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1. Problem #5 :

An 80 kg bungee jumper jumps from a bridge at point A with no bungee cord attached to the ankles. The jumper falls 20m before the 10m long elastic portion of the bungee cord begins to stretch. Cord drops a total of 40m before reaching the position shown. Neglecting any energy losses, determine:
(a) The stiffness k of the cord for which the maximum velocity occurs when the jumper is at point B.
(b) The maximum acceleration then.
Start any assumptions.

Solution:

1. Assumptions:
(i) Negligible air resistance and other energy losses
(ii) The bungee cord returns to an ideal spring
(iii) The jumper's mass is concentrated at a point

(a) Stiffness k of the cord:

The cord begins to stretch after 20m of the fall. The cord first is 10m, so the stretch of the cord is 30.

$\Delta L = 40 - 10$
$= 30m$

The potential energy lost by the jumper is converted into elastic potential energy by the cord. At the lowest point, the jumper momentarily comes to rest.

Form principle of conservation of energy:

$mgh = \frac{1}{2}k(\Delta L)^2$

$80(9.81)(40) = \frac{1}{2}k(30)^2$

$k = 104.96 \frac{kN}{m}$

(b) Maximum velocity occurred during the fall:

Maximum velocity occurs when the net force on the jumper is zero. Which is when the upward force equals the net equals the downward force of gravity.

$\Sigma F_{force} = 0$
$+\uparrow \Sigma F_y = 0$
$-kx + mg = 0$
$x = \frac{mg}{k}$
$x = \frac{80(9.81)}{104.96}$
$x = 7.34m$

∴ Total distance fallen to reach v_{max} = 20 m + 30 m + 7.34 m
= 57.34 m

Conservation of energy from the start to point B, then:

$mgx = \frac{1}{2}mv_{max}^2 + \frac{1}{2}k\Delta L^2$

$80(9.81)(57.34) = \frac{1}{2}(80)v_{max}^2 + \frac{1}{2}(104.96)(30)^2$

$\Rightarrow v_{max} = 33.34 \frac{m}{s}$

(b) Maximum acceleration (Amax):

Maximum acceleration occurs at the lowest point of the fall where the cord's stretch is maximum (i.e. 40+30m).

$\Sigma F_{net} = k\Delta L - mg$

$a_{max} = \frac{k\Delta L - mg}{m}$

$a_{max} = \frac{104.96(40+30) - 80(9.81)}{80}$

$a_{max} = 238.4 \frac{m}{s^2}$

2. Problem #7 :

A sprinkler otherwise through a long drain on a level wind at a constant speed of 30 km/hr. The plane disappears 100.00 km at some point ahead. Then the 95% drain with a velocity (5.00) relative to the plane. Calculate the trajectory then on the time on the duration of water necessary to cross the gulch formed, and the lateral field. Also the wind and the time.

Solution:

Given:

- velocity of plane relative to ground:

$v_p = 30 km/hr = 30 \times \frac{5}{18} m/s$
$= 8.33 m/s$

- Drop distance north:

$d = 100.0 \times \frac{km}{1000} = 0.100 \frac{km}{1000}$
$= 1.00 \times 10^{-4} m$

- Gravity: speed relative to the plane:

$|v_{pw}| = 5 m/s$

$v_{pw,x} = 0$ $\quad v_{pw,y} = 5 \cos 90° = 0 \frac{m}{s}$ $\quad v_{pw,z} = 5 \sin 90° = 5 \frac{m}{s}$

Velocity on the ground frame:

- Wind speed (measured at rest, ahead of plane):

$v_w = (x, y, z)$

- Each speed (ground frame) = plane velocity + relative velocity

$v_{wx} = (v_p + 0 + 0 + 0) = (0 + 0 + 0)$

$= (8.33 \frac{m}{s}, 0 + 0 + 0, 0 + 0)$

$= (8.33 \frac{m}{s}, 0 \frac{m}{s}, 0 \frac{m}{s})$

Thus,

$v_{wx,y,z} = 8.33 \frac{m}{s}, 0 \frac{m}{s}, 0 \frac{m}{s}$

Maximum distance in y direction (Ymax):

$\Sigma F_y = ma(v_{wy} - v_{py})$

$= (m)(0 + 0 + 0 - 0)$

$= 5.56 \frac{m}{s}$ (Downward)

Maximum distance in y direction (Ymax):

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In order to allow for an upward motion, the cannon must be inclined. I assume that the cannon is a hollow sphere.

Solution:

Equations below:

Block has spheres with by slide on the top of an inclined plane (angle θ) and a finite fall of distance L down the ramp, and hollow. Then simultaneously without slipping.

The box must reach the bottom that is, the cord spheres and the second box's hollow sphere.

Given:

- A rotary body's translational acceleration is down on inclined plane (with θ):

$a = \frac{g \sin\theta}{1 + \frac{I}{mr^2}}$

where, I = Rotational inertia
m = Mass of the sphere
r = Distance of rotation

For a solid sphere:

$I_{solid} = \frac{2}{5}mr^2$

$\Rightarrow a_{solid} = \frac{g \sin\theta}{1 + \frac{2}{5}}$

$= \frac{5}{7} g \sin\theta$

For a hollow sphere:

$I_{hollow} = \frac{2}{3}mr^2$ (Non-rotated hollow sphere)

$\Rightarrow a_{hollow} = \frac{g \sin\theta}{1 + \frac{2}{3}}$

$= \frac{3}{5} g \sin\theta$

Since, $\frac{5}{7} g \sin\theta > \frac{3}{5} g \sin\theta$

$\Rightarrow a_{solid} > a_{hollow}$

So, solid sphere reaches bottom first.

Equations Required: Since, moment of inertia of solid sphere is less than moment of inertia of hollow sphere this means, solid sphere will require less rotational inertia to get it moving, and thus will accelerate faster.

On the other hand, the hollow sphere with large moment of inertia, will resist changes in angular velocity and accelerate more slowly.

4. Problem #8 :

The uniform 20-kg gate ABC is balanced in the horizontal position when a 50N is horizontal force is suddenly applied at B. If the coefficients of the hinge, friction are 0.25 and 0.20 respectively, determine the initial acceleration of point A.

Solution:

Let's first find out the direction of rotation.

Torque generated by force force at B: $\tau_{B,cw} = (50)(0.3)$
$= 15 Nm$ (cw)
Force tends to rotate clockwise.

$\tau_{B,ccw} = 0 Nm$
No force tends to rotate counter-clockwise.

Torque generated by Gravitational force: $\tau_{G,cw} = (\frac{L}{2})(mg)$
$= (\frac{0.4}{2})(20)(9.81)$
$= (0.2m)(196.2N)$
$= 39.24 Nm$
(cw)

∴ Net Torque = 39.24 - 15
= 24.5 Nm
The net will rotate clockwise.

We know that,

$\tau_{net} = I\alpha$

$24.5 = (\frac{1}{3}ml^2)\alpha$

$\Rightarrow \alpha = \frac{24.5 Nm}{\frac{1}{3}(20)(0.4)^2}$ (counter clockwise)

$= 91.875 \frac{rad}{s^2}$ (CCW)

∴ Net force acting in x direction:

$\Sigma F_{net,x} = 50 - \mu_s mg$

$= (0.25)(20)(9.81)$

$(F_{net})_x = 32.95 N$

$(F_{net})_x = 7.95 N$
(acceleration in point A)

Let's find out the tangential acceleration at A.

- The tangential acceleration can be found out using point B, by taking the product of the net acceleration times the distance of rotation.

- Distance of rotation will be 0.4 m, the tangential acceleration is therefore constant.

$\Rightarrow a_{tan} = r\alpha$

$= (0.4)$
$= 91.875 \frac{m}{s^2}$

∴ Acceleration at point A:

Net acceleration at A is obtained as

$(a_{net})_A = \sqrt{a_{tan}^2 + (a_{net})^2}$

$= \sqrt{91.875^2 + 7.95^2}$

$= 92.22 \frac{m}{s^2}$

THE END