Laplace Transform Part 2

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1. Properties of Laplace Transform :-

→ **LINEARITY** : is combination of **homogeneity** principle and **superposition** principle.

if ,

Time domain ↙   Laplace domain ↙

$$f_1(t) \rightleftharpoons F_1(s)$$
$$f_2(t) \rightleftharpoons F_2(s)$$

then,

$$a \cdot f_1(t) + b \cdot f_2(t) \rightleftharpoons a \cdot f_1(s) + b \cdot f_2(s)$$

This addition is because of the superposition principle

These constant multiplications are because of the homogeneity proper

Example : Find the laplace transform of the following function :

$$f(t) = u(t) + 2 \cdot e^{-3t} + 3\sin(2t)$$

Solution :

$$L[f(t)] = F(s) = L[u(t)] + L[2 \cdot e^{-2t}] + L[3\sin(2t)]$$

$$F(s) = \frac{1}{s} + 2 \cdot \frac{1}{(s+3)} + 3 \cdot \frac{2}{s^2 + 2^2}$$

$$\boxed{F(s) = \frac{1}{s} + \frac{2}{s+3} + \frac{6}{s^2 + 4}}$$

→ **TIME SCALING :**

If ,

$$f(t) \rightleftharpoons F(s)$$

then,

$$f(at) \rightleftharpoons \frac{1}{|a|} F\left(\frac{s}{a}\right)$$

Example : If the Laplace transform (L.T) of

$$f(t) \rightleftharpoons \frac{1}{s+1} ,$$

the find the L.T of $f(2t)$.

Solution :

$$L[f(t)] = F(s) = \frac{1}{s+1}$$

Using the Time Scale property :

$$L[f(2t)] = \frac{1}{|2|} F\left(\frac{s}{2}\right)$$

$$= \frac{1}{2} \cdot \frac{1}{\left(\frac{s}{2}\right) + 1} \qquad [\because |2| = 2]$$

$$= \frac{1}{2} \cdot \frac{2}{s+2}$$

$$\boxed{L[f(2t)] = \frac{1}{s+2}}$$

2. References :

1. Neso Academy

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