Time Constant Form of a Control System

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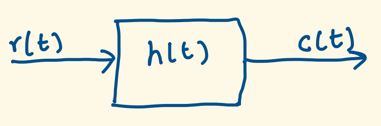
1. Introduction :-

- The time constant of a system decides how fast or slow is the response of the system.
  in other words, it tells the SPEED of the system.



- If the Time Constant of the system is high then the response is slow or the speed of the system is slow.

- And if Time Constant of the system is low then the response of the system is fast or we can say, the speed of the system is high.

2. Definition of Time Constant :-

- The time constant of a system is defined as the time required by the system to achieve 63.2% of the final value.

- It is denoted by letter τ.

3. Time Constant of a Control System :

- It is the characteristic of a first order system.

- It is defined as :

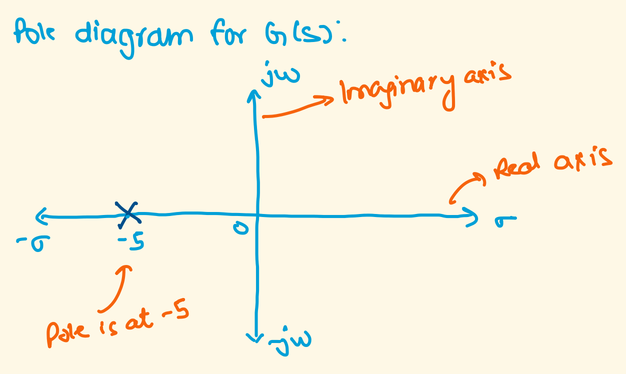
$$\boxed{\tau = -\frac{1}{\text{Real part of pole location}}}$$

Example: $G(s) = \frac{25(s+1)}{(s+5)}$; Determine the time constant of the system

Solution: As we know,

$$\tau = -\frac{1}{\text{Real part of pole location}}$$

Pole diagram for $G(s)$:



So, real part of pole location is -5.

$$\therefore \quad \tau = -\frac{1}{-5} = \frac{1}{5} = 0.2$$

$\therefore$ $\tau = 0.2$ seconds

4. Time Constant Form of a control system :-

- It is the standard form of transfer function of a 1st order control system.

- It is defined as: $\boxed{G(s) = \frac{K}{s\cdot\tau + 1}}$

  where: $K \rightarrow$ DC gain of the system
  $\tau \rightarrow$ Time Constant

Example: Find the time constant of the function $G(s) = \frac{5}{s+2}$

Solution: Currently $G(s)$ is not in the standard Time Constant form, so let's convert:

$$G(s) = \frac{5}{s+2}$$

$$G(s) = \frac{5}{2\left(\frac{s}{2}+1\right)}$$

$$G(s) = \frac{(5/2)}{\left(\frac{1}{2}\right)\cdot s + 1}$$

$\therefore$ $\tau = \frac{1}{2} = 0.5$ seconds. $\left[\because G(s) = \frac{K}{s\cdot\tau+1}\right]$

Example: The impulse response of a system is $y(t) = 2\cdot e^{-50t}\cdot u(t)$.
Find the time constant of the system.

Solution: Given: $y(t) = 2e^{-50t}u(t)$

this is impulse response.
"Response" means output.
It also means impulse signal was the input.

Input: $x(t) = \delta(t)$ impulse signal

We know that,

$$\text{Transfer function (TF)} = \frac{L[y(t)]}{L[x(t)]} = \frac{L[2\cdot e^{-50t}u(t)]}{L[\delta(t)]} = \frac{2\cdot {}^{1}/_{(s+50)}}{1}$$

$$\boxed{TF = \frac{2}{s+50}}$$

Now, let's make the TF of system in standard time constant form

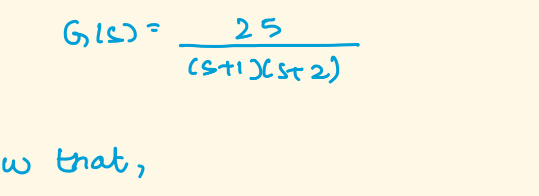
$$TF = \frac{2}{50\left(\frac{s}{50}+1\right)} = \frac{(2/50)}{\left(\frac{1}{50}\right)\cdot s + 1}$$

$\therefore$ $\tau = \frac{1}{50} = 0.02$ seconds $\left[\because \text{Comparing with } \frac{K}{s\tau+1}\right]$

5. Solved Problem of Time Constant Form :-

Example: Given $G(s) = \frac{25}{(s+1)(s+2)}$ & $x(t) = \delta(t)$.
Find the time constant of the output.

Solution:



Let's convert the input to laplace domain:

$$L[x(t)] = L[\delta(t)] = 1$$

$\therefore$ $\boxed{X(s) = 1}$

Given: $G(s) = \frac{25}{(s+1)(s+2)}$

We know that,

$$Y(s) = G(s)\cdot X(s)$$

$$Y(s) = \frac{25}{(s+1)(s+2)}\cdot 1$$

$$Y(s) = \frac{25}{(s+1)(s+2)}$$

Since there are 2 poles of this system. So, there is going to be 2 time constants for this system.

So, let's convert $Y(s)$ into standard Time Constant form.

$$Y(s) = \frac{25}{(s+1)\cdot 2\cdot\left(\frac{s}{2}+1\right)}$$

This one is already in standard form

We had to take 2 as common factor to make it to standard form.

$$Y(s) = \frac{25/2}{(s+1)\left(\frac{1}{2}\cdot s+1\right)}$$

Time constants :-
$\tau_1 = 1$ second
$\tau_2 = \frac{1}{2} = 0.5$ seconds

6. General Time Constant form of a Control System :-

$$\boxed{G(s) = \frac{K\cdot(1+s\tau_1)(1+s\tau_2)(1+s\tau_3)\cdots(1+s\tau_n)}{s^n\cdot(1+s\tau_1')(1+s\tau_2')(1+s\tau_3')\cdots(1+s\tau_n')}}$$

This Transfer Function is representing:
(i) n - zeros . $s = -\frac{1}{\tau_1}, -\frac{1}{\tau_2}, -\frac{1}{\tau_3}, \ldots$
(ii) n-poles at the origin $s = 0 \rightarrow n$ times
(iii) n-poles in the left halfplane
$s = -\frac{1}{\tau_1'}, -\frac{1}{\tau_2'}, -\frac{1}{\tau_3'}, \ldots$

7. References:

1. Neso Academy

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