DC Gain of a System

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1. Introduction:

- DC Gain is the gain of the system when frequency is zero.
- DC Gain of the system is also called as the zero frequency gain.

2. Example:

Problem: Given $G(s) = \frac{2(s+2)}{(s+3)(s+4)}$, find the DC gain:

Solution:

→ **Method 1:** Comparison with the time constant form

$$G(s) = \frac{2(s+2)}{(s+3)(s+4)}$$

$$= \frac{\cancel{2}\cdot\cancel{2}\left(\frac{s}{2}+1\right)}{3\left(\frac{s}{3}+1\right)\cancel{4}\left(\frac{s}{4}+1\right)}$$

$$G(s) = \frac{\left(\frac{s}{2}+1\right)}{3\left(\frac{s}{3}+1\right)\left(\frac{s}{4}+1\right)}$$

Now, if we compare of G(s) with General Time constant form:

$$G(s) = \frac{K\cdot(1+s\tau_1)(1+s\tau_2)\cdots(1+s\tau_n)}{s^n\cdot(1+s\tau_1')(1+s\tau_2')\cdots(1+s\tau_n')}$$

By comparing, we get:

$$\boxed{K = \frac{1}{3} = 0.33}$$

DC Gain

→ **Method 2:** For a Type 0 system

$$\text{DC Gain, } K = \lim_{s\to 0} G(s) \quad \left[\because \text{by definition of DC Gain of system}\right]$$

$$= \lim_{s\to 0}\frac{2(s+2)}{(s+3)(s+4)}$$

$$= \frac{\cancel{2}\cdot\cancel{2}}{3\cdot\cancel{4}}$$

$$= \frac{1}{3}$$

$$= 0.33$$

$$\therefore \boxed{K = 0.33}$$

NOTE: Remember, we can only apply this formula when we have Type-0 system. That means, there should not be any pole at the origin. However, we can always apply Method 1.

Problem: Given $G(s) = \frac{2\cdot(s+2)}{s(s+3)(s+4)}$. Find DC Gain.

Solution:

From the transfer function, we can see that it is Type-1 system as it has 1 pole at the origin.

We will again solve this problem using 2 methods:

→ **Method-1:** Comparison with general time constant form:

$$G(s) = \frac{2\cdot(s+2)}{s(s+3)(s+4)}$$

$$= \frac{\cancel{2}\cdot 2\,(1+s/2)}{s\cdot 3\left(1+\frac{s}{3}\right)4\left(1+\frac{s}{4}\right)}$$

$$= \frac{(1/3)\,(1+s/2)}{s\cdot\left(1+\frac{s}{3}\right)\left(1+\frac{s}{4}\right)}$$

Comparing it with general time constant form:

$$G(s) = \frac{K\cdot(1+s\tau_1)(1+s\tau_2)\cdots(1+s\tau_n)}{s\cdot(1+s\tau_1')(1+s\tau_2')\cdots(1+s\tau_n')}$$

Note: The primes (') does not mean derivative.

By comparing, we got:

$$\text{DC gain, } \boxed{K = \frac{1}{3} = 0.33}$$

→ **Method-2:** For Type-1 system

$$\text{DC Gain, } K = \lim_{s\to 0} s\cdot G(s)$$

$$= \lim_{s\to 0}\cancel{s}\cdot\frac{2(s+2)}{\cancel{s}(s+3)(s+4)}$$

$$= \lim_{s\to 0}\frac{2(s+2)}{(s+3)(s+4)}$$

$$= \frac{2\cdot 2}{3\cdot 4}$$

$$= \frac{1}{3}$$

$$= 0.33$$

$$\therefore \boxed{K = 0.33}$$

BONUS: for a Type-2 system:

$$\text{DC gain, } \boxed{K = \lim_{s\to 0} s^2\cdot G(s)}$$

3. Solved Problem:-

The open loop DC gain of a unity feedback system with closed loop transfer function $\frac{s+4}{s^2+7s+13}$ is :

(a) $\frac{4}{13}$ (b) $\frac{4}{9}$ (c) 4 (d) 13

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Solution: Given: $CLTF = \frac{s+4}{s^2+7s+13}$

Closed loop transfer function

Since, we are required to find out the DC gain of open loop transfer function (OLTF), so let's first find out OLTF:

We know, the relationship b/w CLTF & OLTF:

$$CLTF = \frac{G(s)}{1 \pm OLTF}, \text{ where } OLTF = G(s)\cdot H(s)$$

Since, it's given that, it is unity feedback system which means:

$$H(s) = 1$$

$$\therefore \text{So, } OLTF = G(s) \Rightarrow CLTF = \frac{G(s)}{1 \pm G(s)}$$

Also, by default, it is Negative feedback system. So,

$$CLTF = \frac{G(s)}{1+G(s)}$$

Substituting value of CLTF:

$$\frac{s+4}{s^2+7s+13} = \frac{G(s)}{1+G(s)}$$

$$[1+G(s)](s+4) = G(s)\left[s^2+7s+13\right]$$

$$s+4 + s\cdot G(s) + 4G(s) = G(s)\left[s^2+7s+13\right]$$

$$s+4 = G(s)\left[s^2+7s+13-s-4\right]$$

$$G(s) = \frac{s+4}{s^2+6s+9}$$

We can see that it is Type-0 system as it has no poles at the origin.

Therefore,

$$\text{DC gain, } K = \lim_{s\to 0} G(s)$$

$$= \lim_{s\to 0}\frac{s+4}{s^2+6s+9}$$

$$= \frac{4}{9}$$

$$\therefore \boxed{K = \frac{4}{9}}$$

⇒ Option (b) is correct answer ✓

4. References:

1. Neso Academy

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