

## Invert the Matrix

Thursday, June 12, 2025 10:19 PM

Author: Kumar Anurag.

We know about functions,  $f(x)$   
and we also know that they have inverse  $f^{-1}(x)$

Some concepts can be applied to matrices as well.

Let's calculate inverse of a function, first:

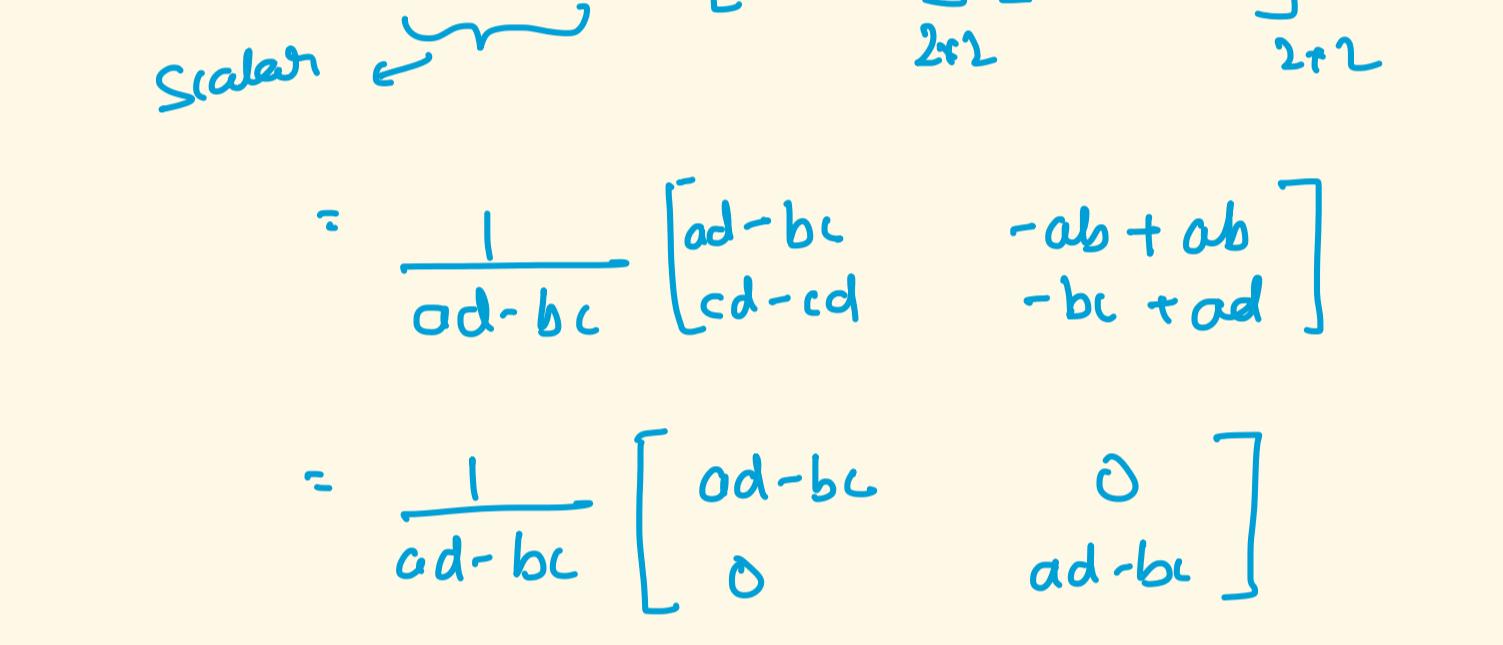
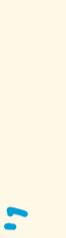
$$f(x) = 2x + 3$$

Solution:

$$\begin{aligned} y &= 2x + 3 \\ \text{swap} &\quad \uparrow \\ x &= 2y + 3 \\ \text{Now solve for } y &\quad \rightarrow y = \frac{(x-3)}{2} \\ \text{This is } f^{-1}(x) &\rightarrow f^{-1}(x) = \frac{(x-3)}{2} \end{aligned}$$

Let's verify:

$$f(x) = 2x + 3 \qquad f^{-1}(x) = \frac{(x-3)}{2}$$



Hence, verified 😊

Now, let's talk about MATRICES

let A be a matrix,

Normally, for a variable!  $x^{-1} = \frac{1}{x}$

But this is not the case for matrices:

$$A^{-1} \neq \frac{1}{A} \quad \left\{ \begin{array}{l} \text{there is no concept} \\ \text{of matrix division} \end{array} \right\}$$

Moving on, we know that

$$x \cdot x^{-1} = 1$$

In case of matrices,

$$A \cdot A^{-1} = I \quad \xrightarrow{\text{Identity Matrix}}$$

\* Finding the inverse of a Two-by-Two Matrix:-

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

determinant

Let's verify:

$$A \cdot A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \left( \frac{1}{ad-bc} \right) \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{ad-bc} \begin{bmatrix} ad-bc & -ab+ab \\ cd-cd & -bc+ad \end{bmatrix}$$

$$= \frac{1}{ad-bc} \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

Hence proved.

Q: Why do we need inverse matrix?

Since there is no concept of matrix division  
but there will be lot of situations where  
we need division from algebraic point of view  
for example,

$$XA = B$$

and we are tasked to solve for X.

As we cannot divide both sides by A but  
we can multiply both sides by  $A^{-1}$ .

$$XAA^{-1} = BA^{-1}$$

$$XI = BA^{-1}$$

$$\boxed{X = BA^{-1}}$$

( $\because$  provided B &  $A^{-1}$   
are compatible for  
multiplication)

Notice:

$$AX = B$$

$$A^{-1}AX = A^{-1}B$$

$$I X = A^{-1}B$$

$$\boxed{X = A^{-1}B}$$

( $\because$  order matters)

STEP-01: find the matrix of minors

$$\text{minor matrix of } A = \begin{bmatrix} -24 & -20 & -5 \\ -18 & -15 & -4 \\ 5 & 4 & 1 \end{bmatrix} \quad \text{--- } \oplus \quad \text{block lines}$$

STEP-02: matrix of cofactors

$$\text{cofactor matrix} = \begin{bmatrix} -24 & -20 & -5 \\ -18 & -15 & -4 \\ 5 & 4 & 1 \end{bmatrix} \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$= \begin{bmatrix} -24 & 20 & -5 \\ 18 & -15 & 4 \\ 5 & -4 & 1 \end{bmatrix}$$

Trick: Corner and center entries will  
remain as they are and other entries  
will have sign inverted.

STEP-03: find the adjugate / adjoint

$$\text{adj}(A) = \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix}$$

Keep diagonal  
same and rest  
are swapped.

STEP-04: find determinant  $\det(A)$

$$\det(A) = 1 \begin{vmatrix} 1 & 4 & -2 \\ 6 & 0 & 5 \\ 0 & 5 & 0 \end{vmatrix} + 3 \begin{vmatrix} 0 & 1 & 0 \\ 5 & 0 & 1 \\ 0 & 5 & 6 \end{vmatrix}$$

$$= -24 - 2(-20) + 3(0-5)$$

$$= -24 + 40 - 15$$

$$= -39 + 40$$

$$= 1$$

STEP-05: find the inverse

$$A^{-1} = \frac{1}{\det(A)} \text{adj}(A)$$

$$= \frac{1}{1} \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{bmatrix}}$$