Fall 2018

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Author: Kumar Anurag

1. Problem 01:

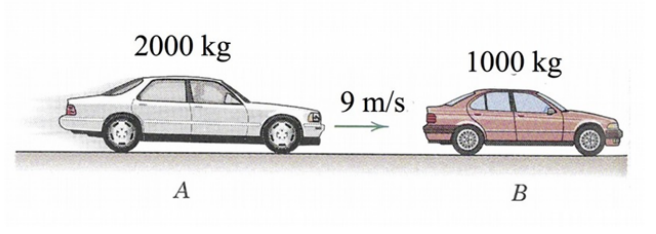
Car B is initially stationary and is struck by a car A moving with initial speed $v_1 = 9\,m/s$. The cars became entangled and move together with speed v' after the impact. If the duration of the collision is 0.1 sec. Determine.

(a) the common final speed v'

(b) the average acceleration of each car during the collision.

(c) the magnitude R of the average force exerted by each car on the other car during impact.

Assume that the breaks are released during the collision. State any other assumptions.



Solution:

Assumptions

① Neglecting External Impulses

Given:

$m_A = 2000\,kg$ $\quad v_{A0} = 9\,m/s$

$m_B = 1000\,kg$ $\quad v_{B0} = 0\,m/s$

a) Conservation of Linear Momentum:

$$m_A v_{A0} + m_B v_{B0} = (m_A + m_B)v'$$

$$2000(9) + 1000(0) = 3000 \cdot v'$$

$$v' = \frac{18000}{3000}$$

$$\boxed{v' = 6\,m/s}$$ ← common final speed

b) Average Acceleration:

$$(a_{average})_A = \frac{\Delta v_A}{\Delta t} = \frac{v' - v_{A0}}{\Delta t} = \frac{6-9}{0.1} = -30\,m/s^2$$

given $[\because \Delta t = 0.1s]$

negative sign means it is decelerating

$$(a_{average})_B = \frac{\Delta v_B}{\Delta t} = \frac{v' - v_{B0}}{\Delta t} = \frac{6-0}{0.1} = 60\,m/s^2$$

(c) Magnitude of average force exerted

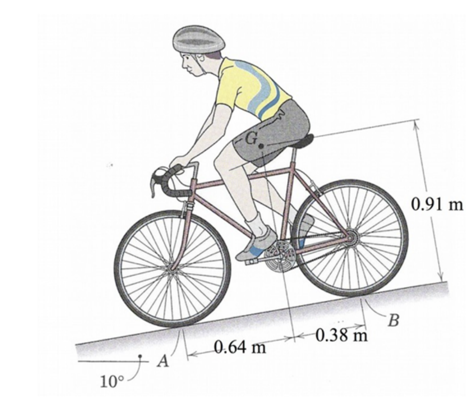
on car B $= |m_B a_B| = |1000 \cdot 60| = 60000\,N$

on car A $= |m_A a_A| = |2000(-30)| = |-60000| = 60000\,N$

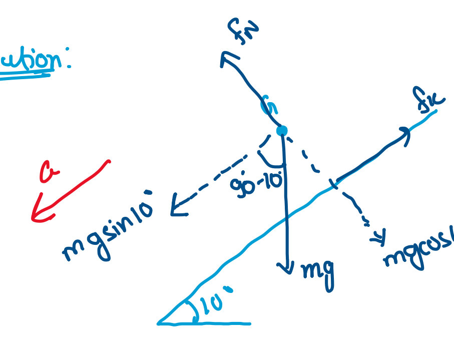
Newton 3rd law following.

2. Problem 02:

The bicyclist applies the brakes as he descends the 10° incline. What deceleration 'a' would cause the dangerous condition of tipping about the front wheel (a.k.a. front wheelie or endo)? The combined center of mass of the rider and bicycle is at G.



Solution:



$mg\cos 10°$ → this will create clockwise torque at point A (−ve)

$mg\sin 10°$ → this will create ccw torque at point A (+ve)

Net torque at point A = 0

$$+mg\sin(10°)\cdot(0.91) - mg\cos 10°(0.64) - ma(0.91) = 0$$

divide both sides by m

$$g\sin(10)(0.91) - g\cos(10)(0.64) - a(0.91) = 0$$

$$\boxed{a = -5.09\,m/s^2}$$

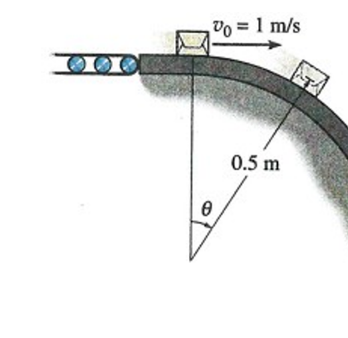
this shows deceleration.

3. Problem 03:

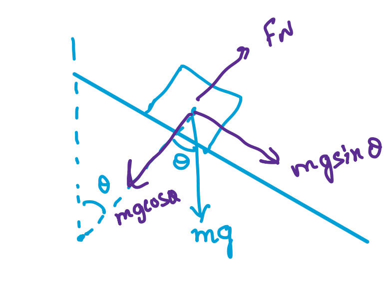
<already done in Fall 2017 - Problem 03>

4. Problem 04:

Packages having a mass of 2 kg are delivered from a conveyor belt to a smooth circular ramp with a velocity $v_0 = 1\,m/s$ as shown in the figure. If the radius of the ramp is 0.5 m, determine the angle $\theta = \theta_{max}$ at which each package begins to leave the surface. State any necessary assumptions.



Solution:



Smooth

No friction

$$\therefore \Sigma(f_{net})_y = ma_y$$

$$mg\cos\theta - f_N = ma_y$$

centripetal acceleration

$$mg\cos\theta - f_N = m\frac{v^2}{r}$$

at $v_{max} \Rightarrow f_N = 0$

$$mg\cos\theta_{max} - 0 = m\frac{v_{max}^2}{r}$$

$$\frac{v_{max}^2}{0.5} = g\cos\theta_{max}$$

$$v_{max}^2 = (0.5)(9.8)\cos\theta_{max}$$

$$v_{max}^2 = 4.9\cos\theta_{max} \quad \text{------} ①$$

$$\therefore \Sigma(f_{net})_x = ma_x$$

tangential acceleration a_t

$$mg\sin\theta = ma_t$$

$$a_t = g\sin\theta$$

Now, we know that.

$$a_t = v\frac{dv}{ds}$$

$$v\frac{dv}{ds} = g\sin\theta_{max}$$

$$\frac{v\,dv}{r\,d\theta} = 9.8\sin\theta_{max} \quad [\because s = r\,d\theta]$$

$$\frac{v\,dv}{(0.5)d\theta} = 9.8\sin\theta_{max}$$

$$v\,dv = 9.8(0.5)\int \sin\theta_{max}\,d\theta$$

$$\int_1^{v_{max}} v\,dv = 4.9\int_0^{\theta_{max}} \sin\theta_{max}\,d\theta$$

$$\left|\frac{v^2}{2}\right|_1^{v_{max}} = 4.9\left|-\cos\theta_{max}\right|_0^{\theta_{max}}$$

$$\frac{v_{max}^2 - 1}{2} = 4.9(-\cos\theta_{max} + 1)$$

$$\boxed{\theta_{max} = 42.7°}$$

—— x —— THE END —— x ——