Transfer Function of a Closed Loop System

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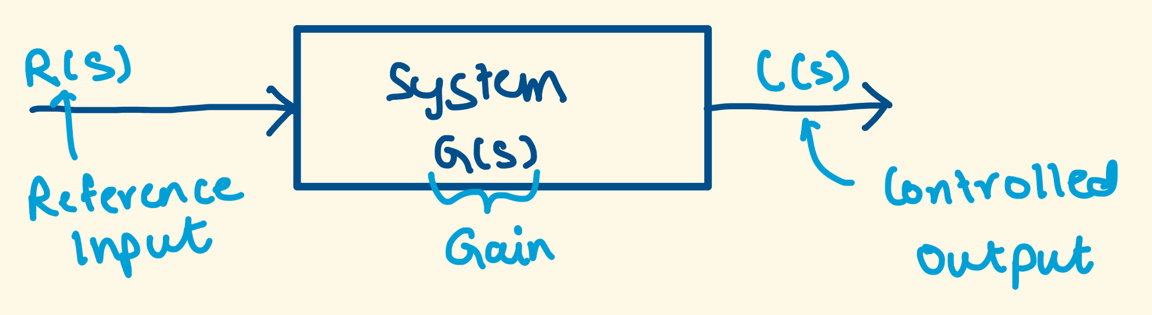
1. Transfer Function of an Open Loop System :-



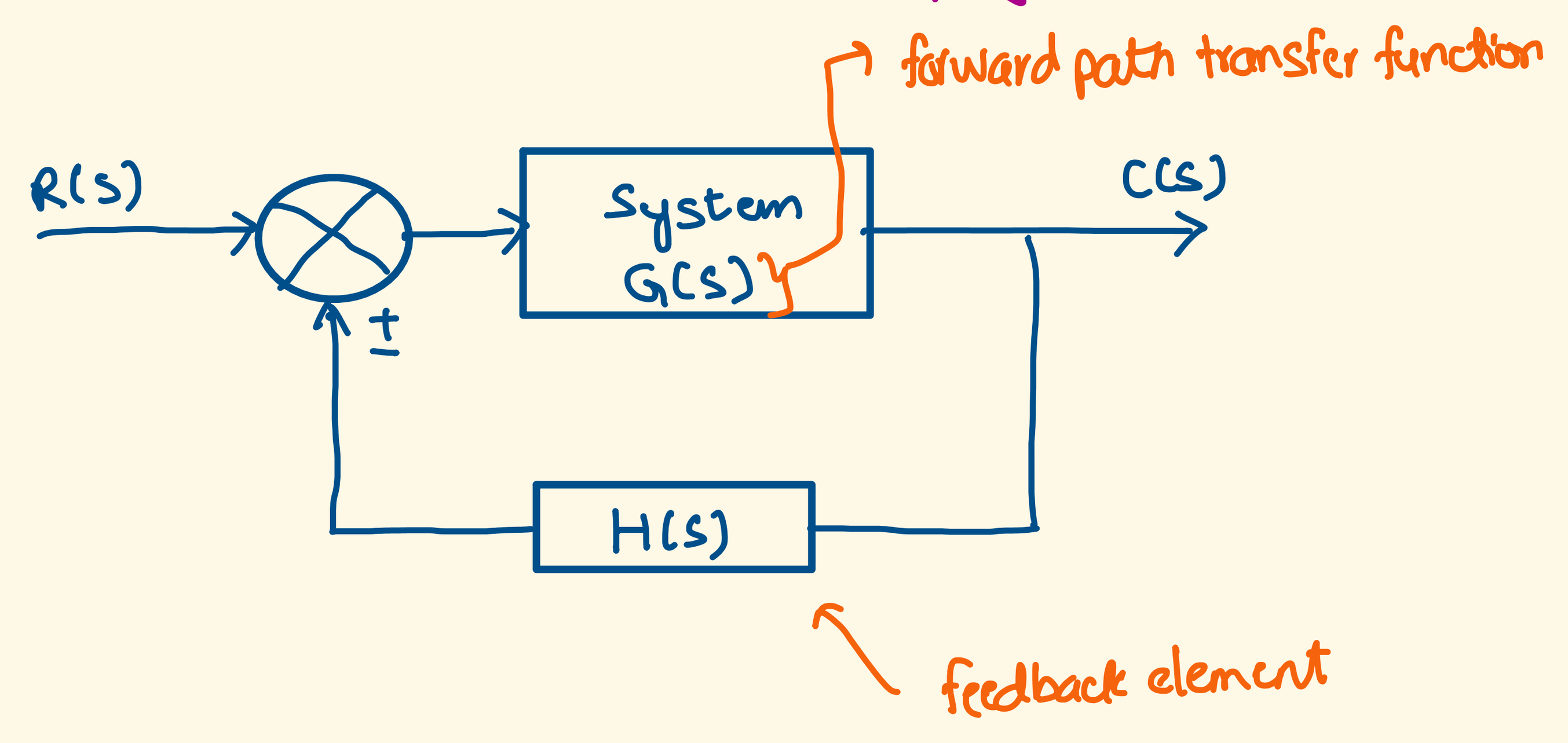
$C(s) = R(s) \cdot G(s)$

$$\Rightarrow \quad G(s) = \frac{C(s)}{R(s)}$$

C(s) → o/p
R(s) → i/p

Transfer function of open loop system

2. Transfer function of a closed loop System :-

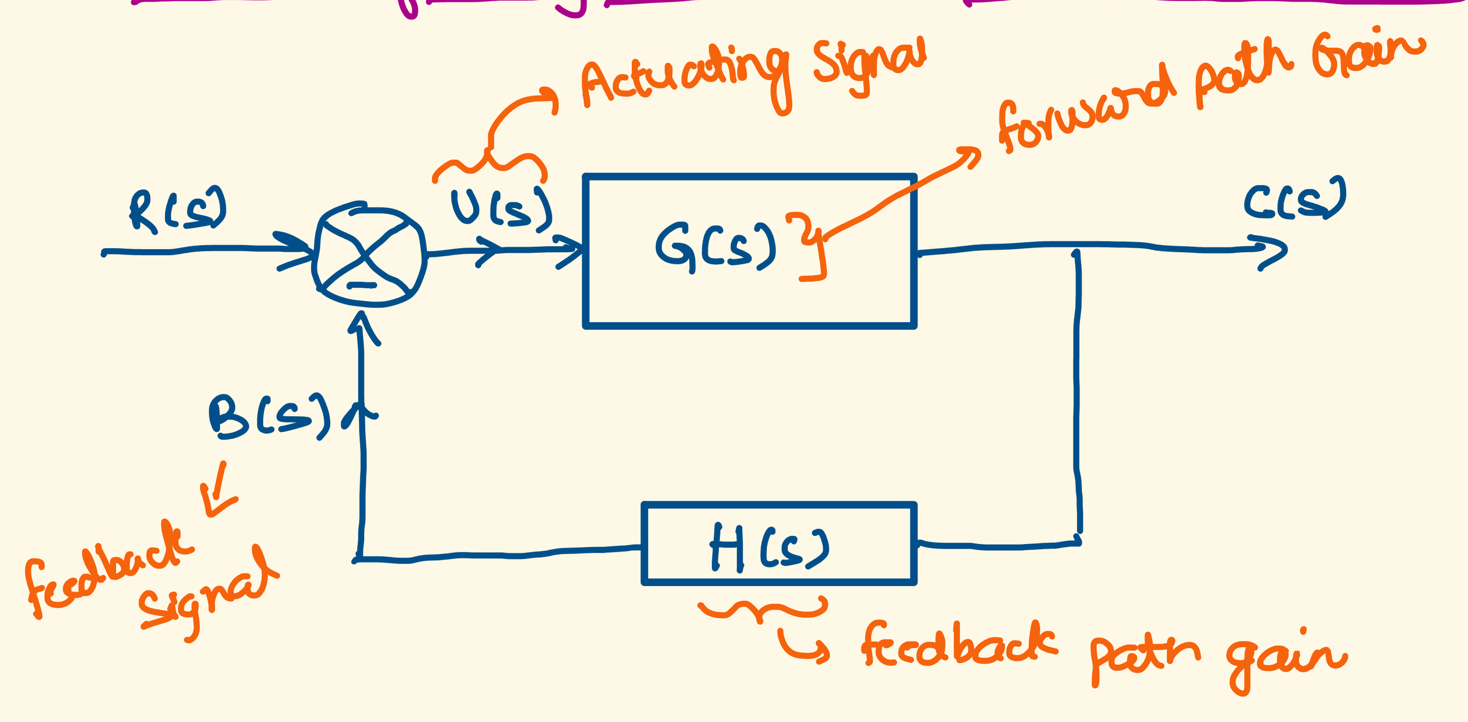


- There are two types of feedback for a closed loop system:

  (i) Positive Feedback: in which the feedback signal is in phase with the Reference Input R(s).

  (ii) Negative Feedback: in which the feedback signal is 180° out of phase with respect to the Reference Input Signal R(s).

3. Deriving Transfer Function for a closed loop system :- in case of Negative Feedback



$U(s)$ = Actuating Signal (Error signal in case of Unity Feedback System).
$B(s)$ = Feedback Signal

Since, $B(s)$ is negative feedback signal

$$U(s) = R(s) - B(s)$$
$$= R(s) - H(s) \cdot C(s) \quad [\because B(s) = C(s) \cdot H(s)]$$

We also know,

$$C(s) = G(s) \cdot U(s)$$
$$C(s) = G(s) \cdot [R(s) - H(s) \cdot C(s)]$$
$$C(s) = G(s) \cdot R(s) - G(s) \cdot H(s) \cdot C(s)$$
$$C(s)[1 + G(s)H(s)] = G(s) \cdot R(s)$$
$$C(s) = \frac{G(s) \cdot R(s)}{1 + G(s) \cdot H(s)}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)}}$$

→ Transfer function of closed loop system with negative feedback

For positive feedback:

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s) \cdot H(s)}}$$

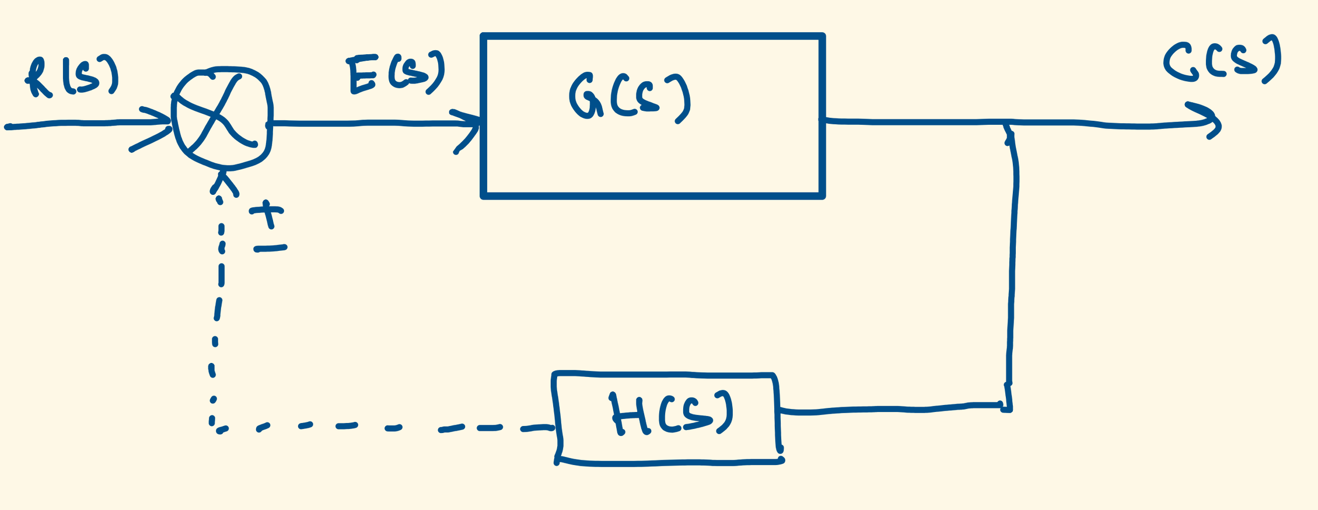
Combined Transfer function for Closed loop system:

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}}$$

→ Closed loop Transfer function (CLTF)

4. Open Loop Transfer Function (OLTF):

↳ OLTF is a characteristic of closed loop system



- In OLTF, we do not consider the effect of feedback in the input junction. That means the feedback is present because it is a closed loop system but we are not adding feedback signal into the input junction.

- There is a misconception that OLTF is for open loop system but it is a characteristic of closed loop system.

$$OLTF = G(s) \cdot H(s)$$

We already know,

$$CLTF = \frac{G(s)}{1 \pm G(s) \cdot H(s)}$$

$$\boxed{CLTF = \frac{G(s)}{1 \pm OLTF}} \quad [\because OLTF = G(s) \cdot H(s)]$$

→ we can use this formula to calculate CLTF when OLTF is given (or vice-versa).

5. Applications:

Closed loop Transfer function	Open loop Transfer function
1. To analyze the stability of the system by Routh-Hurwitz Criteria.	1. To analyze the Steady State Errors.
	2. To draw the Root Locus
	3. To draw the Bode Plot
	4. To draw the Nyquist Plot.

* Note: The OLTF is used for all the purposes except to analyze the Stability.

6. References:

1. Neso Academy

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