Routh-Hurwitz Criteria

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Author: Kumar Anurag

1. Introduction:

Necessary conditions of Stability

If we know the poles of closed loop transfer function, we can comment on the stability of the system.

The closed loop poles are the roots of the characteristic system.

Challenge: If the characteristic equation is of higher order, then it is not possible to calculate its roots to determine the closed loop poles.

The transfer function of any linear closed loop system can be represented as:

Controlled Signal   Numerator

$$\frac{C(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_n} = \frac{N(s)}{D(s)}$$

Reference Signal   denominator

The characteristic equation will be:

$$D(s) = 0$$

$$a_0 s^n + a_1 s^{n-1} + \cdots + a_n = 0$$

* Necessary (but not sufficient) conditions for Stable System:

1. All the coefficents of the polynomial must have same sign.

   $a_0, a_1, a_2, \ldots, a_n$: All should be either positive or negative

2. None of the coefficients vanishes, i.e. all the powers of 's' must be present in characteristic equation $D(s)$.

The above two conditions are necessary for a system to be stable, but not sufficient

If any polynomial satisfies the above two conditions, then it is called HURWITZ polynomial.

2. Routh-Hurwitz Criterion:-

- R-H criterion is a mathematical test that is a necessary and sufficient condition for the stability of an LTI system.
- All the roots of a characteristic equation lie in the Left half plane (LHP), if and only if, a certain set of algebraic combinations of its coefficients have same sign.
  - → Tabulating the coefficients of characteristic equation in a particular way.
  - → Tabulation of coefficients gives an array called Routh Array.
  - → Interpret the Routh table to determine the number of poles in the right-half of s-plane.

3. Method of forming Routh's Array:-

$$F(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n = 0$$

s^n	a_0	a_2	a_4	a_6
s^{n-1}	a_1	a_3	a_5	a_7
s^{n-2}	b_1	b_2	b_3	
s^{n-3}	c_1	c_2		
$\vdots$	$\vdots$			
s^0	a_n			

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}; \quad b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}; \quad b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}; \quad c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

Routh's Stability Criterion: All the terms in the first column of the Routh's array must have same sign. There should not be any sign change in the first column of Routh's array.

4. Examples:

Problem 01: Examine the stability of given equation using Routh's method:

$$s^3 + 6s^2 + 11s + 6 = 0$$

Solution: First, let's check for Hurwitz polynomial

→ all coefficients have same sign (positive) ✓
→ there is no missing power of s ✓

Since, both checks have passed, so given polynomial is a Hurwitz polynomial.

s^3	1	11
s^2	6	6
s^1	$\frac{6(11) - 1(6)}{6} = 10$	$\frac{6(0) - 1(0)}{6} = 0$
s^0	$\frac{10(6) - 6(0)}{10} = 6$	

If we see the first column of Routh's array, there is no change of sign (all have same sign, i.e. +ve), therefore the system is STABLE.

Problem 02: Examine the stability of given equation using Routh's method:

$$s^3 + 4s^2 + s + 16 = 0$$

Solution: As given polynomial has no missing power of s and also all coefficients of s have same power, so it is a HURWITZ polynomial.

s^3	1		1
s^2	4	change of sign	16
s^1	$\frac{4(1) - 1(16)}{4} = -3$		$\frac{4(0) - 1(0)}{4} = 0$
s^0	$\frac{-3(16) - 4(0)}{-3} = 16$	change of sign	

∴ No. of sign changes in 1st column of Routh' array = 2

Therefore, the system is UNSTABLE.

> **Knowledge Cloud:**
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> No. of sign changes in 1st column of Routh' array = No. of poles in Right Hand Plane

5. References:

1. Neso Academy

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