Angular Momentum
Lec 1

Rotation / Rumm Aksing

1. Introduction:

Before we talk about Angular Momentum, let's talk about Linear Momentum.

Linear Momentum, $\vec{p} = m\vec{v}$ ← Mass / Velocity vector

Anything that moves has a Momentum.

A train moving with high speed has larger momentum because it has more mass and high velocity.

It's difficult to stop things which have high momentum.

It's not the linear momentum → by rotation.

This is about the linear momentum.

Let's talk about Angular Momentum now:

Any object that rotates or revolve around another object has an angular momentum.

Angular Momentum, $\vec{L} = \vec{r} \times \vec{p}$

→ mass of rotation

Another common expression for Angular Momentum:

We know that:

Moment of Inertia, $I = \beta m R^2$
& Angular velocity, $\omega = \frac{v}{r}$

$$L = I\omega$$
$$L = (mR^2)\left(\frac{v}{r}\right)$$
$$L = mvR$$

Relationship the linear and angular momentum:

$$L = mvR$$
$$L = (mv)R$$
$$L = \vec{p} \times \vec{r} \Rightarrow \vec{L} = \vec{r} \times \vec{p}$$

Cross product

2. Conservation of Angular Momentum:

Let's find out conservation of linear momentum.

Newton Second's 2nd law of motion:

$$\vec{F} = m\vec{a}$$
$$= m\left(\frac{\Delta \vec{v}}{\Delta t}\right)$$
$$= (m\Delta \vec{v})\frac{1}{\Delta t}$$
$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t} \quad \left[\because \vec{p} = m\vec{v}\right]$$

which means the net force acting on the object is basically the rate of change of linear momentum.

If the net force acting on the object is zero, which means the change in momentum of that object is zero.

$$\vec{F}_{net} = 0 \Rightarrow \Delta \vec{p} = 0$$

which means momentum has to be conserved. In other the initial momentum will be equal to final momentum.

$$\boxed{\text{Initial} = \text{Final}}$$
$$\vec{p}_i = \vec{p}_f$$

Now, let's talk about Angular Momentum.

We know that:

$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$= \vec{r} \times \left(\frac{\Delta \vec{p}}{\Delta t}\right)$$
$$= (\Delta \vec{p} \times \vec{r})\frac{1}{\Delta t}$$
$$\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t} \quad \left[\because \vec{L} = \vec{r} \times \vec{p}\right]$$

which means the net torque acting on the system is basically the rate of change of angular momentum.

So, if net torque acting on system is zero, then change in angular momentum must be zero.

$$\vec{\tau}_{net} = 0 \Rightarrow \Delta \vec{L} = 0$$

which means the angular momentum must be conserved.

$$\boxed{\text{Initial} = \text{Final}}$$
$$\vec{L}_i = \vec{L}_f$$
$$\boxed{\vec{L}_{net} = \vec{L}_i = \vec{L}_f}$$

Initial: $\vec{L}_{net} = 0$

Lf = Li
Lf = Li
Lf = Li