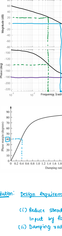
Matlab : Kumar Anang

1. **Problem 53 :**

Using the open loop phase Bode plot is given below design a lag compensator such that the performance of the closed loop system meets both of the following criteria: (a) the steady state error to a ramp is decreased to $\frac{1}{10}$ of the uncompensated steady state error.
(b) The damping ratio of the system is around 0.6



Solution: Design Requirements:

(1) Reduce steady state error to a ramp input by factor of 10
(2) Damping ratio = 0.6

→ **Steady-state Error Requirement:**

For ramp input, $K_{v,1} = \frac{1}{K_v}$ where $K_v = \lim_{s\to 0} sG(s)$

To reduce the error by factor of 10, we must increase K_v by a factor of 10:

$K_v^{new} = 10 \cdot K_v^{old}$

→ **Gain Margin:** From current bode, $\omega_{pc} = 1$ rad/sec
At ω_{pc}, magnitude = −20dB

From margin = 0 − (−20) = +20 dB

This means, we must increase by 20dB.

→ **Damping Ratio — Phase margin Graph:**

From graph, we can see that at $\zeta = 0.6$, the value of Phase Margin = 90°.

$\zeta = 0.6 \Rightarrow$ Phase margin $\cong 60°$
$\xrightarrow{\text{lead}}$
we need to add 30° for this phase margin while designing the lag compensator

From Bode Plot:

Gain crossover frequency, $\omega_{gc} = 16$ rad/s

$\angle G(j\omega_{gc}) = -170°$

Uncompensated Phase Margin = $\angle G(j\omega_{gc}) - (-180°)$
$= -170° + 180°$
$= 10°$

This very small, we need 60°, so we will apply phase lead of 50°

→ **Lag Compensator:**

$G_c(s) = \frac{s+z}{s+p}$, where $z < p$

and $\frac{z}{p} = \beta > 1$

→ **Selecting New Crossover Frequency:**

We will shift it from 16 to 0.6 (MPC 2.3.1),

New new phase = −140°

New Phase Margin = −140° + 180°
= 40°
This was too marginal

New Gain Margin (at $\omega = 0.6$) = $\underbrace{+20\,dB}_{\text{from graph}}$

This was too large.

$20\log_{10}\beta > 20$

$\Rightarrow \boxed{\beta = 10}$

$\frac{z}{p} = \beta = 10$

$\boxed{p = 10\,z}$

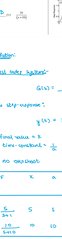
Choosing z:

Let $z = 0.06$, $\Rightarrow p = 10(0.06)$
$= 0.6$

$\therefore \boxed{G_c(s) = \frac{s+0.06}{s+0.6}}$

2. **Problem 54 :**

Match the transfer function on the left with its unit step response on the right.



Solution:

→ **First order system:**

$G(s) = \frac{K}{\tau s + 1}$

Unit step response:

$y(t) = K(1 - e^{-\frac{t}{\tau}})$

- Final value = K
- time-constant = $\frac{1}{\tau}$
- no oscillations

	τs	K	a	$\sqrt{a}$	Envelope	Expand 5 time time
(1) $\frac{1}{s+1}$	5	1	5m	5		~5 sec
(2) $\frac{10}{s+10}$	⇒	10	0.1m	10^{-2}		~0.5 sec

(3) $\frac{10}{s^2+2s+10}$ → result must be in about 5 seconds
⇒ Curve (B)

→ is extremely fast (ξ=0.1m), no overshoot final magnitude = 10 ⇒ Curve (C)

Underdamped Second Order:

$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$\Rightarrow y(t) = 1 - \cos(\omega_d t)$

$\frac{1}{s^2+1}$ → sinusoidal wave which oscillates now 0 and 2 ⇒ Curve (D)

Underdamped Second Order:

$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

→ a small ξ (0.1m) gives large overshoot and slowly decaying oscillations.

→ a large ξ (0.9) gives small overshoot and quickly decaying oscillations.

$\frac{10}{s^2+2(0.1)10s+10}$ → ξ (0.1) ⇒ Curve (B)

$\frac{10}{s^2+2(0.9)10s+10}$ → $\xi(0.9)$
$= 2(0.9)\omega_n = 9$ → Curve (C)

3. **Problem 55 :**

Continuing solved in Spring 2018 - Problem 55 ?

4. **Problem 56 :**

Consider the closed loop system in the figure. Determine the critical value of K. Is the stability using Routh's stability criterion.

Solution:

Open-loop Transfer function:

$G(s) = \frac{K}{s-1}$

Note: $\frac{s+1}{s-1} = \frac{s+1}{s-1} \Rightarrow$ Since this pole is on right half plane in open-loop system is UNSTABLE
$\frac{s+1}{s-1}$

→ Substitute $(s = j\omega)$

$G(j\omega) = \frac{K}{j\omega - 1}$

$|G(j\omega)| = \frac{K}{\sqrt{\omega^2+1}}$

$\angle G(j\omega) = -\tan^{-1}\left(\frac{\omega}{-1}\right)$
$= -\tan^{-1}(-\omega)$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	K	0°
50	$\frac{K}{\sqrt{50+1}}$	−88°
100	$\frac{K}{\sqrt{100+1}}$	−89°
∞	0	−180°

Since,

$G(s) = \frac{K}{s-1}$

$G(j\omega) = \frac{K}{j\omega - 1}$

$= \frac{K}{j\omega - 1} \cdot \frac{j\omega + 1}{j\omega + 1}$

$= \frac{K(j\omega + 1)}{(j\omega)^2 - 1}$

$= \frac{K\omega + K}{-\omega^2 - 1}$

$G(j\omega) = \underbrace{\frac{-K}{\omega^2+1}}_{\text{real part}} - j\underbrace{\frac{K\omega}{\omega^2+1}}_{\text{imaginary part}}$

- At $\omega = 0$:

$G(j0) = \frac{-K}{0+1} - j\frac{K \cdot 0}{0+1}$
$= -K$

- At $\omega = \infty$:

$G(j\infty) \to 0$

Since, we have P = 1

$Z = N + P$

To ensure stability, Z must be 0

$0 = N + 1$

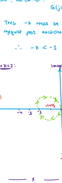
$\Rightarrow N = -1$

This means the trajectory plot of $G(j\omega)$ must encircle −1 once in the clockwise direction

Since, at $(\omega = 0)$:

$G(j\omega) = -K$

This → must be less than −1 so that trajectory plot encircles (−1,0) critical point

$\therefore -K < -1 \Rightarrow \boxed{K > 1}$

Plot: