Fall 2017

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1. Problem 01:

<already solved in Spring 2018 - Problem 03>

2. Problem 02:

Sketch the root locus for the system with transfer function:

$$G(s) = \frac{6(s+1)}{s^2(s+2)(s+3)}$$

Explain the rules you applied.

Solution: $G(s) = \frac{6(s+1)}{s^2(s+2)(s+3)}$

* **Poles and zeros:**

Poles: $s^2(s+2)(s+3) = 0$

$\Rightarrow s = 0, 0, -2, -3$ $\quad n = 4$

Zeros: $(s+1) = 0$

$\Rightarrow s = -1$ $\quad m = 1$

* **Asymptotes:**

No. of asymptotes $= n - m$
$= 4 - 1$
$= 3$ asymptotes

Centroid of Asymptotes, $\sigma_0 = \frac{\Sigma(\text{poles}) - \Sigma(\text{zeros})}{n-m}$

$$= \frac{(0+0-2-3)-(-1)}{4-1}$$

$$= \frac{-5+1}{3} = \frac{-4}{3}$$

$$= -1.33$$

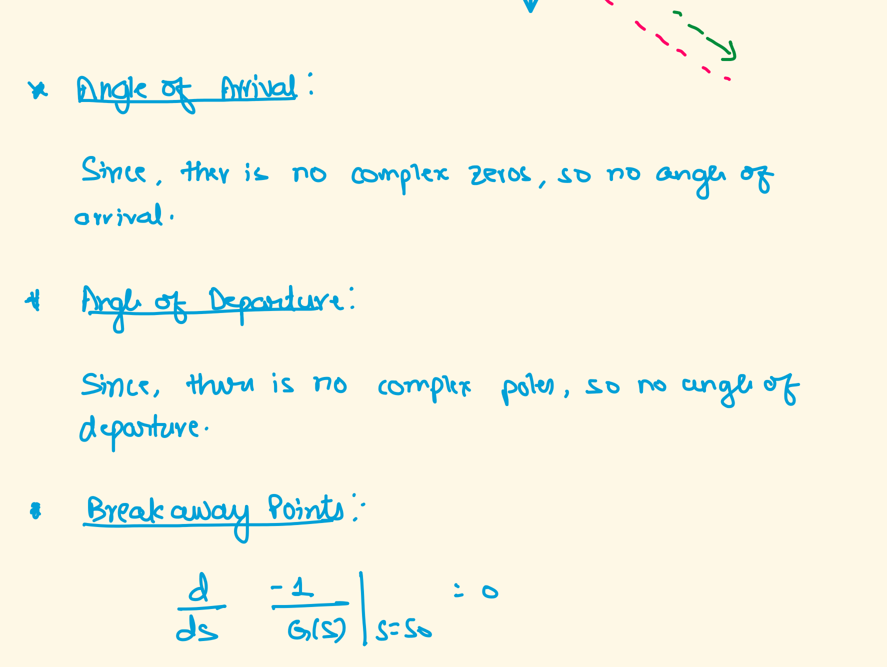
Angle of Asymptotes, $\theta_{asym} = \frac{180^\circ + 360^\circ(l)}{n-m}$

$$= \frac{180^\circ + 360^\circ(l)}{4-1}$$

$$= 60^\circ + 120^\circ(l)$$

$0 \le l \le (n-m)-1$
$0 \le l \le (4-1)-1$
$0 \le l \le 2$
$\therefore l = 0, 1, 2$

when $(l=0)$: $\theta_{as} = 60^\circ + 120^\circ(0) = 60^\circ$
when $(l=1)$: $\theta_{as} = 60^\circ + 120^\circ(1) = 180^\circ$
when $(l=2)$: $\theta_{as} = 60^\circ + 120^\circ(2) = 300^\circ$



* **Angle of Arrival:**

Since, there is no complex zeros, so no angle of arrival.

* **Angle of Departure:**

Since, there is no complex poles, so no angle of departure.

* **Breakaway Points:**

$$\frac{d}{ds}\left.\frac{-1}{G(s)}\right|_{s=s_0} = 0$$

$$\frac{d}{ds}\left.\frac{-(s^3+2s^2)(s+3)}{6(s+1)}\right|_{s=s_0} = 0$$

$$\frac{d}{ds}\left.\frac{s^4+3s^3+2s^3+6s^2}{(s+1)}\right|_{s=s_0} = 0$$

$$\frac{d}{ds}\left.\frac{s^4+5s^3+6s^2}{(s+1)}\right|_{s=s_0} = 0$$

$$\frac{(s+1)(4s^3+15s^2+12s) - (s^4+5s^3+6s^2)(1)}{(s+1)^2} = 0$$

$4s^4 + 15s^3 + 12s^2 + 4s^3 + 15s^2 + 12s - s^4 + 5s^3 + 6s^2 = 0$
$3s^4 + 24s^3 + 33s^2 + 12s = 0$
$3s(s^3 + 8s^2 + 11s + 4) = 0$

$s \approx 0 \ldots$

$\theta_{break} = 90^\circ, 270^\circ$

* **Crossing at Imaginary Axis:-**

$1 + K\,G(s) = 0$

$$1 + \frac{6K(s+1)}{s^2(s+2)(s+3)} = 0$$

$s^2(s+2)(s+3) + 6K(s+1) = 0$
$(s^3+2s^2)(s+3) + 6Ks + 6K = 0$
$s^4 + 5s^3 + 6s^2 + 6Ks + 6K = 0$

s^4	1	6	6K
s^3	5	6K	0
s^2	$\frac{30-6K}{5}$	$\frac{30K}{5} = 6K$	0
s^1	$\frac{\left(\frac{30-6K}{5}\right)6K - 30K}{\frac{30-6K}{5}}$	0	0
s^0	6K		

for system to stable:

$6K > 0$

$K > 0$

$\frac{30-6K}{5} > 0$

$30 > 6K$

$\boxed{K < 5}$

$$\frac{\left(\frac{30-6K}{5}\right)6K - 30K}{\left(\frac{30-6K}{5}\right)} > 0$$

$30K < 6K\left(\frac{30-6K}{5}\right)$

$5 < \left(\frac{30-6K}{5}\right)$

$30 - 6K > 25$
$5 > 6K$
$K < \frac{5}{6}$

$K < 0.833$

$0 < K < 0.833$

3. Problem 03:

<already solved in Spring 2018 - Problem 02>

4. Problem 04:

For a unity feedback system, evaluate the following with a ramp input $r(t) = t u(t)$, so that the state error is 0.01. The natural frequency of the closed-loop system $\omega_n = 5$ rad/s.

$$G(s) = \frac{Y(s)}{E(s)} = \frac{K}{s(s+a)}$$

a) The system type
b) The values of K and a
c) The closed-loop system's damping ratio, ξ
d) If K is reduced to 4 and $a = 0.4$, find the corresponding new values of ω_n and ξ

Solution: Given:

$$G(s) = \frac{K}{s(s+a)}, \quad H(s) = 1$$

Input: $r(t) = t\,u(t)$

Transfer function, $TF = \frac{G(s)}{1 + G(s)\cdot H(s)}$

$$= \frac{K/s(s+a)}{1 + \frac{K}{s(s+a)}}$$

$$= \frac{K}{s(s+a)+K}$$

$$\boxed{TF = \frac{K}{s^2 + as + K}}$$

a) **Type of System:-**

$$G(s) = \frac{K}{s(s+a)}$$

$\therefore$ Since, G(s) has one pole at origin, so this is Type-1 system.

b) Comparing it with the general 2^{nd} order transfer f^n:

$$\text{General, } TF = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$\therefore \omega_n^2 = K$ $\qquad 2\xi\omega_n = a$ ---- (1)
$\omega_n = \sqrt{K}$
$5 = \sqrt{K}$ ($\because$ given)
$\boxed{K = 25}$

Error = 0.01

$$\lim_{s\to 0} \frac{s\cdot R(s)}{1 + G(s)H(s)} = 0.01$$

Since, $r(t) = t\,u(t) \Rightarrow R(s) = 1/s^2$

$$\lim_{s\to 0} \frac{s\cdot\frac{1}{s^2}}{1 + \frac{K}{s(s+a)}} = 0.01$$

$$\lim_{s\to 0} \frac{1}{s + \frac{K}{s+a}} = 0.01$$

$$\frac{1}{\lim_{s\to 0}\frac{K}{s+a}} = 0.01$$

$$\lim_{s\to 0} \frac{s+a}{K} = 0.01$$

$$\frac{a}{K} = 0.01$$

$a = K(0.01)$
$= 25(0.01)$
$\boxed{a = 0.25}$

c) $2\xi\omega_n = a$ [$\because$ from eq (1)]

$2\xi(5) = 0.25$

$\xi = \frac{0.25}{10}$

$\boxed{\xi = 0.025}$ → Damping Ratio

d) New K, $K_{new} = 4$
$a_{new} = 0.4$

$(\omega_n)^2_{new} = K_{new}$

$(\omega_n)_{new} = \sqrt{K_{new}}$
$= \sqrt{4}$

$\boxed{(\omega_n)_{new} = 2 \text{ rad/sec}}$

$\therefore 2\xi_{new}(\omega_n)_{new} = a_{new}$

$$\xi_{new} = \frac{a_{new}}{2\cdot(\omega_n)_{new}}$$

$$= \frac{0.4}{2(2)}$$

$$\boxed{\xi_{new} = 0.1}$$

——x—— THE END ——x——