Laplace Transform Part 1

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1. Introduction of laplace transform :-

given by Pierre-Simon Laplace — mathematician & astronomer

- Laplace transform play a very important role in the modelling of control systems.
- Because when we do the mathematical modelling of the control systems we get differential equations and the laplace transform converts the complex differential equations to simple algebraic equations.
- Laplace transform is the tool to represent the frequency domain of a time domain function.
- Laplace transform is an integral transform

General Integral Transform: $\underbrace{g(\alpha)}_{\text{output}} = \int_a^b \underbrace{f(t)}_{\text{input}} \cdot \underbrace{K(\alpha, t)}_{\text{integral kernel}}\, dt$

Laplace Transform:

$$\underbrace{L[f(t)] = F(s)}_{\text{output}} = \int_{-\infty}^{\infty} \underbrace{f(t)}_{\text{input}} \cdot \underbrace{e^{-st}}_{\text{integral kernel}}\, dt$$

where, $s = \underbrace{\sigma}_{\text{damping factor of the control system}} + \underbrace{j\omega}_{\text{Angular frequency}}$

* The stability of the control system depends upon the damping factor(σ)

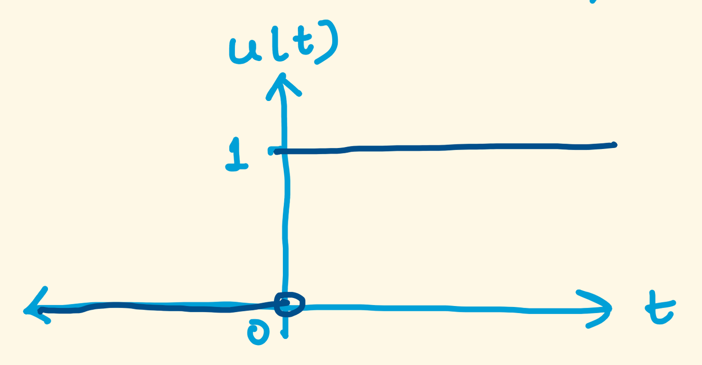
2. Example:

Find the laplace transform of unit step function u(t).

Solution: Given: $f(t) = u(t)$

We know that,

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$



$$F(s) = \int_{-\infty}^{\infty} f(t) \cdot e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} u(t) \cdot e^{-st}\, dt$$

$$= \int_0^{\infty} u(t) \cdot e^{-st}\, dt \qquad [\because u(t) = 0,\ t < 0]$$

$$= \int_0^{\infty} 1 \cdot e^{-st}\, dt$$

Put $-st = x$ $\qquad t = 0 \rightarrow x = 0$

$-s\, dt = dx$ $\qquad t = \infty \rightarrow x = \infty$

$dt = \frac{-dx}{s}$

$$= \int_0^{\infty} e^{x} \frac{-dx}{s} = \frac{-1}{s} \int_0^{\infty} e^{x} dx$$

$$= \frac{-1}{s} \left[e^{x}\right]_0^{\infty}$$

Back substitution

$$= \frac{-1}{s} \left[e^{-st}\right]_0^{\infty}$$

$$= \frac{-1}{s} \left[e^{-\infty} - e^{-0}\right]$$

$$= \frac{-1}{s} \left[\cancel{\frac{1}{e^{\infty}}}^{\,0} - 1\right]$$

$$= \frac{1}{s}$$

$$F(s) = L[u(t)] = \frac{1}{s}$$

3. Laplace transform of some standard signals :

	Signal	Laplace Transform
1.	$u(t)$ (unit step signal)	$1/s$
2.	t (Ramp Signal)	$1/s^2$
3.	t^n	$n!/s^{n+1}$
4.	e^{-at}	$1/s+a$
5.	$\delta(t)$ (Impulse Signal)	1
6.	$\cos(\omega t)$	$s/s^2+\omega^2$
7.	$\sin(\omega t)$	$\omega/s^2+\omega^2$

4. Inverse laplace transform (ILT):

The time domain signal can be obtained from the frequency domain signal by using ILT.

$$\boxed{\underbrace{f(t)}_{\text{output}} = \frac{1}{2\pi j} \int_{\sigma - j\omega}^{\sigma + j\omega} \underbrace{F(s)}_{\text{input}} \cdot e^{st}\, ds}$$

5. References:

1. Neso Academy

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