Introduction to Transfer Function

Saturday, June 14, 2025 2:49 AM

Author : Kumar Anurag

1. Definition:

- Transfer function is the ratio of Laplace transform of output to the Laplace transform of input, when all the initial conditions are assumed to be zero.

- Transfer function is an important parameter of an LTI system. We use the transfer function to define the LTI system.

Input $x(t)$ → [LTI system, Impulse Response : $h(t)$] → Response $y(t)$

$$y(t) = x(t) \underset{\text{convolution}}{\underset{\uparrow}{*}} h(t)$$ (Time domain)

By Convolution Property:

$$Y(s) = X(s) \cdot H(s)$$ (frequency domain)

where, $Y(s)$, $X(s)$ and $H(s)$ are Laplace transforms of $y(t)$, $x(t)$ and $h(t)$ respectively.

$$\boxed{H(s) = \frac{Y(s)}{X(s)}}$$

Transfer function

2. Example:

Find the transfer function of the LTI system given by:

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2 \cdot y(t) = x(t)$$

where,

$x(t) \rightarrow$ input

$y(t) \rightarrow$ output

Solution: We know from time differentiation property of LT:

$$L\left[\frac{d}{dt}[y(t)]\right] = s \cdot Y(s) - \cancel{y(0^-)}^{0}$$

[∵ Since LTI system so, initial condition zero]

$$L\left[\frac{d^2}{dt^2}[y(t)]\right] = s^2 \cdot Y(s) - s \cdot \cancel{y(0^-)}^{0} - \cancel{y'(0^-)}^{0}$$

$$= s^2 \cdot Y(s)$$

$$L[y(t)] = Y(s)$$

Apply LT both sides of differential equation:

$$s^2 \cdot Y(s) + 3 \cdot s \cdot Y(s) + 2 \cdot Y(s) = X(s)$$

$$Y(s)[s^2 + 3s + 2] = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2+3s+2} = \frac{1}{s^2+2s+s+2}$$

$$H(s) = \frac{1}{(s+1)(s+2)}$$

3. References:

1. Neso Academy

—x— THE END —x—