Inverse Laplace Transform Part 2

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1. Example of ILT with repeated roots :-

→ Example 01: Find the ILT of $F(s) = \dfrac{2}{(s+1)(s+2)^2}$

Roots of denominator of F(s) are real & repeating

Solution:

Using Partial Fraction Decomposition:

$$F(s) = \frac{2}{(s+1)(s+2)^2} = \frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2}$$

→ multiplying both sides by (s+1) and set (s+1) = 0

$$\frac{2}{\cancel{(s+1)}(s+2)^2}\cancel{(s+1)} = A + \frac{B}{(s+2)}\cancel{(s+1)}^{0} + \frac{C}{(s+2)^2}\cancel{(s+1)}^{0}$$

$$A = \frac{2}{(s+2)^2} = \frac{2}{(-1+2)^2} = 2$$

$$\boxed{A = 2}$$

→ Multiplying both sides by (s+2)

$$\frac{2}{(s+1)(s+2)^{\cancel{2}}} \cdot \cancel{(s+2)} = \frac{A}{(s+1)}(s+2) + B + \frac{C}{(s+2)^{\cancel{2}}}\cancel{(s+2)}$$

$$\frac{2}{(s+1)(s+2)} = \frac{A}{(s+1)}(s+2) + B + \frac{C}{(s+2)}$$

Notice, we can't substitute s = −2, as we will get zero in denominator.

So, let's set s = 0:

$$\frac{2}{1\cdot 2} = \frac{A}{1}\cdot 2 + B + \frac{C}{2}$$

$$\frac{2}{2} = 2\cdot 2 + B + \frac{C}{2} \qquad [\because A = 2]$$

$$1 - 4 = B + \frac{C}{2}$$

$$2B + C = -6 \quad \text{---- (1)}$$

Now, let's set s = 1:

$$\frac{2}{2\cdot 3} = \frac{2}{2}\cdot 3 + B + \frac{C}{3}$$

$$\frac{1}{3} = 3 + B + \frac{C}{3}$$

$$1 = 9 + 3B + C$$

$$3B + C = -8 \quad \text{--- (2)}$$

Eq^n (2) − (1):

$$B = -8 + 6$$

$$\boxed{B = -2}$$

Substituting B in Eq^n (2):

$$3(-2) + C = -8$$

$$C = -8 + 6$$

$$\boxed{C = -2}$$

Therefore:

$$F(s) = \frac{2}{(s+1)} - \frac{2}{(s+2)} - \frac{2}{(s+2)^2}$$

Apply inverse laplace transform both sides:

$$f(t) = 2\cdot e^{-t}u(t) - 2e^{-2t}u(t) - 2e^{-2t}t\,u(t)$$

$$\boxed{f(t) = \left[2e^{-t} - 2e^{-2t} - 2t\,e^{-2t}\right]u(t)}$$

2. References:

1. Neso Academy

—— x —— THE END —— x ——