Rotational Motion Basics

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1. Introduction:

- If an object is rotating or spinning, then we can say that object is in rotational motion.
- Rotational motion is different from linear or straight motion, angular speed is not a straight line.
- Linear motion and rotational motion are same.

2. Angular Position & Angular Displacement:

Let's first talk about linear position & displacement.

x_1 and x_2 are linear positions of an object at points A and B

In linear displacement (d) will be:

$$d = x_2 - x_1$$
$$= (\text{final position}) - (\text{initial position})$$

In Rotational Motion:

θ_A → Angular position at A

θ_B → Angular position at B

So, Angular Displacement, $\Delta\theta = \theta_B - \theta_A$ → (final angular position) − (initial angular position)

- Standard unit for Angular Displacement → radians

3. Angular velocity:

- It tells how fast an object is rotating

- Average linear velocity, $\bar{v} = \frac{\text{distance}}{\text{time}} = \frac{d}{t}$ or $\frac{x_2 - x_1}{t}$

- Average Angular velocity, $\bar{\omega} = \frac{\text{Angular displacement}}{\text{time taken}}$

$$= \frac{\Delta\theta}{\Delta t} \text{ or } \frac{\theta_2 - \theta_1}{t}$$

- Relationship b/w linear & angular velocity:

Linear velocity = (angular velocity) × (distance from axis of rotation)

$$v = r\omega$$

- Important Example:

Consider a wheel is rotating in fans marked 2 points on the wheel A and B.

So which point has greater Angular velocity, A or B?

Ans: Same as both points A and B are on same wheel so, $\omega_A = \omega_B$

So, that has about the linear velocities, we may say as the fix A and B.

Ans: No, their linear velocities will be different because their distance from center is different.

$$v_A = r_A \omega_A \quad ; \quad v_B = r_B \omega_B$$

Since $r_A > r_B$, therefore,

$$v_A > v_B$$

So, A point will have greater linear velocity.

Conversion Sum: Since both points are on the same wheel, point A has to cover bigger circumference than point B, therefore, A has to have greater linear velocity than B.

4. Time Period and Frequency:

- Time period, T → time taken by an object to complete one cycle → time / cycle → seconds

- Frequency, f → $\frac{\text{cycles}}{\text{time}} = \frac{1}{T}$ → Hertz

- Angular velocity, $\omega = 2\pi f = \frac{2\pi}{T}$

5. Angular Acceleration:

Average Linear acceleration, $\bar{a}$ → change in velocity / time in time

$$= \frac{v_2 - v_1}{\Delta t} \text{ m/s}^2$$

- Average Angular acceleration, $\bar{\alpha} = \frac{\omega_2 - \omega_1}{\Delta t}$ rad/s²

6. Centripetal & Tangential Acceleration:

→ Centripetal acceleration

- Centripetal acceleration, $a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r}$

$$= r\omega^2$$

- Imp: If an object is moving with constant speed around an arc, then it will only have centripetal acceleration.

- Now, what if object is not moving with constant speed around an arc, then it will have both.

- In this case, object will have 2 accelerations, centripetal as well as tangential acceleration.

$$a_{net} = \vec{a}_c + \vec{a}_t$$

a_{net} & a_c are always perpendicular to each other.

$$\therefore a_{net} = \sqrt{a_c^2 + a_{net}^2}$$

7. Important Stuff:

LINEAR	ROTATIONAL
1. If object is moving with constant speed. $\Delta v = 0$	1. $\Delta\omega = 0$
2. object is constant acceleration. $v_f = v_i + at$ $v_f^2 = v_i^2 + 2as$ $s = v_i t + \frac{1}{2}at^2$	2. $\omega_f = \omega_i + \alpha t$ $\omega_f^2 = \omega_i^2 + 2\alpha\theta$ $\theta = \omega_i t + \frac{1}{2}\alpha t^2$
3. $v_f^2 - v_i^2 = 2as$ or $v_f^2 - v_i^2 = 2as$	3. $\omega_f^2 - \omega_i^2 = 2\alpha\theta$
4. $s = v_{avg} t$ or $s = \frac{1}{2}(v_i + v_f)t$	4. $\theta = \omega_{avg} t$ or $\theta = \frac{1}{2}(\omega_i + \omega_f)t$
5. $s = \left(\frac{v_i + v_f}{2}\right)t$ → average velocity	5. $\theta = \left(\frac{\omega_i + \omega_f}{2}\right)t$ → Average angular velocity

$\boxed{d = r\theta}$ → Linear displacement → Angular displacement

$\Rightarrow v = r\omega$

$\Rightarrow a = r\alpha$

8. Problems:

1. A wheel spins at a constant angular speed of 2π radian/sec and makes 4π turns in 2 min.
(a) Find angular velocity. (with the dial go through in 2 minutes)
(b) How far, along the road will the wheel travel at the end in 2 minutes?

Solution: Given: $\omega = 2\pi$ rad/sec

(a) In 1 second, dial comes 2π radians
In 60 seconds → → → 2π×60 radians
1 minute → → → 120π radians
2 minutes → → → 2×120π radians

So, total no. of revolutions = $\frac{2\times120\pi\text{ rad}}{2\pi}$
$= 120$ revolutions
$\frac{\text{total revolutions}}{\text{time}}$ → $\frac{120}{2}$ revolutions/min
$= 1140$ revolutions.

(b)

$d = ?$
$t = 2\text{ min}$
$= 2\times60 = 120$
$\omega = 2\pi$ rad/sec
$\theta = 4\pi$ rad

$d = \theta R$
$= 4\pi R$
$= 2\times\frac{22}{7}\times R$
$= 12\frac{4}{7}$ meter

2. A disc spins by from rest at a constant rate of 3.5 rad/s.
(a) What is the final angular speed of the disk after 12 seconds?
(b) How many revolutions will the disk go through during this time period?
(c) What is the linear speed at a point at the rim of the disk after 12 seconds?
Hint: Radius of disk is 50 cm.

Solution: $R = 50\text{ cm} = \frac{50}{100} = 0.5\text{ m}$

(a) $\omega_0 = 0$ rad/s
$\alpha = 3.5$ rad/s²
$\omega_f = \omega_i + \alpha t$
$= 0 + (3.5)(12)$
$= 42\frac{\text{rad}}{\text{s}}$

(b) $\theta = \omega_i t + \frac{1}{2}\alpha t^2$
$= 0 + \frac{1}{2}(3.5)(12)^2$ radians
$= \frac{3.5\times144}{2}$ revolutions
$= 252$ revolutions
$[\because 1 = 2\pi]$
$= 40.14$ revolutions.

(c) linear speed at the edge after 12 seconds:
Since, we are calculating the speed at the edge that means the distance from the center will equal to R.

$\therefore v = \omega R$
$= (42)\left(\frac{50}{100}\right)$
$= 21\frac{\text{m}}{\text{s}}$

3. A disk with a diameter of 60 cm starts up from 20 rad/s to 40 rad/s in 5 seconds.
(a) How many revolutions will the disk go through during that time period?
(b) What is the average linear acceleration of the disk?
(c) What is the average linear acceleration of the disk?

Solution: $R = \frac{60}{2}\text{ cm} = \frac{60}{200} = \frac{3}{10}$ m
$= 0.3$ m

(a) $\omega_i = 20$ rad/s, $\omega_f = 40$ rad/s, $t = 5$ seconds

$\omega_f = \omega_i + \alpha t$
$40 = 20 + \alpha(5)$
$\alpha = \frac{20}{5}$
$= 4\text{ rad/s}^2$

$\therefore \theta = \omega_i t + \frac{1}{2}\alpha t^2$
$= 100 + 50$
$= 150$ radians
$= \frac{150}{2\pi}$ revolutions
$= 23.87$ revolutions

(b) $\bar{a} = \frac{\omega_2 - \omega_1}{\Delta t}$
$= \frac{40 - 20}{5}$
$= 4\text{ m/s}^2$

(c) $\bar{a}_c = \frac{v^2}{R}$
$= \frac{\left(\frac{3}{10}\right)^2}{\left(\frac{3}{10}\right)}$
$= 1.2\text{ m/s}^2$

4. A wheel with a diameter of 60 cm starts up from 20 rad/s to 80 rad/s. The linear acceleration of the edge of the wheel is 15 m/s². How many revolutions will the wheel go through during this period?

Solution:

$R = \frac{60}{2}\text{ cm} = \frac{60}{200}$
$= \frac{3}{10}$
$= 0.3$ m

$\omega_i = 20$ rad/s, $\omega_f = 80$ rad/s, $a_{linear} = 15$ m/s²

$a_{linear} = \alpha R$
$\alpha = \frac{a_{linear}}{R}$
$\alpha = \frac{15}{0.3}$ rad/s²

$\therefore \omega_f^2 - \omega_i^2 = 2\alpha\theta$
$(80)^2 - (20)^2 = \left(\frac{15\times2}{0.3}\right)\theta$

$\frac{6400 - 400}{15\times2} = \theta$
$\theta = \frac{6000}{30}$ radians
$= \frac{3000}{15\times2\pi}$ revolutions
$= 31.83$ Revolutions

9. References:

- The Physics Classroom, Video

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