Spring 2018

Control Systems - Exam

**Problem - Part 1**

**Given the system shown:**

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)}$$

(a) Write the system's state-space representation in canonical form. Hint: Use the "Diagonalized representation" method. Show all steps. Hint: Use the "Diagonalized representation" method. Show all steps. Hint: Use the "Diagonalized representation" method. Show all steps.

(b) Find the poles of $\frac{C(s)}{R(s)}$ in terms of $K$. Hint: Use your answer from part (a) to check consistency between both approaches.

**Solution:**

- Open-Loop Transfer Function and Closed-Loop Transfer Function

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)}$$

$$\frac{C(s)}{R(s)} = \frac{K}{s^2+3s+2}$$

$$\frac{C(s)}{R(s)} = \frac{K}{s^2+3s+2}$$

- Canonical Realization, In the Laplace Form

$$\frac{C(s)}{R(s)} = \frac{K}{s^2+3s+2}$$

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)}$$

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1}$$

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1} \cdot \frac{1}{1}$$

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1}$$

- Time, in Series:

Given:

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)}$$

Hence:

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1}$$

Hence:

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1}$$

- Diagonalized:

Canonical:

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)}$$

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1}$$

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1}$$

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1}$$

- Canonical Form, Diagonalized, In the Laplace Form

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1}$$

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1}$$

- Time, in Series:

Given:

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1}$$

Hence:

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1}$$

Hence:

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1}$$

- Equivalent:

Canonical:

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1}$$

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1}$$

- Canonical Form, Diagonalized, In the Laplace Form

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1}$$

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1}$$

- Time, in Series:

Given:

$$\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1} \cdot \frac{1}{1