1 Semiconductor Physics

Charge neutrality, law of mass action (in T.E.):

$$p_0 - n_0 + N_d - N_a = 0 \tag{1}$$
$$n_0 p_0 = n_i^2(T) \tag{2}$$

Carrier drift:

$$J_n^{\text{drift}} = qn\mu_n\mathcal{E} \tag{3}$$
$$J_p^{\text{drift}} = qn\mu_p\mathcal{E} \tag{4}$$

Carrier diffusion:

$$J_n^{\text{diff}} = qD_n\frac{\mathrm{d}n}{\mathrm{d}x} \tag{5}$$
$$J_p^{\text{diff}} = -qD_p\frac{\mathrm{d}p}{\mathrm{d}x} \tag{6}$$
$$\tag{7}$$

Einstein's Relation:

$$\frac{D}{\mu} = \frac{kT}{q} \tag{8}$$

Boltzmann's relations (60 mV rule) + 6.012 reference ($\phi_{ref} = 0$ at $n_0 = n_i$):

$$n_0 = n_i e^{\frac{q\phi}{kT}} \tag{9}$$
$$p_0 = n_i e^{-\frac{q\phi}{kT}} \tag{10}$$

Or:

$$\phi = \frac{kT}{q}\ln\frac{n_0}{n_i} = -\frac{kT}{q}\ln\frac{p_0}{n_i} \tag{11}$$

Note : $-550\text{mV} < \phi < 550\text{mV}$.
Quasi-neutrality: slowly varying doping profile $\Rightarrow n_0(x) \approx N_D(x)$.

2 PN Junction electrostatics

2.1 No bias

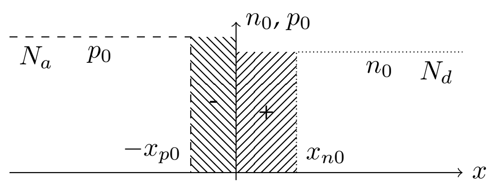


Figure 1: Carrier concentration (depletion approximation)

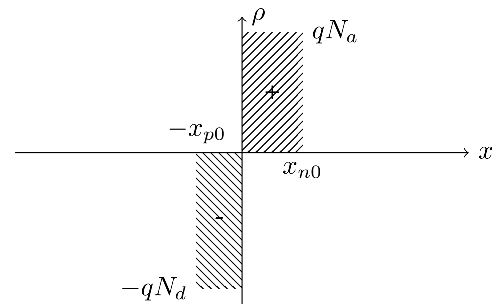


Figure 2: Charge density

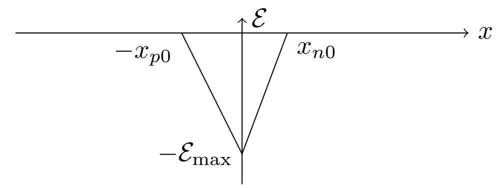


Figure 3: Electric field

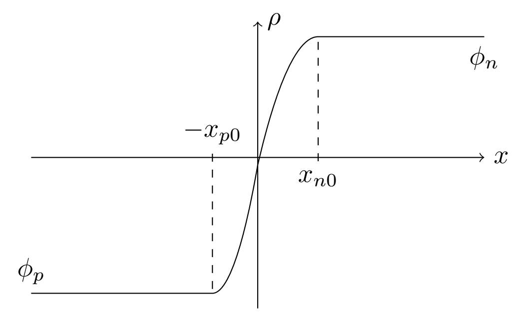


Figure 4: Electrostatic potential

2.2 Reverse bias ($V_d < 0$)

$$x_n = \sqrt{\frac{2\epsilon_s(\phi_b - V_d)N_a}{q(N_a+N_d)N_d}} \quad \text{and} \quad x_p = \sqrt{\frac{2\epsilon_s(\phi_b - V_d)N_d}{q(N_a+N_d)N_a}} \tag{12}$$

$$\mathcal{E}_{\max} = \sqrt{\frac{2q(\phi_b - V_d)N_aN_d}{\epsilon_s(N_a+N_d)}} \tag{13}$$

3 MOS electrostatics

3.1 Shapes

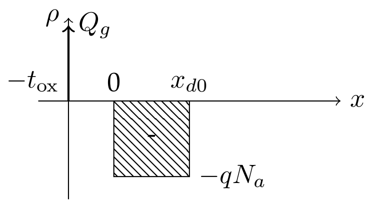


Figure 5: Charge density

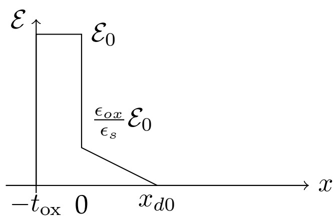


Figure 6: Electric field

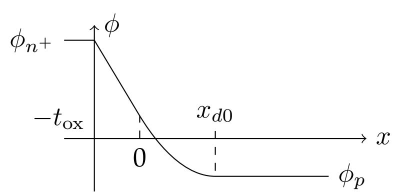


Figure 7: Electrostatic potential

$$\phi_b = \phi_{n^+} - \phi_p = \phi_{n^+} + \frac{kT}{q} \ln \frac{N_a}{n_i} \tag{14}$$

$$x_d(V_{GB}) = \frac{\epsilon_s}{C_{ox}} \sqrt{1 + \frac{2C_{ox}^2(\phi_b + V_{GB})}{\epsilon_s q N_a}} \tag{15}$$

$$\phi(0) = \phi_{n^+} + \frac{qN_a x_d^2(V_{GB})}{2\epsilon_s} \tag{16}$$

$\phi(0) \Rightarrow$ carrier concentration @ boundary.

3.2 Regimes

3.2.1 Depletion

Depletion region : V_{GB} increases $\Rightarrow$ D.R. widens.

3.2.2 Flatband

@ Flatband, no more depletion region : $V_{FB} = -\phi_b$.

3.2.3 Accumulation

$V_{GB} < V_{FB} \Rightarrow$ carrier accumulation @ boundary.

3.2.4 Threshold

Beyond threshold, depletion approximation invalid.

$$V_T = V_{FB} - 2\phi_p + \sqrt{2\epsilon_s q N_a(-2\phi_p)} \tag{17}$$

3.2.5 Inversion

$V_{GB} > V_T$. Then, $x_d = x_d(V_T)$, and charge accumulation : $Q_n = C_{ox}(V_{GB} - V_T)$.

4 MOSFET transistor

4.1 Regions of operation

Initially, $V_{SB} = 0$.

4.1.1 Cutoff

$V_{GS} < V_T$, $V_{DS} \geq 0$. No inversion charge $\Rightarrow I_D = 0$

4.1.2 Triode

$V_{GS} > V_T$, $V_{DS} > 0$ but $V_{DS} < V_{GS} - V_T$

$$I_D = \frac{W}{L}\mu_n C_{ox}(V_{GS} - \frac{V_{DS}}{2} - V_T)V_{DS} \tag{18}$$

4.1.3 Saturation

$V_{GS} > V_T$, $V_{DS} > V_{GS} - V_T$.

$$I_D = \frac{W}{2L}\mu_n C_{ox}(V_{GS} - V_T)^2 \tag{19}$$

4.2 Other effects

4.2.1 Channel Length Modulation

In saturation region, channel length decreases when V_{DS} increases $\rightarrow$ I_D increases. $\lambda = \dfrac{0.1\mu\mathrm{mV}^{-1}}{L}$.

$$I_D = \frac{W}{2L}\mu_n C_{ox}(V_{GS} - V_T)^2[1 + \lambda V_{DS}] \tag{20}$$

4.2.2 Backgate characteristics

Backgate effect parameter $\gamma = \dfrac{1}{C_{ox}}\sqrt{2\epsilon_s q N_a}$, $V_{T0} = V_T(V_{BS} = 0)$

$$V_T(V_{BS}) = V_{T0} + \gamma\left(\sqrt{-2\phi_p - V_{BS}} - \sqrt{-2\phi_p}\right) \tag{21}$$

4.3 Small signal model

Transconductance

$$g_m = \frac{\partial i_D}{\partial v_{GS}} = \sqrt{2\frac{W}{L}\mu_n C_{ox} I_D} \tag{22}$$

Output conductance

$$g_0 = \frac{\partial i_D}{\partial v_{DS}} \simeq \lambda I_D \tag{23}$$

Backgate transconductance

$$g_{mb} = \frac{\partial i_D}{\partial v_{BS}} = \frac{\gamma g_m}{2\sqrt{-2\phi_p - V_{BS}}} \tag{24}$$

Capacitances:

- C_{gs} channel charge (intrinsic) $\frac{2}{3}WLC_{ox}$+ overlap capacitance, WC_{ov}
- C_{gd} overlap capacitance, WC_{ov}
- C_{sb} source junction depletion capacitance $WL_{\text{diff}}C_j$ (+sidewall $(2L_{\text{diff}} + W)C_{jsw}$) (caps @ V_{SB})
- C_{db} drain junction depletion capacitance (+sidewall) (same as above, @ V_{DB})

5 CMOS

5.1 Definitions

5.1.1 Noise margins

$NM_H = V_{OH} - V_{IH}$ and $NM_L = V_{IL} - V_{OL}$

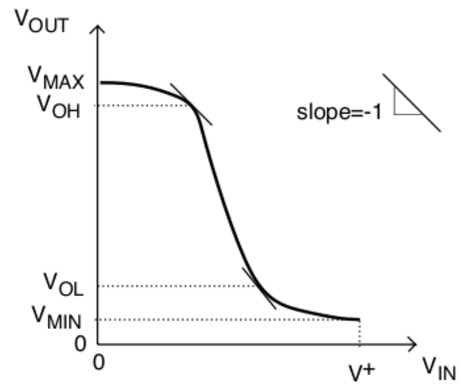


Figure 8: Inverter Input-Output characteristics

5.1.2 Delays

- t_R rise time between 10% and 90% of total swing
- t_F fall time between 90% and 10% of total swing
- t_{PHL} propagation delay from high-to-low between 50% points
- t_{PLH} propagation delay from low-to-high between 50% points
- Propagation delay $t_p = \frac{1}{2}(t_{PHL} + t_{PLH})$.

5.2 CMOS inverter

5.2.1 Noise margins

$$V_M = \frac{V_{Tn} + (V_{DD} + V_{Tp})\sqrt{\dfrac{k_p}{k_n}}}{1 + \sqrt{\dfrac{k_p}{k_n}}} \tag{25}$$

$$A_v = -(g_{mn} + g_{mp})(r_{0n}//r_{0p}) \tag{26}$$

5.2.2 Transient

Power dissipation: $C_G V_{DD}^2 f$

6 PN Diode

6.1 I-V characteristics

$$I = qAn_i^2\left(\frac{1}{N_a}\frac{D_n}{W_p - x_p} + \frac{1}{N_d}\frac{D_p}{W_n - x_n}\right)\left(e^{\frac{qV}{kT}} - 1\right) \tag{27}$$

- Under forward bias: Minority carriers are injected across the junction and diffuse to the contact where they recombine
- Under reverse bias: Minority carriers are generated at the contact and diffuse to the junction where they are extracted across the junction

6.2 Small-Signal model

6.2.1 Conductance

$$g_d = \frac{q(I_D + I_0)}{kT} \tag{28}$$

6.2.2 Capacitances

- Junction (Reverse Bias, Forward-Bias $V_d = \phi_b/2$)

$$C_j = A\sqrt{\frac{q\epsilon_s N_a N_d}{2(N_a + N_d)(\phi_b - V_D)}} \tag{29}$$

- Diffusion ($\tau_{Tn} = \dfrac{(W_p - x_p)^2}{2D_n}$ transit time)

$$C_d = \frac{q}{kT}\left(\tau_{Tn} I_{Dn} + \tau_{Tp} I_{Dp}\right) \tag{30}$$

7 NPN Bipolar Junction transistor

7.1 Regions of operation

7.1.1 Forward-Active region

- $V_{BE} > 0$: injection of electrons from emitter to base, injection of holes from base to emitter.
- $V_{BC} < 0$: extraction of electrons from base to collector, extractions of holes from collector to base.
- Collector current: focus on electron diffusion in base

$$I_C = qA_E\frac{D_{nB}}{W_B}n_{pB0}\exp\left(\frac{V_{BE}}{V_{Th}}\right) \tag{31}$$

- Base current: focus on hole injection and recombination at emitter contact.

$$I_B = qA_E\frac{D_{pE}}{W_E}p_{nE0}\left(\exp\left(\frac{V_{BE}}{V_{Th}}\right) - 1\right) \tag{32}$$

- Current gains
  - $\alpha_F = \dfrac{I_C}{I_E} = \dfrac{1}{1 + \dfrac{N_{aB}D_{pE}W_B}{N_{dE}D_{nB}W_E}} \simeq 1$ (typ. 0.99)
  - $\beta_F = \dfrac{I_C}{I_B} = \dfrac{N_{dE}D_{nB}W_E}{N_{aB}D_{pE}W_B}$ big $\Rightarrow N_{dE} \gg N_{aB}, \quad W_E \gg W_B$ and NPN better than PNP because $D_n \simeq 2D_p$.

7.1.2 Other regions

Reverse-Active Region Like Forward active, but reversed. Poor $\beta_R = \dfrac{N_{dC}D_{nB}W_C}{N_{aB}D_{pC}W_B}$.

Cutoff region $V_{BE} < 0, V_{BC} < 0 \Rightarrow$ small leakage currents

Saturation region $V_{BE} > 0, V_{BC} > 0 \Rightarrow$ superposition of FAR and RAR.

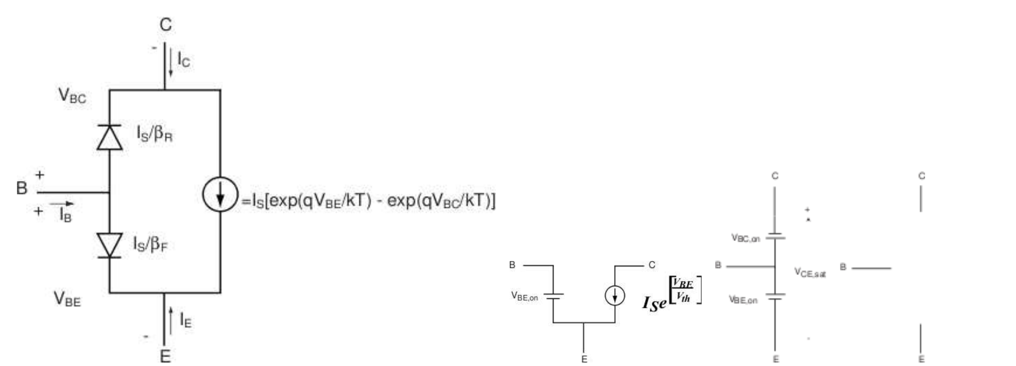
7.1.3 Circuit models

Large signal

$$I_C = I_S\left(\exp\frac{V_{BE}}{V_{Th}} - \exp\frac{V_{BC}}{V_{Th}}\right) - \frac{I_S}{\beta_R}\left(\exp\frac{V_{BC}}{V_{Th}} - 1\right) \tag{33}$$

$$I_B = \frac{I_S}{\beta_F}\left(\exp\frac{V_{BE}}{V_{Th}} - 1\right) + \frac{I_S}{\beta_R}\left(\exp\frac{V_{BC}}{V_{Th}} - 1\right) \tag{34}$$

$$I_E = -I_S\left(\exp\frac{V_{BE}}{V_{Th}} - \exp\frac{V_{BC}}{V_{Th}}\right) - \frac{I_S}{\beta_F}\left(\exp\frac{V_{BE}}{V_{Th}} - 1\right) \tag{35}$$



(a) Ebers-Moll (b) FAR (c) Sat (d) C-O