Nyquist Plot

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1. Introduction:

The Nyquist Stability Criteria is a graphical method used in control systems to determine the stability of a closed-loop system by analyzing its open-loop frequency response.

It is based on plotting the Nyquist plot, which is a plot of the open-loop transfer function $G(s)H(s)$ evaluated along the imaginary axis ($s = j\omega$) and its encirclements of the critical point $-1 + j0$ in the complex plane.

KEY IDEA: Instead of solving for the closed-loop poles directly, we analyze how the Nyquist plot of the open-loop system interacts with the critical point -1.

The RULE:

Let,

P : number of open-loop poles in the Right half Plane (RHP)

N : no of clockwise encirclements of -1 by the Nyquist Plot

Z : Number of closed-loop poles in RHP.

Then, the Nyquist Criterion states:

$$Z = N + P$$

- for a stable closed-loop system, we want no RHP poles (i.e. $Z = 0$), which implies:

$$\Rightarrow \boxed{N = -P}$$

2. Example:

A unity feedback system has a loop transfer function:

$$G(s) = \frac{50}{(s+1)(s+2)}$$

Use Nyquist Criterion to determine the system stability in the closed loop configuration. Is the open system stable.

Solution: $G(s) = \frac{50}{(s+1)(s+2)}$

- STEP-01: Substitute $s = j\omega$

$$G(j\omega) = \frac{50}{(j\omega+1)(j\omega+2)}$$

- STEP-02: Write the equations for magnitude $|G(j\omega)|$ and phase angle $\angle G(j\omega)$

Magnitude, $M = |G(j\omega)|$

$$\boxed{M = \frac{50}{\sqrt{\omega^2+1}\,\sqrt{\omega^2+4}}}$$

[$\because$ magnitude of $a + jb = \sqrt{a^2+b^2}$]

Phase Angle, $\phi = \angle G(j\omega)$

$$= \underbrace{+}_{\text{for numerator we use positive sign}} \tan^{-1}\left(\frac{0}{50}\right) \underbrace{-}_{} \tan^{-1}\left(\frac{\omega}{1}\right) \underbrace{-}_{} \tan^{-1}\left(\frac{\omega}{2}\right)$$

for denominators we use negative sign

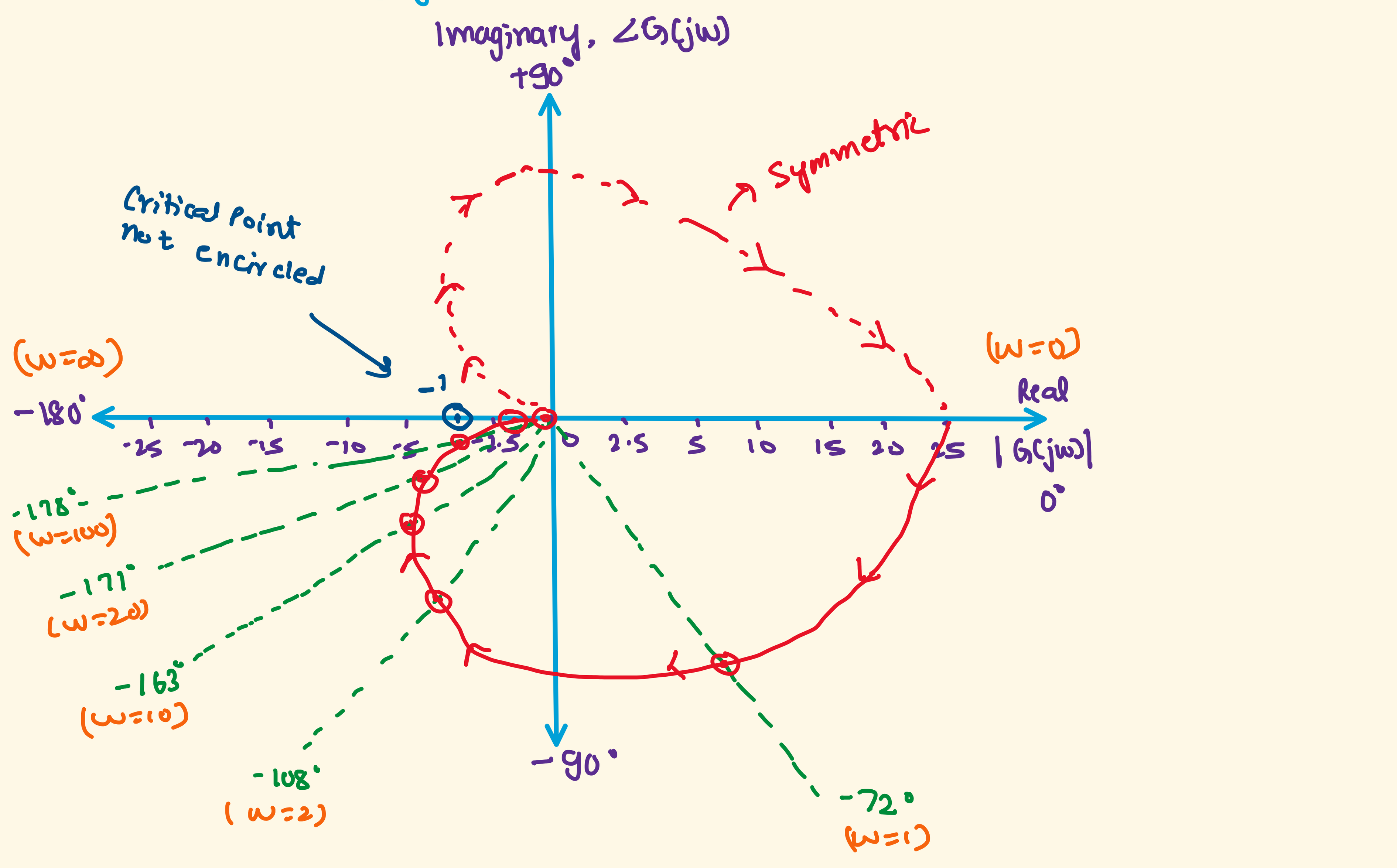
[$\because$ for $a + bj \rightarrow$ angle $= \tan^{-1}\left(\frac{b}{a}\right)$]

$$\boxed{\phi = -\tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{2}\right)}$$

- STEP-03: We will draw a table where 'ω' varies from 0 to ∞.

ω	$M = \lvert G(j\omega)\rvert$	$\phi = \angle G(j\omega)$
0	25	0°
1	16	-72°
2	8	-108°
10	0.5	-163°
20	0.1	-171°
100	0.005	-178°
∞	0	-180°

- STEP-04: Plotting



- STEP-05: Stability Check

$$Z = N + P$$

N = no. of encirclements of critical point $(-1 + j0)$

So,

$\Rightarrow \boxed{N = 0}$

P = no. of poles of open loop transfer function $G(s)H(s)$ on the Right-Half Plane (RHP)

Since, all poles (-1 and -2) of $G(s)H(s)$ are on the Left Half Plane (LHP).

$\Rightarrow$ So, $\boxed{P = 0}$ → means open loop system is STABLE.

$\therefore$ $Z = N + P$

$= 0 + 0$

$\boxed{Z = 0}$ → means closed-loop system is STABLE

3. References:

1. Ekeeda

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