Parallel Axis Theorem

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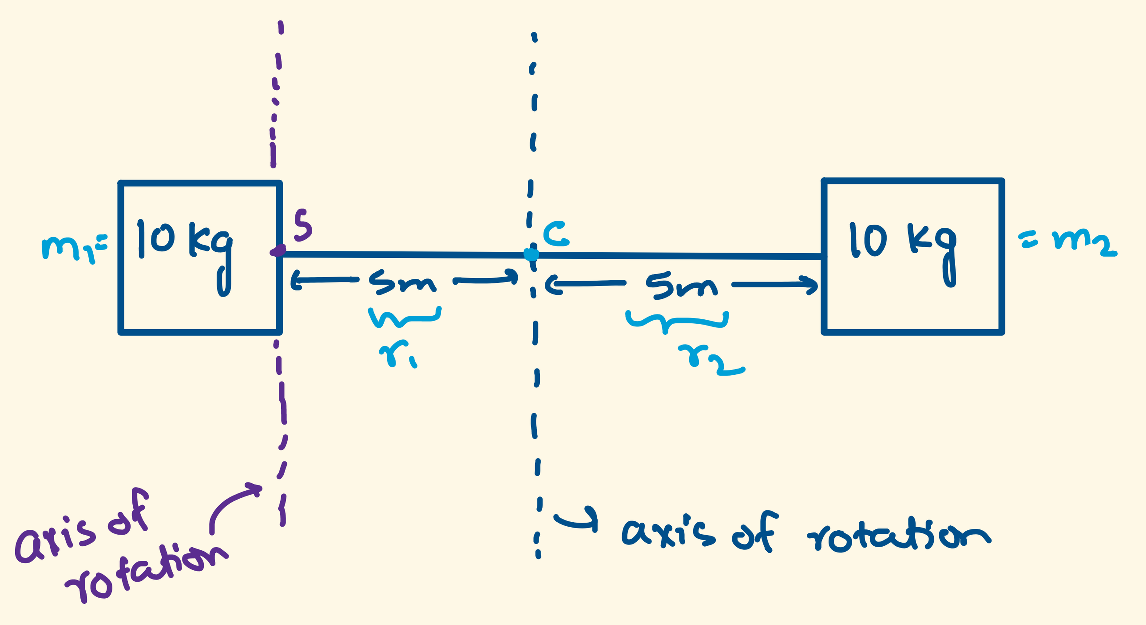
1. Introduction:

Let's say, we have two blocks (10 kg each) seperated by 10 m distance.



What is the total inertia of the system?

* Moment of Inertia about center of mass C:

$$\sum I_c = m_1 r_1^2 + m_2 r_2^2$$
$$= 10(5)^2 + (10)(5)^2$$
$$= 500 \text{ kg m}^2$$

* Moment of Inertia about Axis of Rotation S:

$$\sum I_s = \left(\begin{array}{c}\text{Moment of Inertia}\\ \text{of object 1}\end{array}\right) + \left(\begin{array}{c}\text{Moment of Inertia}\\ \text{of object 2}\end{array}\right)$$
$$= 10\cdot(0)^2 + 10(10)^2$$
$$= 1000 \text{ kg}\cdot\text{m}^2$$

Hmm, is there any other way to calculate the same.

Yes, there is. We can use Parallel Axis Theorem.

* Assumptions of Parallel Axis Theorem:

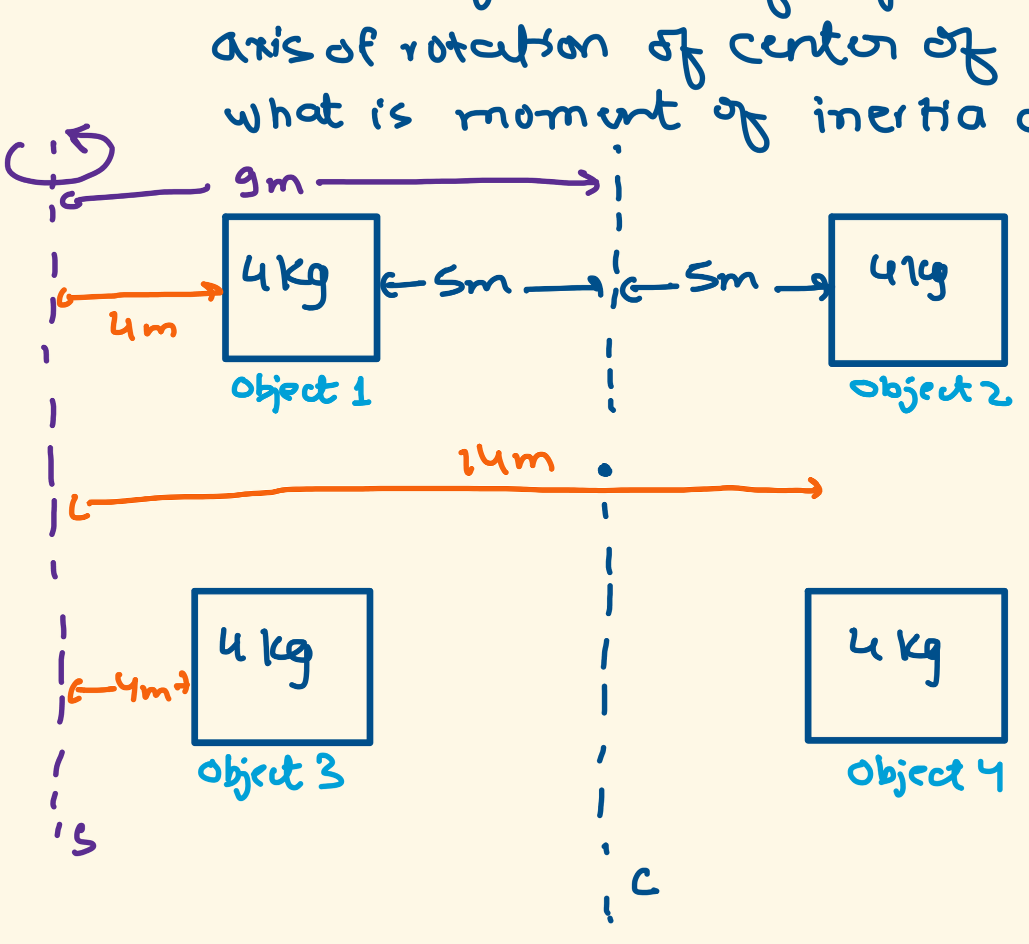
- The new axis should be parallel to the axis of rotation about the center of mass.
- System should be symmetric (in our case it is as they both have same mass 10 kg.).

$$I_{New} = I_c + \underbrace{M}_{\text{total mass of the system}}\underbrace{d^2}_{\text{displacement b/w new axis and axis of rotation of center of mass}}$$

$$= 500 + (10+10)(5)^2$$
$$= 500 + 20(25)$$
$$= 1000 \text{ kg}\cdot\text{m}^2$$

2. Examples:

Example: We have 4 blocks kept equidistant from each other as shown below. Calculate the moment of inertia of system about axis of rotation of center of mass. Also, what is moment of inertia about axis S.



Solution: $I_c = \sum (mr^2)$

$$= 4\cdot(mr^2) \quad [\because \text{identical}]$$
$$= 4\cdot(4\cdot 5^2)$$
$$= 400 \text{ kg}\cdot\text{m}^2$$

for calculating, moment of inertia about axis S, let's use parallel axis theorem.

$$I_s = I_c + Md^2$$
$$= 400 + (4+4+4+4)(9)^2$$
$$= 400 + 16\cdot(81)$$
$$= 400 + 1296$$
$$= 1696 \text{ kg}\cdot\text{m}^2$$

You know what, let's try to calculate I_s without using parallel axis theorem and see the PAIN ☹

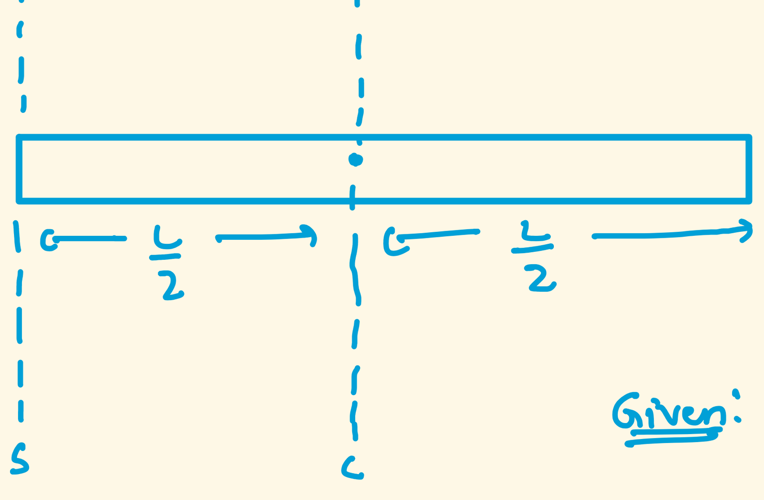
$$I_s = (MOI)_1 + (MOI)_2 + (MOI)_3 + (MOI)_4$$
$$= 4(4)^2 + 4(14)^2 + 4(4)^2 + 4(14)^2$$
$$= 2[4\cdot 4^2] + 2[4\cdot(14)^2]$$
$$= 2\cdot 64 + 2[4\cdot 196]$$
$$= 2[64 + 784]$$
$$= 2[848]$$
$$= 1696 \text{ kg}\cdot\text{m}^2$$

It was complicated, wasn't it.

Example: We know that for a rod, the moment of inertia about the axis of rotation of center of mass is $\frac{1}{12}ML^2$.

Using parallel axis theorem, prove that MOI about the axis of rotation at the edge of the rod is $\frac{1}{3}ML^2$.

Solution:



Given: $I_c = \frac{ML^2}{12}$

$$I_s = I_c + Md^2$$
$$= \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2$$
$$= ML^2\left(\frac{1}{12} + \frac{1}{4}\right)$$
$$= ML^2\left(\frac{1+3}{12}\right)$$
$$= \frac{1}{3}ML^2$$

Hence, proved

3. References:

1. The Organic Chemistry Tutor

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