n factor

{ M_j = mass of story 'j' ; ϕ_j = disp of story 'j' }

→ R_i is obtained from response spectrum curve. Hence, this way $S_{a_{max}}$ is obtained for one mode.

Implementation

→ Classical lumped mass shear building model. — 3 stories.

⇒ 3D → NDOF – 1D lateral system.

→ Mathematical model : $M\ddot{x} + C\dot{x} + Kx = F(t)$

(*) Imp. parameters : Interstory stiffness – K_j

& Rayleigh damping : $\zeta = 0.05 \equiv 5\%$ {Assuming}
→ α, β = —

→ $f_{natural}$, Γ_i, ζ_i, M_i, m_{M_i}, ϕ_i {Mode shape}

[Rel. amplitude of displ. when building vibrates in jth mode (OR) eigenvector corr. to eigenvalue w_j^2 : Natural freq of mode 'j'.]

ϕ_i → how str. deforms ; q_{ij} → how much it deforms @ that instant

↳ Eigenvector of $[K] - w^2[M]$

i.e. : $\boxed{[[K] - w^2[M]].[\phi] = 0}$: $w_j = \sqrt{\lambda_j}$ (eigenvalues)

($f_j = w_j / 2\pi$)

(*) Uncoupling : To uncouple [M] & [K], we use the orthogonality prop, with help of [ϕ] :–

CONVERTING PHY SPACE to MODAL coord.

CODE : Mn = diag(Vectors' * M * Vectors) ; {uncoup modal mass M}

$[\phi]^T[M][\phi] = [M^*]$; $[\phi]^T[K][\phi] = [K^*]$

[coupled] Modal mass matrix ; ↳ Modal stiffness matrix

↳ {uncoupled ≡ diagonalized}

Now, we also need to uncouple [C] ; but can't do it

Reference-Chopra, A. K. (2017). Dynamics of Structures: Theory and Applications to Earthquake Engineering (5th Edition). Pearson.

OBTAINING VELOCITY AND ACCELERATION FROM THE DISPLACEMENT

with $[\phi]$ for any random $[C]$: So use Rayleigh Damping {Not physical law}

$\Rightarrow \boxed{C = \alpha[M] + \beta[K]}$

mass prop. damping coeff — stiffness prop. "

& $\boxed{\zeta_i = \frac{\alpha}{2w_i} + \frac{\beta w_i}{2}}$: Find α, β by setting ζ_i values for any 2 modes, say 1 & 3 : $\Rightarrow$ 2 eq^{n}s $\boxed{\text{2 variables } \alpha, \beta}$

$\boxed{\begin{array}{l} \alpha : w_i \uparrow \text{ses} \Rightarrow \zeta_i \downarrow \text{ses} \\ \beta : w_i \uparrow \text{ses} \Rightarrow \zeta_i \uparrow \text{ses} \end{array}}$

→ Now $[C]$ can be uncoupled to $[C^*]$ using $[\phi]$:

$\boxed{[C^*] = \alpha[M^*] + \beta[K^*]}$: Can be proved

∴ MDOF system $\xrightarrow{\text{uncoupled}}$ N - SDOF systems.

For a mode "i" : $\boxed{M_i^* \ddot{q}_i + C_i^* \dot{q}_i + K_i^* q_i = F_i^*(t)}$

To obtain $[F^*]$; $[F^*] = [\phi]^T [F]$

(*) Computing the SDOF response from above eq^n :-

Analytical solution by convolving : $q_i(t) = (F_i^* * h_i)(t) \cdot \Delta t$

{ where $h_i(t) = \frac{1}{M_i^* w_{d,i}} e^{-\zeta_i w_i t} \sin(w_{d,i}\, t)$ }

called impulse response f^n -

2. Generalized Duhamel's integral

A linear SDOF system is governed by the following initial value problem

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = p(t), \quad u(0) = \dot{u}(0) = 0, \tag{2.1}$$

where m, c and k are, respectively, mass, damping coefficient and stiffness of the system, while $u(t)$ and $p(t)$ denote the time-dependent displacement and force (load) functions with dot representing, as usual, differentiation with respect to time $t > 0$. The solution of the initial value problem defined by equation (2.1) is the convolution integral

$$u(t) = \int_0^t h(\tau)p(t-\tau)\,d\tau, \quad h(t) = e^{-\xi\omega t}\sin(\omega_d t)\,\omega_d^{-1}m^{-1}, \tag{2.2}$$

also known as *Duhamel's integral* [2], in which the parameters ω, ξ and ω_d are defined by the relations

$$\omega^2 = m^{-1}k, \quad \xi\omega = \frac{1}{2}m^{-1}c, \quad \omega_d^2 = \omega^2 - (\xi\omega)^2, \tag{2.3}$$

denoting the natural frequency, the damping ratio and the damped frequency of the SDOF system, respectively.

[CODE : conv(fn(i,:), h(i,:) * dt)]

Similarly convolute for vel. $\dot{q}_i(t)$ & accl^n $\ddot{q}_i(t)$.

$\boxed{\text{Total Response :- } x(t) = \Phi\, q(t) \; ; \; v(t) = \Phi\, \dot{q}_i(t) ; \quad a(t) = \Phi\, \ddot{q}_i(t)}$

Mode shape matrix ← (Transform modal resp. back to physical space)

CODE :
{ x = vectors * q ;
v = " * qd ; }

→ Convolution :

$\boxed{q(t) = (h * f)(t) = \int_0^t h(\tau) f(t-\tau)\,d\tau}$

q → weighted sum of all past forces ⇐

Current displ. is affected by entire history of applied force, depending on how system weighs past inputs.

References:
https://doi.org/10.1098/rspa.2021.0576
https://doi.org/10.3390/buildings14051286

MATLAB CODE

```matlab
clear; clc; close all;

%Input
N=5;                                %Number of stories
Ks=1.44e8;                           %Stiffness
Ms=2.0e8;                           %Mass of each floor
floor_height = 3;

%Using Ground Motion Data (Acceleration(m/s2) vs Time)
data = readmatrix('Dataset.csv');
t = data(:,1);
ag = data(:,2);

dt = mean(diff(t));   % Time step (s)
fs = 1/dt;            % Sample Frequency (Hz)

%Plot 1: Input Ground Acceleration (from CSV)
figure;
plot(t, ag);
xlabel('Time (s)'); ylabel('a_g (m/s^2)');
title('Input Ground Acceleration (from CSV)');
grid on;


M=Ms*diag(ones(N,1));                  %Mass matrix
K=MultiStory_Stiffness(Ks,N);          %Stiffness matrix

%Damping matrix
[V,D] = eig(K,M);
wn = sqrt(diag(D));                 % natural frequencies (rad/s)
w1 = wn(1); w2 = wn(min(2,N));
zeta = 0.05;                        % 5% damping
A = [1/(2*w1), w1/2; 1/(2*w2), w2/2];
x = A\[zeta; zeta];
alpha = x(1); beta = x(2);
C = alpha*M + beta*K;

% Force matrix
ag = ag(:)';
f = zeros(N, length(ag));
for i = 1:N
    f(i,:) = -Ms * ag;
end

Result=MDOF_simulation(M,C,K,f,fs);    %MDOF solver

%Acceleration, Velocty, Time Response Plot
t=[0:1/fs:(length(ag)-1)/fs];
figure;
for i=1:1:N
    subplot(N,3,3*i-2)
    plot(t,Result.Acceleration(i,:)); xlabel('Time(s)'); ylabel(strcat('Acc',num2str(i)));
    subplot(N,3,3*i-1)
    plot(t,Result.Velocity(i,:)); xlabel('Time(s)'); ylabel(strcat('Vel',num2str(i)));
    subplot(N,3,3*i)
    plot(t,Result.Displacement(i,:)); xlabel('Time(s)'); ylabel(strcat('Disp',num2str(i)));
end

%Modeshapes Plot
figure;
xscale = 10.0e5;
yscale = 25;
for i=1:1:N
    plot([0 ;xscale*Result.Parameters.ModeShape(:,i)],yscale*(0:N),'*-');
    hold on
end
hold off
xlabel('Amplitude');
ylabel('Floor');
grid on
daspect([1 1 1]);
title('Modeshapes');



%Interstory Drift
u = Result.Displacement;
interstory_drift = diff(u,1,1);
max_drift = max(abs(interstory_drift),[],2);
drift_ratio = max_drift / floor_height;

%Drift Ratios Plot
figure;
bar(1:length(drift_ratio), drift_ratio);
xlabel('Story Level');
ylabel('Max Drift Ratio');
title('Interstory Drift Ratio per Story');
grid on;



a = Result.Acceleration;            % (N x time) floor accelerations
m = Ms * ones(N,1);                 % (N x 1) mass per floor
base_shear = (m' * a);              % (1 x time) total shear at base
base_shear = base_shear(:)';        % ensure row vector for plotting
max_base_shear = max(abs(base_shear));

%Base Shear Time History Plot
figure;
plot(t(1:length(base_shear)), base_shear, 'LineWidth', 1.2);
xlabel('Time (s)');
ylabel('Base Shear (N)');
title('Base Shear Time History');
grid on;

```

```matlab
function Result=MDOF_simulation(M,C,K,f,fs)

n=size(f,1);
dt=1/fs; %sampling rate
[Vectors, Values]=eig(K,M);
Freq=sqrt(diag(Values))/(2*pi); % undamped natural frequency
steps=size(f,2);

Mn=diag(Vectors'*M*Vectors); % uncoupled mass
Cn=diag(Vectors'*C*Vectors); % uncoupled damping
Kn=diag(Vectors'*K*Vectors); % uncoupled stifness
wn=sqrt(diag(Values));
zeta=Cn./(2*sqrt(Mn.*Kn));   % damping ratio
wd=wn.*sqrt(1-zeta.^2);

fn=Vectors'*f; % generalized input force matrix

t=[0:dt:dt*steps-dt];

%forced vibration

for i=1:1:n

    h(i,:)=(1/(Mn(i)*wd(i))).*exp(-zeta(i)*wn(i)*t).*sin(wd(i)*t);
    hd(i,:)=(1/(Mn(i)*wd(i))).*(-zeta(i).*wn(i).*exp(-zeta(i)*wn(i)*t).*sin
    hdd(i,:)=(1/(Mn(i)*wd(i))).*((zeta(i).*wn(i))^2.*exp(-zeta(i)*wn(i)*t).

    qq=conv(fn(i,:),h(i,:))*dt;
    qqd=conv(fn(i,:),hd(i,:))*dt;
    qqdd=conv(fn(i,:),hdd(i,:))*dt;

    q(i,:)=qq(1:steps); % modal displacement
    qd(i,:)=qqd(1:steps); % modal velocity
    qdd(i,:)=qqdd(1:steps); % modal acceleration


end

x=Vectors*q;   %Displacement
v=Vectors*qd;  %Vecloity
a=Vectors*qdd; %Acceleration
```

```matlab
% Free vibration

xi=zeros(n,1);  % Displacement initial condition
vi=zeros(n,1);  % Velocity initial condition

xno=Vectors'*M*xi./Mn;
vno=Vectors'*M*vi./Mn;

for i=1:1:n

AA=(vno(i)+xno(i).*zeta(i).*wn(i))./wd(i);
BB=xno(i);

qf(i,:)=exp(-zeta(i)*wn(i)*t).*(AA.*sin(wd(i)*t)+BB.*cos(wd(i)*t));
qdf(i,:)=wd(i)*exp(-zeta(i)*wn(i)*t).*(AA.*cos(wd(i)*t)-BB.*sin(wd(i)*t))-z
qddf(i,:)=wd(i)^2*exp(-zeta(i)*wn(i)*t).*(-AA.*sin(wd(i)*t)-BB.*cos(wd(i)*t


end

x=x+Vectors*qf;
v=v+Vectors*qdf;
a=a+Vectors*qddf;

Result.Displacement=x;
Result.Velocity=v;
Result.Acceleration=a;
Result.Parameters.Freq=Freq;
Result.Parameters.DampRatio=zeta*100;
Result.Parameters.ModeShape=Vectors;
end
```

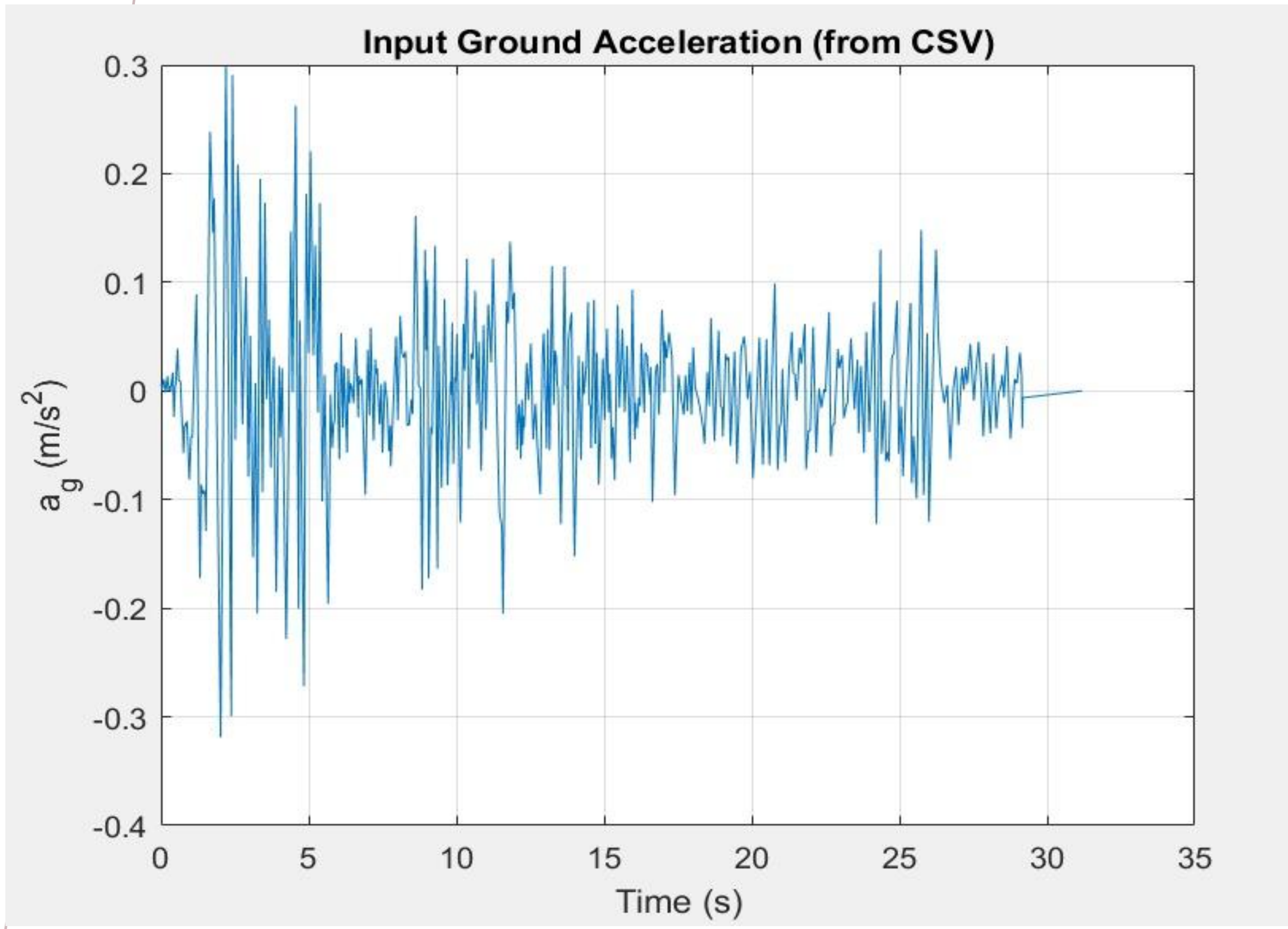
```
function K=MultiStory_Stiffness(Ks,N)

KK=Ks*ones(N+1,1);
KK(N+1)=0;
K=zeros(N,N);

for i=1:1:N
for j=1:1:N
if i==j
K(i,j)=KK(i)+KK(i+1);
end
if i==j+1
K(i,j)=-KK(i);
end
if j==i+1
K(i,j)=-KK(j);
end
end
end

end
```

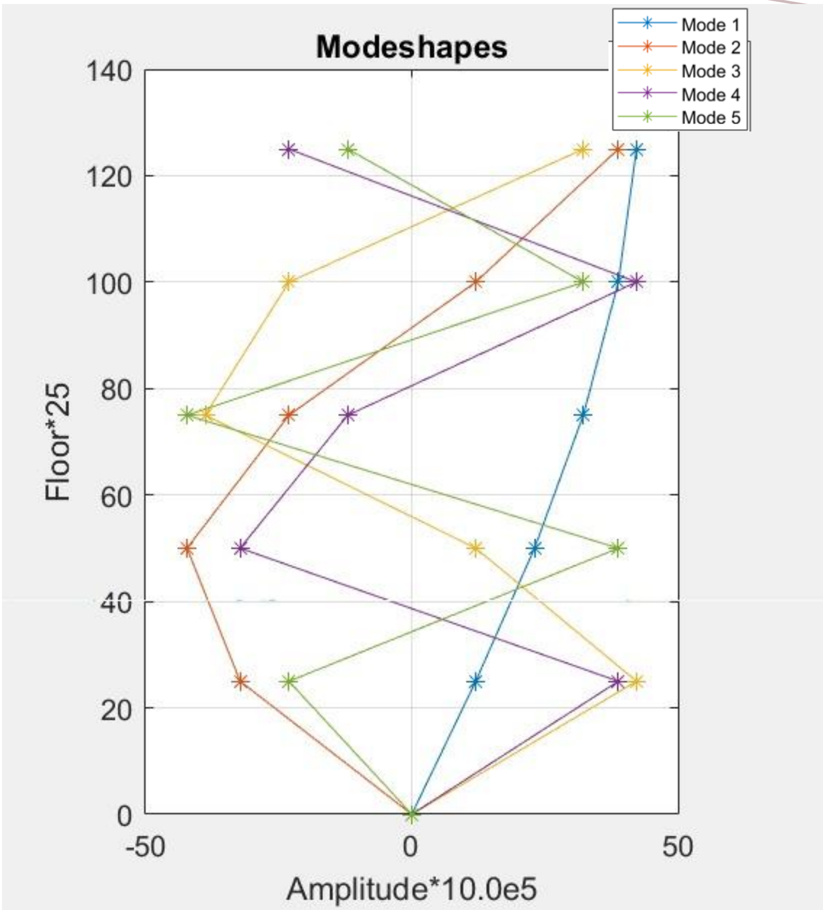
INPUT DATA



El Centro Earthquake Dataset

Reference:- https://www.vibrationdata.com/elcentro.htm

Mode Shapes-(Output)



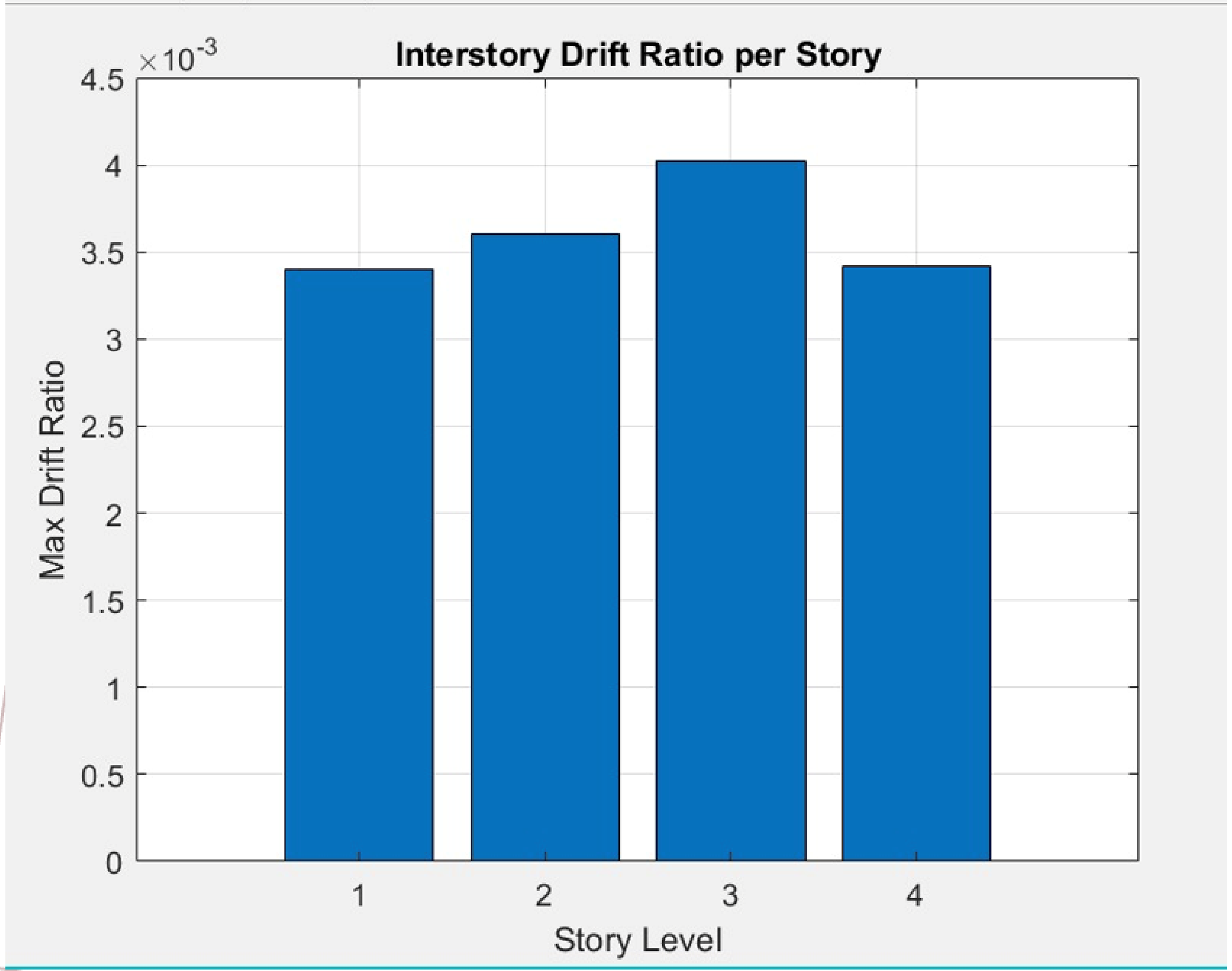
Displacements corresponding to different mode shapes

Mode shapes are crucial in seismic analysis:

•Lower modes usually control overall drift and base shear.

•Higher modes are essential for capturing forces and accelerations at upper floors, especially for short, impulsive ground motions

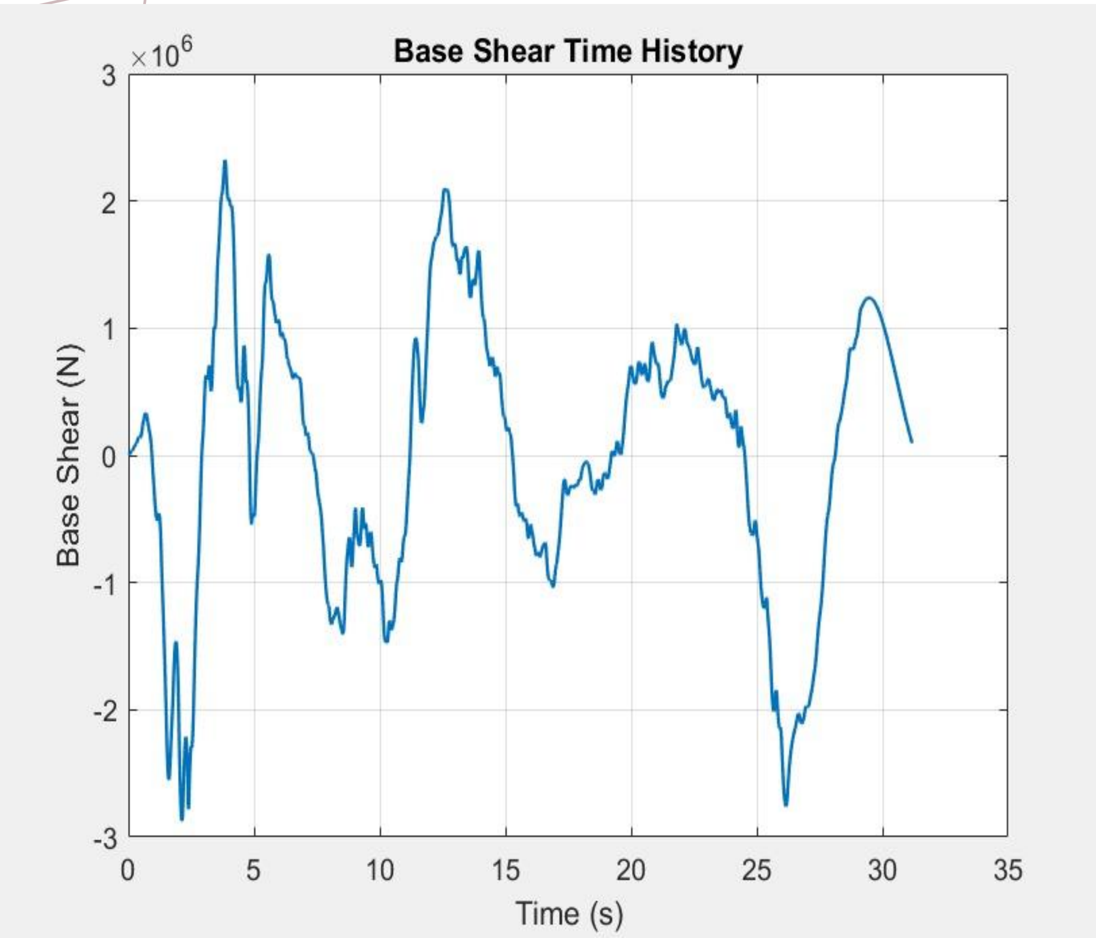
RESULTS



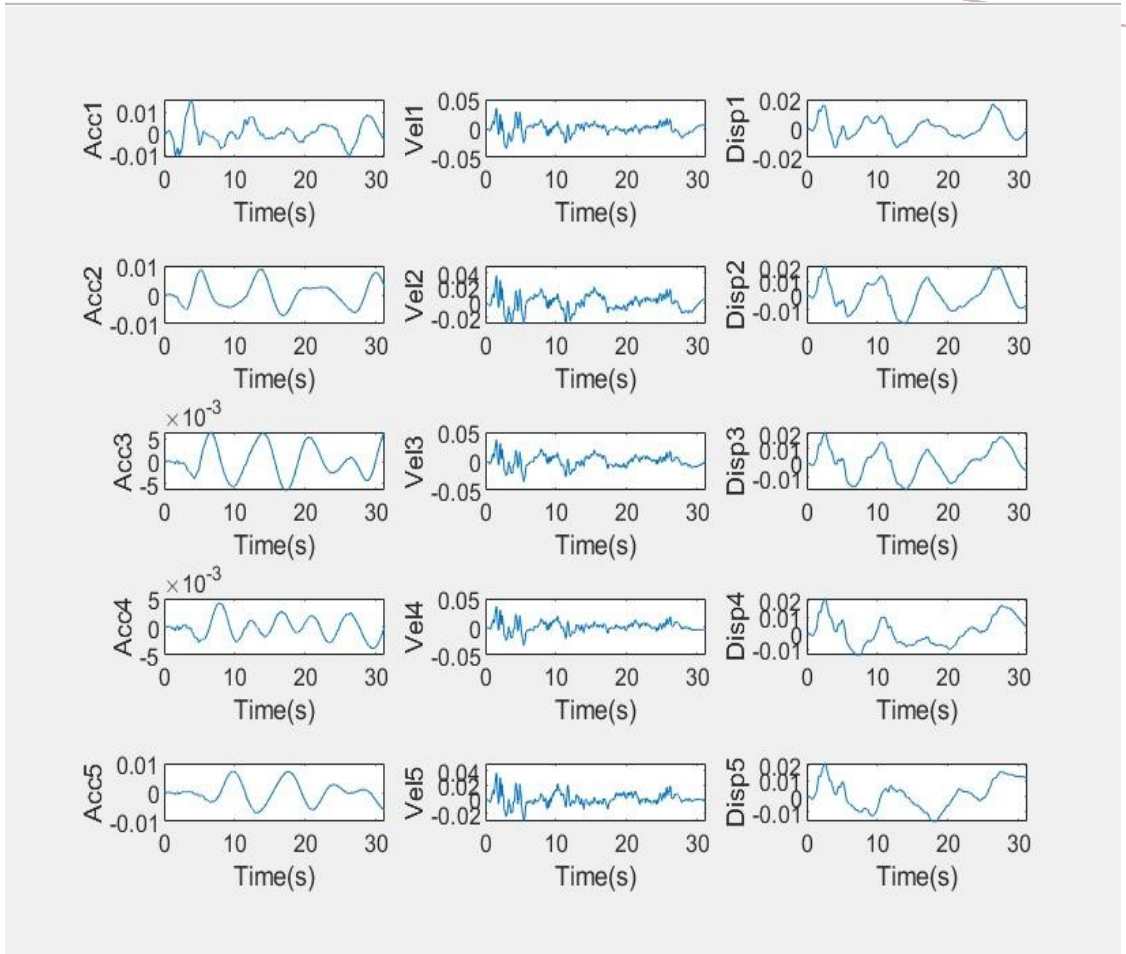
Max drift ratios:
(≈0.34%−0.4%≈0.34%−0.4%) are well below code limits. For elastic response, most seismic codes (e.g., IS 1893, ASCE 7, Eurocode 8) allow up to 2%-3% drift for Life Safety and Collapse Prevention.
Our results indicate an elastic response, with no excessive story drift

Reference: Interstory drift based scaling of earthquake ground motions
Numan Eren, Halûk Sucuoğlu, Rui Pinho

RESULTS



The obtained base shear time-history and its maximum value are in strong agreement with published research and code-predicted results for similar MDOF systems, confirming accurate response capture.



Layered response plots (Acc, Vel, Disp) for each story exhibit typical dynamic behaviour under seismic input, with amplification at upper stories, aligning with benchmarks from academic and software-based studies

THANK YOU

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