

Let  $f(x)$  be an linear function.

Let  $x^*$  be an internal dividing point by ratio  $a:b$

$$\text{Then } x^* = \frac{bx_1 + ax_2}{a+b} = \frac{b}{a+b}x_1 + \frac{a}{a+b}x_2$$

$$\text{Since } f(x) \text{ is an linear function, } f(x^*) = \frac{b}{a+b}f(x_1) + \frac{a}{a+b}f(x_2)$$

$$\text{Thus, when we put } \frac{a}{a+b} = t \text{ and } \frac{b}{a+b} = 1-t$$

$$\text{Then } f(x^*) = (1-t)f(x_1) + tf(x_2)$$

$$\text{Also, when we put } \frac{b}{a+b} = s \text{ and } \frac{a}{a+b} = 1-s$$

$$\text{Then } f(x^*) = sf(x_1) + (1-s)f(x_2)$$