Laplace Transform Part 4

Saturday, June 14, 2025 9:40 PM

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1. Properties of Laplace Transform :-

→ Time Differentiation :-

If,

$$f(t) \rightleftharpoons F(s)$$

then,

$$\frac{d}{dt}[f(t)] \rightleftharpoons s \cdot F(s) - f(0^-)$$

$f(0^-)$ ↳ initial value

$$\frac{d^2}{dt^2}[f(t)] \rightleftharpoons s[s \cdot F(s) - f(0^-)] - f'(0^-)$$

$$\rightleftharpoons s^2 \cdot F(s) - s f(0^-) - f'(0^-)$$

Example: Given the following differential equation, find the L.T. of $y(t)$, if all initial conditions are zero.

$$\frac{d^2}{dt^2}(y(t)) + 12 \cdot \frac{d}{dt}(y(t)) + 32 \cdot y(t) = 32\, u(t)$$

Solution: $L[y(t)] = Y(s)$

Applying second order time differentiation property:

$$L\left[\frac{d^2}{dt^2} y(t)\right] = s^2 \cdot Y(s) - s\, \cancel{y(0^-)}^{\,0} - \cancel{y'(0^-)}^{\,0} = s^2 \cdot Y(s)$$

$$L\left[\frac{d}{dt} y(t)\right] = s \cdot Y(s) - \cancel{y(0^-)}^{\,0} = s \cdot Y(s)$$

$$L[u(t)] = \frac{1}{s}$$

Now, taking L.T. both sides on given differential eq^n:

$$L[y''(t)] + 12 \cdot L[y'(t)] + 32 \cdot L[y(t)] = 32 \cdot L[u(t)]$$

$$s^2 \cdot Y(s) + 12 \cdot s \cdot Y(s) + 32 \cdot Y(s) = 32 \cdot \frac{1}{s}$$

$$s^3\, Y(s) + 12 s^2\, Y(s) + 32 s\, Y(s) = 32$$

$$Y(s) \cdot [s^3 + 12 s^2 + 32 s] = 32$$

$$\boxed{L[y(t)] = Y(s) = \frac{32}{s^3 + 12 s^2 + 32 s}}$$

Note: We converted a complex differential equation to a simple algebraic equation by using Laplace Transform ☺

2. References:

1. Neso Academy

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