Rotational Kinetic Energy Examples

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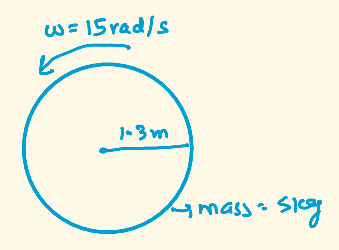
1. Problem 01:

A 5kg disc with a radius of 1.3m is spinning at an angular speed of 15 rad/s.
(a) What is the inertia of the solid disc?
(b) What is the rotational kinetic energy of the disc?

Solution:



a) Moment of Inertia. $I = \frac{1}{2}mR^2 = \frac{1}{2}(5)(1.3)^2$

$$= 4.225 \text{ kg}\cdot\text{m}^2$$

b) Linear Kinetic Energy. $K = \frac{1}{2}mv^2$

$$= \frac{1}{2}m(R\omega)^2 \quad [\because v = r\omega]$$

$$= \frac{1}{2}\underbrace{mR^2}\omega^2$$

This term mR^2 represents the moment of Inertia.

Rotational Kinetic Energy. $RKE = \frac{1}{2}I\omega^2$ Think of 'I' as rotational equivalent of mass

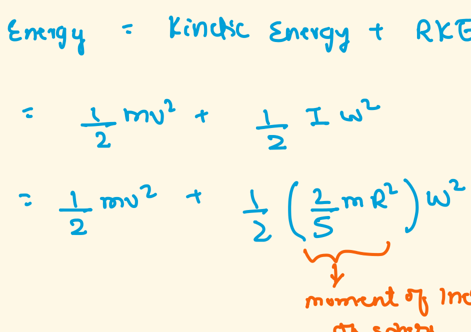
$$RKE = \frac{1}{2}(4.225)(15)^2$$

$$= 475.3\text{J}$$

2. Problem 02:

A sphere rolls down a 20° incline starting from rest at a height of 50m. How fast will it be moving forward when it reaches the bottom of the incline.

Solution:



Conservation of Energy:

Potential Energy = Kinetic Energy + RKE

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\underbrace{\left(\frac{2}{5}mR^2\right)}_{\text{moment of Inertia of sphere}}\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{5}m(R^2\omega^2)$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 \quad [\because v = R\omega]$$

$$gh = \frac{1}{2}v^2 + \frac{1}{5}v^2$$

$$(9.8)(50) = \left(\frac{1}{2} + \frac{1}{5}\right)v^2$$

$$98 \times 5 = \frac{7}{10}v^2$$

$$v^2 = \frac{98 \times 5 \times 10}{7}$$

$$v = \sqrt{700}$$

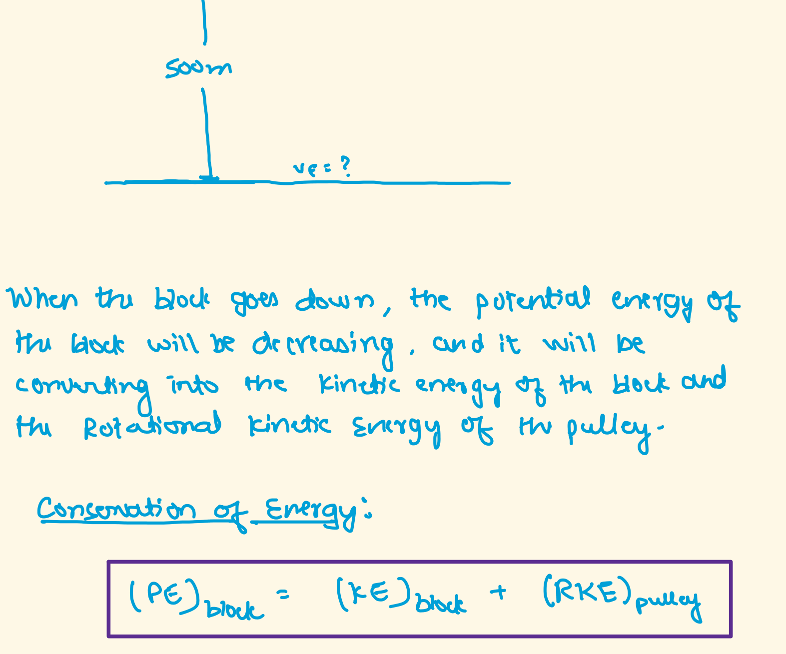
$$v = 10\sqrt{7}$$

$\boxed{v = 26.46 \text{ m/s}}$ → final velocity at the bottom of incline.

3. Problem 03:

A 20kg solid disc/pulley is attached to a hanging 10kg block which is released from rest 500m above the ground. How fast will the block be moving just before it hits the ground?

Solution:



When the block goes down, the potential energy of the block will be decreasing, and it will be converting into the kinetic energy of the block and the Rotational kinetic Energy of the pulley.

Conservation of Energy:

$$\boxed{(PE)_{block} = (KE)_{block} + (RKE)_{pulley}}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\underbrace{\left(\frac{1}{2}MR^2\right)}_{\text{Inertia of disk}}\omega^2$$

$$10(9.8)(500) = \frac{1}{2}10v^2 + \frac{1}{4}(20)(R^2\omega^2)$$

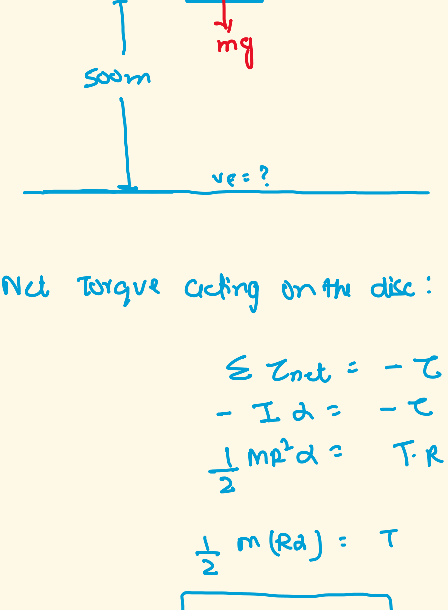
$$49000 = \frac{1}{2}10v^2 + 5v^2 \quad [\because v = R\omega]$$

$$49000 = 5v^2 + 5v^2$$

$$v^2 = 4900$$

$\boxed{v = 70 \text{ m/s}}$ → final velocity of block before it hits the ground.

Another Method



∴ Net Torque acting on the disc:

$$\Sigma \tau_{net} = -\tau \quad \text{(clockwise)}$$

$$-I\alpha = -\tau$$

$$\frac{1}{2}MR^2\alpha = T.R \quad [\because \tau = T.R]$$

Tension

$$\frac{1}{2}m(R\alpha) = T$$

$$\boxed{T = \frac{1}{2}Ma} \quad \text{---- (1)}$$

∴ Net force acting on the block:

$$\Sigma F_{net} = +T - mg$$

$$-ma = T - mg$$

negative sign because block is moving downwards

$$mg - ma = T$$

$$mg - ma = \frac{1}{2}Ma \quad [\because \text{from (1)}]$$

$$mg = a\left(m + \frac{M}{2}\right)$$

$$\boxed{a = \frac{mg}{m + \frac{M}{2}}}$$

$$a = \frac{10(9.8)}{10 + \left(\frac{20}{2}\right)} = \frac{10(9.8)}{20}$$

$$a = 4.9 \text{ m/s}^2$$

$$v^2 - u^2 = 2as$$

$$v^2 - 0 = 2(4.9)(500)$$

$$v^2 = (9.8)(500)$$

$$v^2 = 4900$$

$$\boxed{v = 70 \text{ m/s}}$$

4. References:

1. The Organic Chemistry Tutor

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