Inverse Laplace Transform Part 1

Saturday, June 14, 2025 10:36 PM

Author : Kumar Anurag

1. Introduction of Inverse Laplace Transform (ILT) :-

- Inverse laplace transform is the method to find the time domain function $f(t)$ whenever we are given the frequency domain function $F(s)$.

$$\boxed{f(t) = \frac{1}{2\pi j}\int_{\sigma - j\omega}^{\sigma + j\omega} F(s)\cdot e^{st}\, ds}$$

- Since, the above expression is bit complicated so we will avoid using this expression, insted we will use method of partial fractions and some properties to obtain time domain signal $f(t)$.

2. Examples :

→ Example 01 : Find the ILT of $F(s) = \frac{1}{(s+3)^2}$

Solution:

Let $y(t) = t$, then

$$L[y(t)] = \frac{1}{s^2} \qquad \left[\because \text{using formula Ramp function}\right]$$

Using frequency shifting property of LT:

$$L[e^{-3t}\cdot y(t)] = \frac{1}{(s+3)^2} = F(s)$$

$$L^{-1}\Big[L[e^{-3t}\cdot y(t)]\Big] = L^{-1}[F(s)]$$

$$e^{-3t}\cdot y(t) = f(t)$$

$$\boxed{f(t) = e^{-3t}\cdot t}$$

→ Example 02 : Find ILT of $F(s) = \frac{2}{(s+1)(s+2)}$

Solution: Using partial fraction decomposition:

$$\frac{2}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

Using cover-up method:

→ Multiply both sides by $(s+1)$ and set $(s+1)=0$

$$\frac{2}{\cancel{(s+1)}(s+2)}\cdot\cancel{(s+1)} = A + \frac{B}{(s+2)}\cancel{(s+1)}^{\,0}$$

$$A = \frac{2}{s+2} = \frac{2}{(-1)+2} = \frac{2}{1} \qquad [\because s+1=0 \Rightarrow s=-1]$$

$$\boxed{A = 2}$$

→ Multiply both sides by $(s+2)$ and set $(s+2)=0$

$$\frac{2}{(s+1)\cancel{(s+2)}}\cdot\cancel{(s+2)} = \frac{A}{(s+1)}\cdot\cancel{(s+2)}^{\,0} + B$$

$$B = \frac{2}{s+1} = \frac{2}{(-2)+1} \qquad [\because s+2=0 \Rightarrow s=-2]$$

$$\boxed{B = -2}$$

Therefore,

$$F(s) = \frac{2}{(s+1)(s+2)} = \frac{2}{s+1} - \frac{2}{s+2}$$

$$F(s) = \frac{2}{s+1} - \frac{2}{s+2} \quad \text{-----} \quad \text{(i)}$$

Let $y(t) = u(t)$, then

$$L[u(t)] = \frac{1}{s}$$

Applying the frequency shift property of LT:

$$L[e^{-t}u(t)] = \frac{1}{s+1} \Rightarrow L[2e^{-t}u(t)] = \frac{2}{s+1}$$

$$L[e^{-2t}u(t)] = \frac{1}{s+2} \Rightarrow L[2e^{-2t}u(t)] = \frac{2}{s+2}$$

$$\therefore \quad L[2e^{-t}u(t)] - L[2e^{-2t}u(t)] = \frac{2}{s+1} - \frac{2}{s+2} = F(s)$$

Taking inverse LT both sides, we get:

$$2e^{-t}u(t) - 2e^{-2t}u(t) = L^{-1}[F(s)] = f(t)$$

$$\therefore \quad f(t) = 2u(t)[e^{-t} - e^{-2t}]$$

$$f(t) = 2e^{-t}\cancel{u(t)}^{\,1}[1 - e^{-t}] \qquad \left[\because \text{in } 1^{st} \text{ quadrant}\right]$$

$$\boxed{f(t) = 2e^{-t}[1 - e^{-t}]}$$

3. References :

1. Neso Academy

— x — THE END — x —