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# Towards Resilient Tracking in Autonomous Vehicles

# A Distributionally Robust Input & State Estimation Approach


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# Introduction - Problem & Solution

Image Source: Freepik

**The Problem**

- ➔ Safety
- ➔ State Data
- ➔ Noisy Measurements

**The Solution**

- ➔ Fuse Data
- ➔ Estimate State Using Model
- ➔ Ensure Reliability

# Baseline - Input & State Estimation (ISE)

**Core Idea**

- ➔ Joint Estimation
- ➔ State & Unknown Input
- ➔ Enhance Prediction

**Key Limitation**

Sensitive to:

- ➔ Non-Linearity
- ➔ Non-Gaussian Noise
- ➔ Outliers

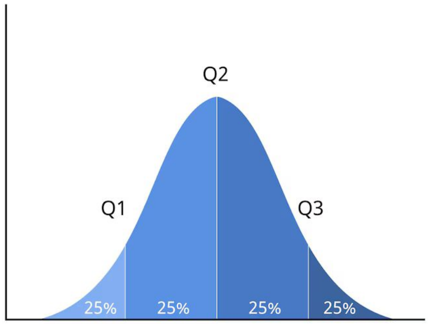



Image Source: Freepik

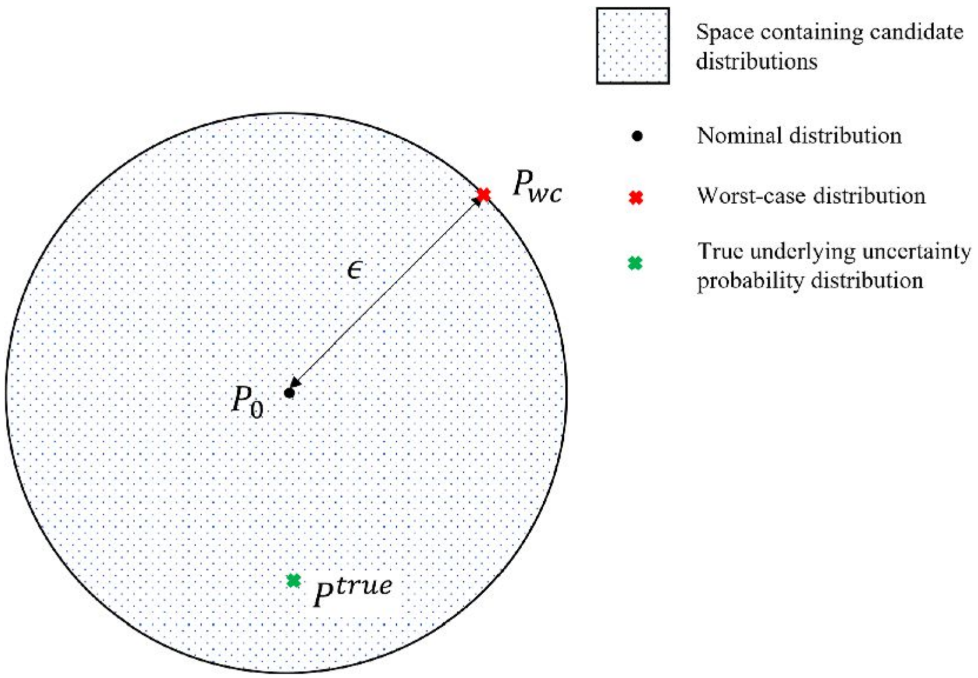
# Baseline - Distributionally Robust Estimation (DRE)

**Core Idea**

- ➔ Robustness
- ➔ Deviating Noise Distributions
- ➔ Ambiguity Sets
- ➔ Worst-Case Optimization

**Key Limitation**

- ➔ Sensitive to Outliers
- ➔ Ignores Unknown Inputs



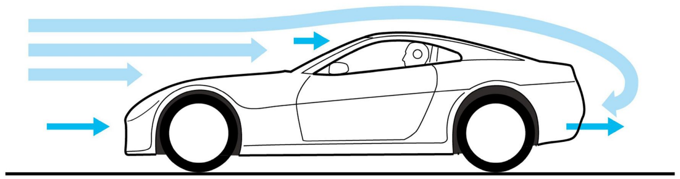



Image Source: Researchgate.net, Gran-Turismo

# Problem Formulation

## System Model

unknown input

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}_k \boldsymbol{x}_k + \boldsymbol{B}_k \boldsymbol{u}_k + \boldsymbol{G}_k \boldsymbol{d}_k + \boldsymbol{w}_k$$

state vector

known input

uncertain process noise distribution

$$\boldsymbol{y}_k = \boldsymbol{C}_k \boldsymbol{x}_k + \boldsymbol{v}_k$$

sensor measurements containing outliers

uncertain measurement noise distribution

## Unknown Forces

actual measurement

$$\hat{\boldsymbol{d}}_{k-1} = \boldsymbol{M}_k(\boldsymbol{y}_k - \boldsymbol{C}_k\hat{\boldsymbol{x}}_k^-)$$

unknown input

gain matrix

expected value

Image Source: Freepik

# Building Block 2: Distributionally Robust Estimation

## Unknown Noises

worst-case scenario

$$\min_{\text{estimator}} \max_{P\in\mathcal{F}} E_P[\text{Error Norm}]$$

best performance estimator

ambiguity set

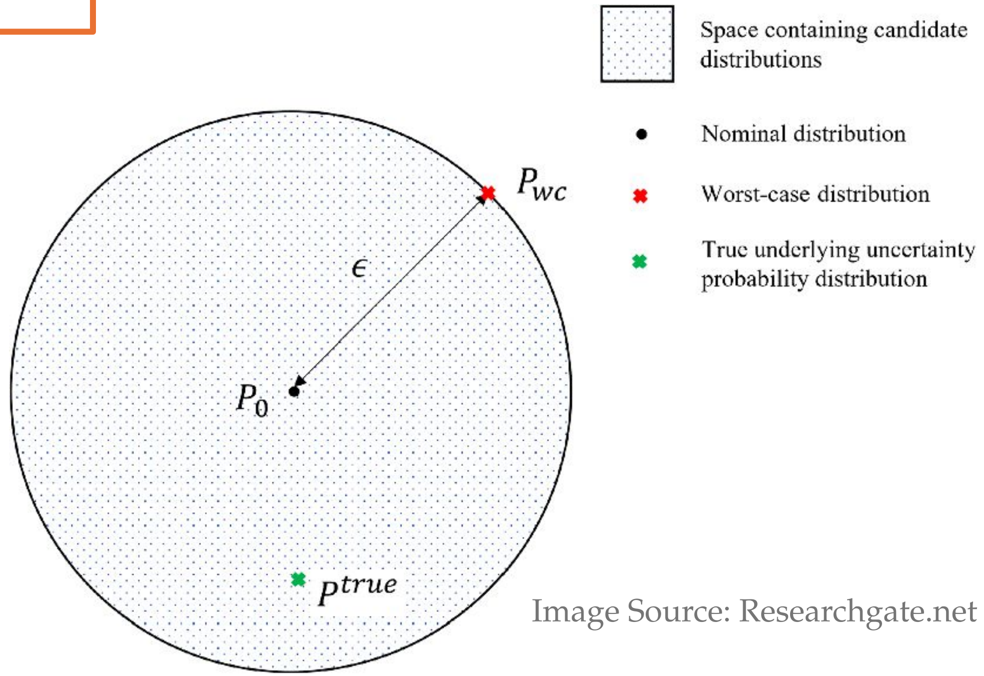



Image Source: Researchgate.net

# Building Block 3: Robust Update

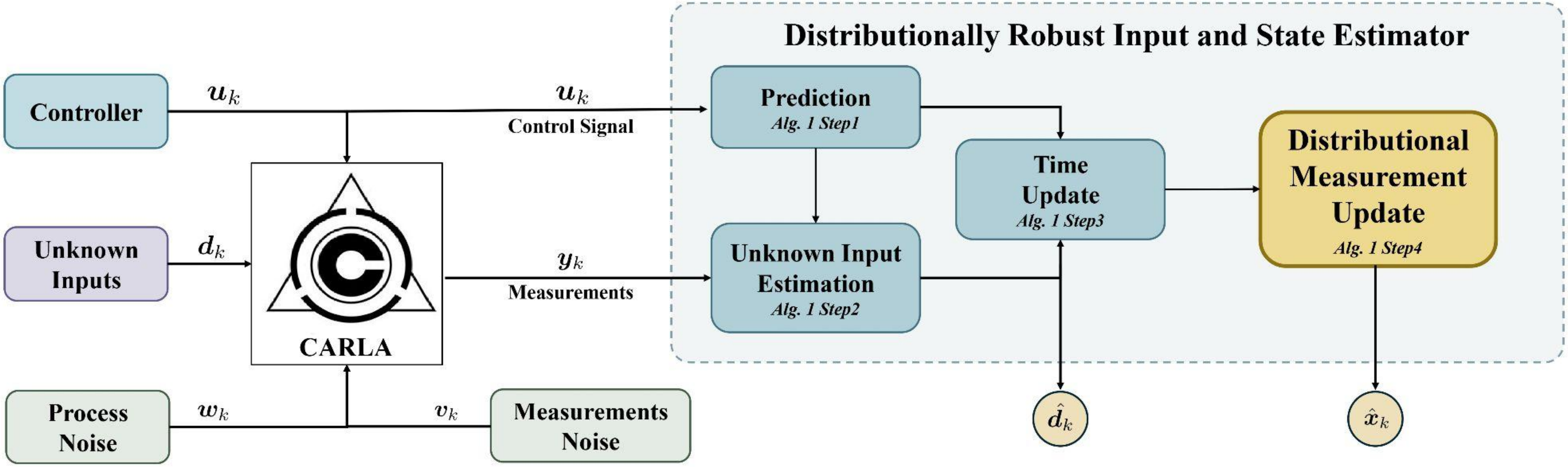
## Outliers

measurement residual

$$\psi(\mu) = \begin{cases} -K, & \mu \leq -K \\ \mu, & |\mu| < K \\ K, & \mu \geq K \end{cases}$$

tuning threshold

# Block Diagram - DRISE Framework



# The DRISE Algorithm Cycle

## Key Steps

1. **Prediction**

$$\hat{\boldsymbol{x}}_k^- = \boldsymbol{A}_{k-1}\hat{\boldsymbol{x}}_{k-1} + \boldsymbol{B}_{k-1}\boldsymbol{u}_{k-1}$$

2. **Input Estimation**

$$\hat{\boldsymbol{d}}_{k-1} = \mathbf{M}_k(\mathbf{y}_k - \mathbf{C}_k\hat{\mathbf{x}}_k^-)$$

3. **Time Update**

$$\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_k^- + \mathbf{G}_{k-1}\hat{\boldsymbol{d}}_{k-1}$$

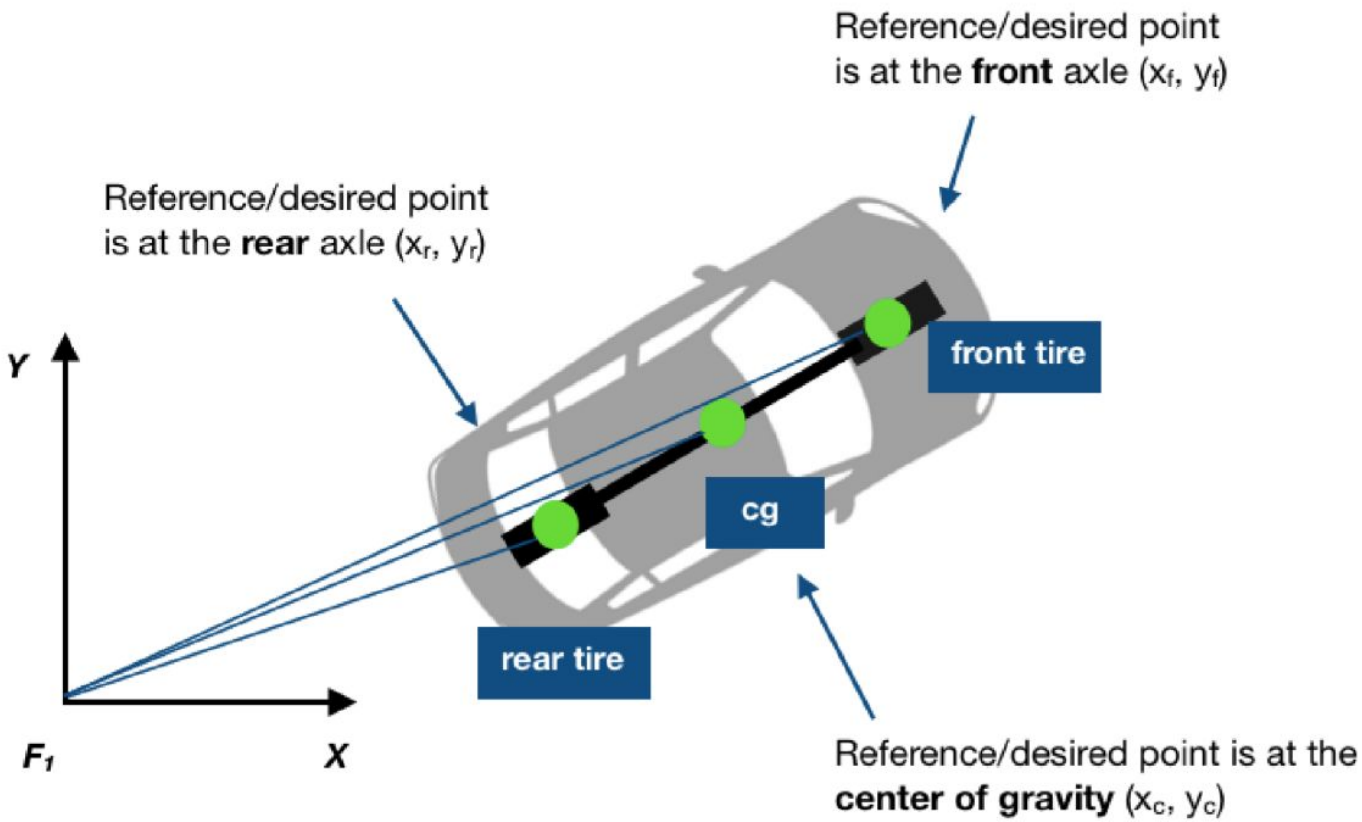
4. **Robust Measurement Update**

$$\hat{\boldsymbol{x}}_k \leftarrow \hat{\boldsymbol{x}}_k + \mathbf{L}_k\psi_k(\mathbf{S}_k^{-1/2}\mathbf{s}_k)$$

## Notation

- $\mathbf{M}_k$: Input est. gain.
- $\mathbf{L}_k$: Robust gain involving ambiguity sets.
- $\psi_k(\cdot)$: Influence function.
- $\mathbf{s}_k = \mathbf{y}_k - \mathbf{C}_k\hat{\mathbf{x}}_k$: Innovation.
- $\mathbf{S}_k$: Robust innovation covariance.

# Simulation Setup




## Simulation Settings

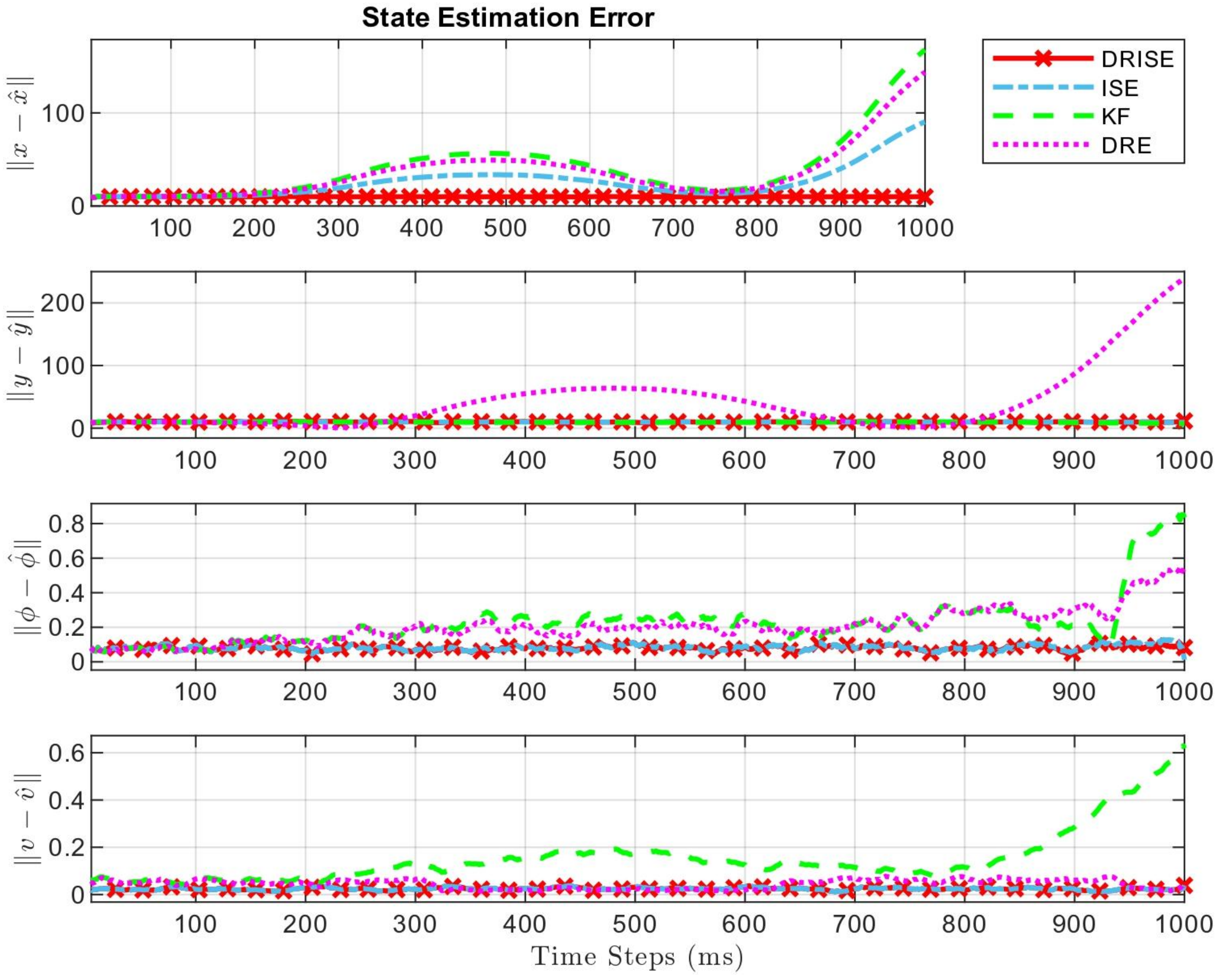
- **Model:** Kinematic Bicycle (LTV)
- **States $\boldsymbol{x}_k$:** Pos, Yaw, Vels
- **Input $\boldsymbol{u}_k$:** Steering, Accel
- **Unknown Input ($\boldsymbol{d}_k$):** Time-Varying Signal
- **Noise:** Proc ($\boldsymbol{Q}_k$), Meas ($\boldsymbol{R}_k$)
- **Outliers/Deviations:** Included in Tests
- **Comparison:** KF, ISE, DRE

# CARLA Simulation Environment


**Testing in CARLA Simulator**

- Open-source, high-fidelity simulator for AV research.
- Provides realistic urban environments, sensors, and physics.
- Challenging testbed for evaluating estimator performance under uncertainty.

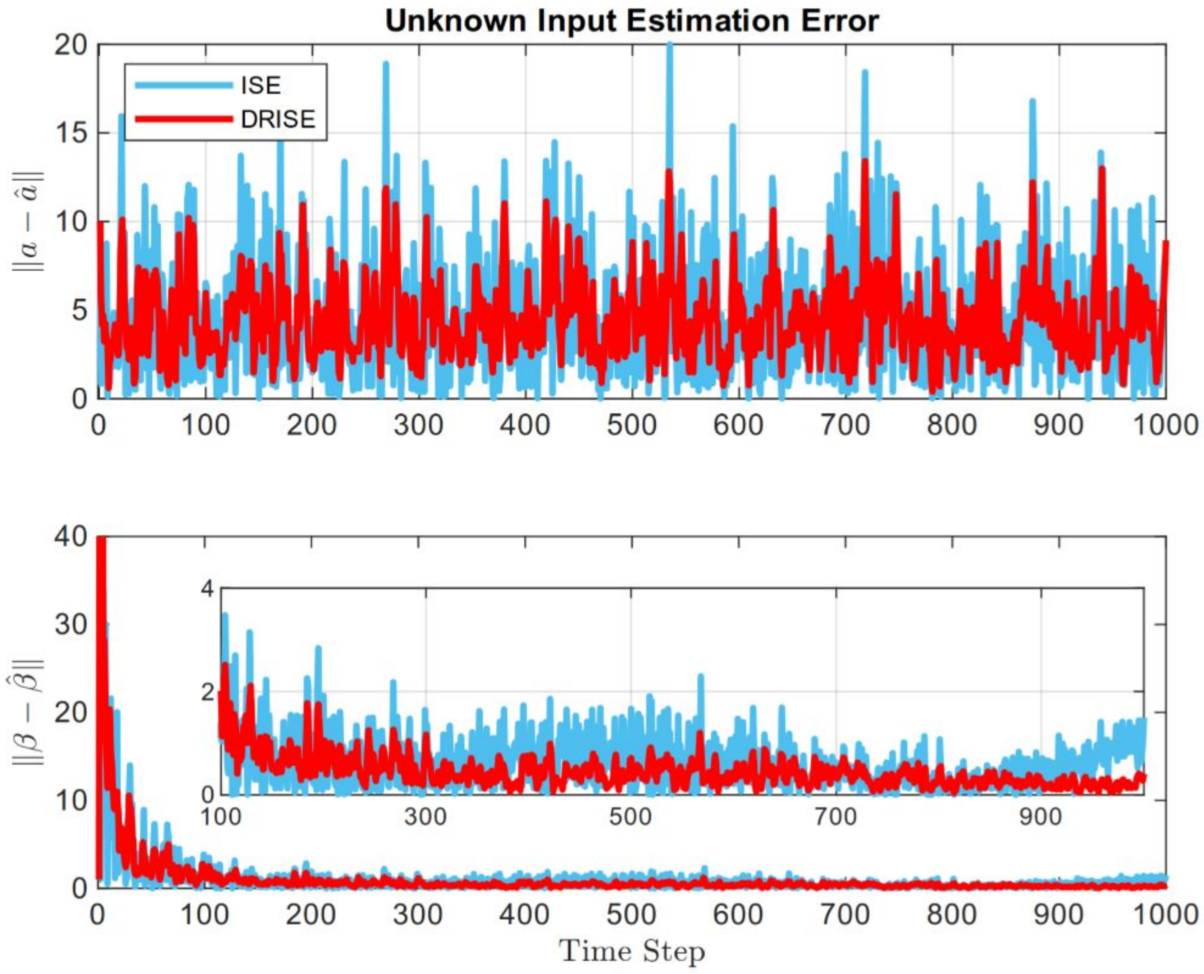
# Results: State Estimation Error




## Analysis

- **DRISE:** Lowest Error
- **KF:** Highest Error/Divergence
- **ISE/DRE:** Moderate Error

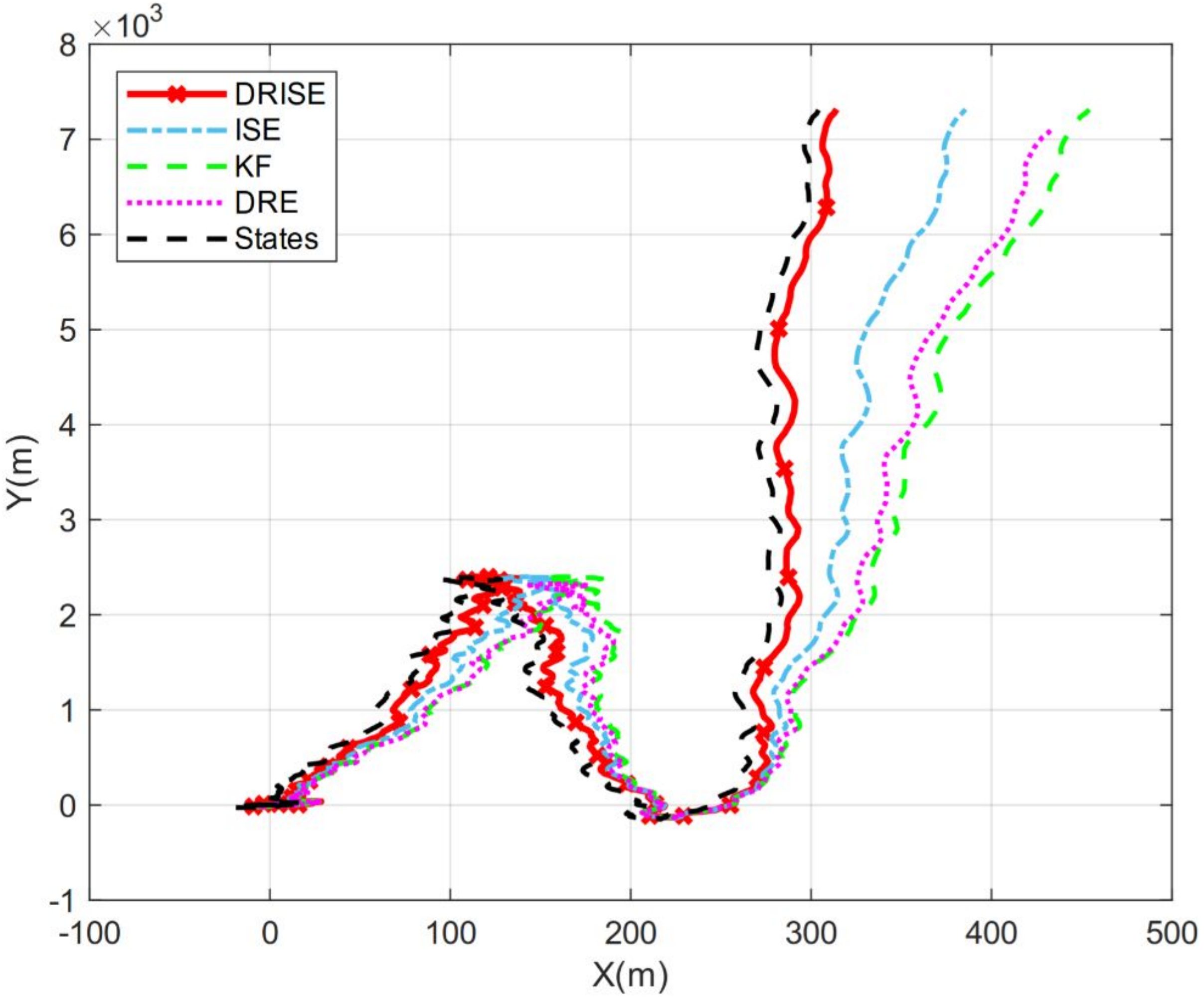
# Results: Unknown Input Error




**Analysis**

- **DRISE:** Lower Error
- **ISE:** Higher Error
- **Benefit:** Robustness Aids Input Est.

# Results: Trajectory Tracking




## Analysis

- **DRISE:** Best Tracking
- **Others:** Show Drift
- **Link:** Accurate Est. $\rightarrow$ Better Tracking

# Future Work

Wasserstein distance

Kullback-Leibler divergence

# Thank you!

Questions?

😊