

Introduction to Transfer Function

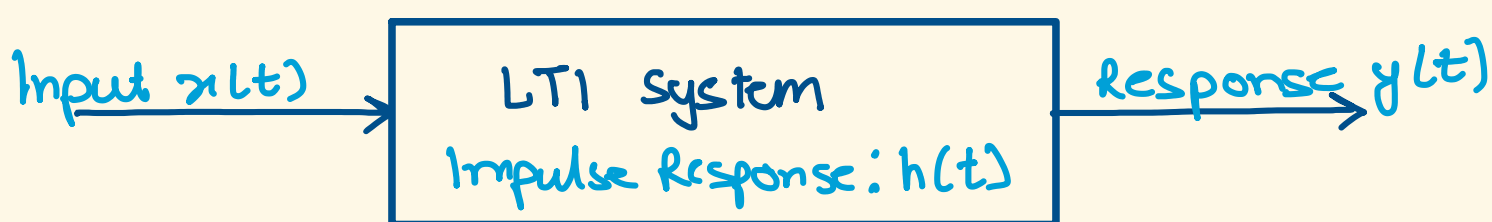
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1. Definition:

- Transfer function is the ratio of Laplace transform of output to the Laplace transform of input, when **all the initial conditions are assumed to be zero**.
- Transfer function is an important parameter of an LTI system. We use the transfer function to define the LTI system.



$$y(t) = x(t) * h(t) \quad (\text{Time domain})$$

↑
convolution

By Convolution Property:

$$Y(s) = X(s) \cdot H(s) \quad (\text{frequency domain})$$

where, $Y(s)$, $X(s)$ and $H(s)$ are Laplace transforms of $y(t)$, $x(t)$ and $h(t)$ respectively.

$H(s) = \frac{Y(s)}{X(s)}$

↑
Transfer function

2. Example:

Find the transfer function of the ^{LTI} system given by:

$$\frac{d^2 y(t)}{dt^2} + 3 \frac{dy(t)}{dt} + 2 \cdot y(t) = x(t)$$

where,

$x(t) \rightarrow$ input

$y(t) \rightarrow$ output

Solution: We know from time differentiation property of LT:

$$L\left[\frac{d}{dt}y(t)\right] = s \cdot Y(s) - \cancel{y(0)}^0 \quad \left[\because \text{Since LTI system} \right. \\ \left. \text{So, initial condition zero} \right]$$

$$L\left[\frac{d^2}{dt^2}y(t)\right] = s^2 \cdot Y(s) - \cancel{s \cdot y(0)}^0 - \cancel{y'(0)}^0 \\ = s^2 \cdot Y(s)$$

$$L[y(t)] = Y(s)$$

Apply LT both sides of differential equation:

$$s^2 \cdot Y(s) + 3 \cdot s \cdot Y(s) + 2 \cdot Y(s) = X(s) \\ Y(s) [s^2 + 3s + 2] = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 + 3s + 2} = \frac{1}{s^2 + 2s + s + 2}$$

$$H(s) = \frac{1}{(s+1)(s+2)}$$

3. References:

- Neso Academy

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