

# CORRECTIONS TO Introduction to Topological Manifolds (Second Edition)

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- (2/25/18) **Page xii, last paragraph:** Allen Hatcher's name is misspelled. (Sorry, Allen.)
- (2/14/15) **Page 23, Exercise 2.6, first line:** Change “collection of topologies” to “nonempty collection of topologies.”
- (6/24/19) **Page 26, just above Exercise 2.12:** Replace the last sentence of the paragraph by “Symbolically, this is denoted by  $x_i \rightarrow x$ .”
- (6/18/19) **Page 27, paragraph before Proposition 2.19:** Just before the last sentence of that paragraph, insert “(Continuity of the restriction of a function to an open subset is understood to be with respect to the topology described in Exercise 2.5.)”
- (3/8/16) **Page 27, last line:** Change two occurrences of  $x$  to  $y$  in the displayed equation, so it reads
- $$(f|_{V_x})^{-1}(U) = \{y \in V_x : f(y) \in U\} = f^{-1}(U) \cap V_x,$$
- (9/26/19) **Page 31, second paragraph below the section heading, second sentence:** Change “two points” to “two distinct points.”
- (6/24/19) **Page 32, just above Exercise 2.38:** Insert the following sentence: “In view of the preceding proposition, in a Hausdorff space we can write  $p = \lim_{i \rightarrow \infty} p_i$  as an alternative notation for  $p_i \rightarrow p$ .”
- (5/17/12) **Page 37, three lines from the bottom:** Change “Exercise 2.49” to “Example 2.49.”
- (11/26/15) **Page 53, part (c) of the proposition continued from the previous page:** Insert another “if” after “if and only.”
- (7/14/18) **Page 58, second display:** Replace  $k$  by  $k/2$  (twice) and  $l$  by  $l/2$  (twice). [The tangent and cotangent functions have period  $\pi$ , not  $2\pi$ .]
- (9/26/19) **Page 60, last paragraph:** In this paragraph and in the first one on page 61, change all subscript  $k$ 's to  $n$ 's (a total of four times).
- (9/26/19) **Page 61, Proposition 3.31, part (c):** Change  $X_k$  to  $X_n$ .
- (11/20/19) **Page 66, after the second line:** Add the following sentence: “We sometimes also say informally that  $Y$  is a quotient space of  $X$  when  $Y$  is a topological space that has the quotient topology with respect to some continuous map from  $X$  to  $Y$ .”
- (3/23/12) **Page 67, Example 3.52, second sentence:** Change this sentence to read “Let  $\sim$  be the equivalence relation on  $X$  such that  $a_1 \sim a_2$  for all  $a_1, a_2 \in A$  and  $x \sim x$  for all other  $x \in X$ ; the partition . . .”
- (5/17/12) **Page 67, Example 3.53, last line:** Change  $\mathbb{B}^n$  to  $\overline{\mathbb{B}}^{n+1}$ .
- (7/17/19) **Page 70, Example 3.66, first paragraph, next-to-last line:** Change  $\theta$  to  $\frac{1}{2\pi}\theta$ .
- (10/7/11) **Page 74, line 5:** Change this statement to read “As a set,  $X \cup_f Y$  is the disjoint union . . .” [The topology on  $X \cup_f Y$  is not the disjoint union topology.]
- (4/1/22) **Page 74, Example 3.78(a):** Change “topological spaces” to “Hausdorff spaces.”
- (11/30/19) **Page 75, proof of Theorem 3.79, second line:** Change  $q(\partial M \cup \partial N)$  to  $q(\partial M \amalg \partial N)$ .
- (6/4/21) **Page 75, proof of Theorem 3.79, second paragraph:** Replace that paragraph with the following: “Suppose  $s \in S$ , and let  $y_0 \in \partial N$  and  $x_0 = h(y_0) \in \partial M$  be the two points in the fiber  $q^{-1}(s)$ . We can choose coordinate charts  $(U, \varphi)$  for  $M$  and  $(V, \psi)$  for  $N$  such that  $x_0 \in U$  and  $y_0 \in V$ , and let  $\hat{U} = \varphi(U)$ ,  $\hat{V} = \psi(V) \subseteq \mathbb{H}^n$  (Fig. 3.13). It is useful in this proof to identify  $\mathbb{H}^n$  with  $\mathbb{R}^{n-1} \times [0, \infty)$  and  $\mathbb{R}^n$  with  $\mathbb{R}^{n-1} \times \mathbb{R}$ . By shrinking  $U$  and  $V$  if necessary, we may assume that  $h(V \cap \partial N) = U \cap \partial M$ . Then we can write the coordinate maps as  $\varphi(p) = (\varphi_0(p), \varphi_1(p))$  and  $\psi(p) = (\psi_0(p), \psi_1(p))$  for some continuous maps  $\varphi_0: U \rightarrow \mathbb{R}^{n-1}$ ,  $\varphi_1: U \rightarrow [0, \infty)$ ,  $\psi_0: V \rightarrow \mathbb{R}^{n-1}$ ,  $\psi_1: V \rightarrow [0, \infty)$ . Our assumption that  $x_0$  and  $y_0$  are boundary points means that  $\varphi_1(x_0) = \psi_1(y_0) = 0$ , and there are open subsets  $U_0, V_0 \subseteq \mathbb{R}^{n-1}$  such that  $\varphi_0(U \cap \partial M) = U_0$ ,  $\psi_0(V \cap \partial N) = V_0$ . (Here we are again using the theorem on invariance of the boundary.) After replacing  $U$  and  $V$  by the preimages

of  $U_0 \times [0, \infty)$  and  $V_0 \times [0, \infty)$ , respectively, we can also assume that  $\hat{U} \subseteq U_0 \times [0, \infty)$  and  $\hat{V} \subseteq V_0 \times [0, \infty)$ ."

- (6/18/19) **Page 76, display in the middle of the page:** Change  $y^n$  to  $y_n$  (twice).
- (7/16/11) **Page 76, last paragraph of the proof of Theorem 3.79:** In the second line of the paragraph, change "embedding of  $N$ " to "embedding of  $M$ ." In the fourth line, change "embedding of  $M$ " to "embedding of  $N$ ."
- (8/8/17) **Page 87, Exercise 4.3:** Insert "nonempty" before "connected."
- (8/23/11) **Page 87, Exercise 4.4:** Insert another "if" after "if and only."
- (8/23/11) **Page 88, proof of Proposition 4.9, fourth paragraph:** In the first sentence of that paragraph, change "open subsets of  $\bigcup_{\alpha \in A} B_\alpha$ " to "open subsets of  $X$  whose union contains  $\bigcup_{\alpha \in A} B_\alpha$ ."
- (10/18/22) **Page 89, line above Proposition 4.11:** Change "Appendix B" to "Appendix A."
- (10/18/22) **Page 93, proof of Proposition 4.26, last sentence:** Change " $X$  is connected ..." to "if  $X$  is nonempty, it is connected ..."
- (8/23/11) **Page 97, line 10:** Change  $B_{n_{\max}}(a)$  to  $B_{n_{\max}}(x)$ .
- (3/9/11) **Page 98, line 3 from bottom:** Change "this proposition" to "this lemma."
- (6/21/20) **Page 99, second paragraph:** Delete the second sentence of the paragraph. [It's not wrong; it's just not needed.]
- (6/21/20) **Page 103, just above Exercise 4.58:** After the word "illustrates," insert "(using the theorem on invariance of the boundary)."
- (6/19/21) **Page 104, proof of Proposition 4.60:** Delete the last sentence in the first paragraph, and in the second paragraph, replace the phrase " $r$  is any positive rational number strictly less than  $r(x)$ " by " $r$  is any positive rational number such that  $B_{2r}(x) \subseteq \hat{U}_i$ ."
- (6/19/21) **Page 106, proof of Theorem 4.68, last paragraph:** After the first sentence of the paragraph, insert: "Begin by setting  $W_0 = U$ ." Then in the third and fourth lines of the paragraph, replace "Choosing  $r_n < \min(\varepsilon_n, 1/n)$ " by "Choosing  $r_n < \min(\varepsilon_n, 1/n)$  and setting  $W_n = B_{r_n}(x_n)$ ."
- (8/23/11) **Page 106, line 3 from the bottom:** Change "countable union" to "countable intersection."
- (8/23/11) **Page 109, statement of Lemma 4.74:** Insert another "if" after "if and only."
- (7/19/15) **Page 110, next-to-last line:** Change  $M$  to  $X$ .
- (8/23/11) **Page 114, proof of Corollary 4.83:** This proof is incorrect. Replace it with the following: "Given a closed subset  $A \subseteq X$  and a neighborhood  $U$  of  $A$ , Lemma 4.80 shows that there is a neighborhood  $V$  of  $A$  such that  $\overline{V} \subseteq U$ . By Urysohn's lemma, there exists a continuous function  $f: X \rightarrow [0, 1]$  such that  $f \equiv 1$  on  $A$  and  $f \equiv 0$  on  $X \setminus V$ . This function satisfies  $\text{supp } f \subseteq \overline{V} \subseteq U$ , so it is the bump function we seek."
- (6/17/19) **Page 119, statement of Proposition 4.93(b):** Add the hypothesis that  $Y$  is Hausdorff.
- (10/24/19) **Page 119, proof of Proposition 4.93, second paragraph:** Change the first sentence to read "To prove (b), assume  $X$  is a second countable Hausdorff space and  $Y$  is Hausdorff, and suppose ...". Then replace the sentence beginning "Suppose on the contrary" by the following: "Suppose on the contrary that  $(x_i)$  is a sequence in  $L$  with no convergent subsequence in  $L$ . Because  $Y$  is Hausdorff,  $K$  is closed and therefore so is  $L$ , which means that  $(x_i)$  has no convergent subsequence in  $X$ ."
- (8/23/11) **Page 121, proof of Lemma 4.94:** Replace the last two sentences of the proof with the following: "Thus  $x$  lies in the closure of  $A \cap K$  in  $K$ . Because  $A \cap K$  is closed in  $K$ , it follows that  $x \in A \cap K \subseteq A$ ."
- (8/23/11) **Page 123, Problem 4-15(d):** Change "every connected neighborhood" to "every neighborhood."
- (9/16/11) **Page 126, Problem 4-30:** Change  $\{A_\alpha\}$  to  $\{X_\alpha\}_{\alpha \in A}$ .
- (4/12/20) **Page 126, Problem 4-31(c):** In the last sentence, change "every element of  $\mathcal{U}$ " to "every nonempty element of  $\mathcal{U}$ ."
- (4/12/20) **Page 128, proof of Prop. 5.1:** Insert before the first sentence of the proof: "The proposition is true by definition when  $n = 0$ , so assume that  $n > 0$ ."
- (7/4/22) **Page 130, third paragraph, lines 5 and 6:** "Homeomorphism" is misspelled.

- (3/20/21) **Page 133, line above Theorem 5.6:** Change “an  $n$ -dimensional subcomplex” to “a subcomplex of dimension at most  $n$ .”
- (1/20/11) **Page 133, proof of Proposition 5.7:** This should refer to Problem 5-8, not 5-7.
- (5/17/12) **Page 136, four lines below the displayed equations:** Change “both  $X'_{n-1}$  and  $X''_{n-1}$  are open” to “both  $X'_n$  and  $X''_n$  are open.”
- (1/20/11) **Page 137, statement of Lemma 5.13:** Change “discrete” to “closed and discrete.”
- (1/20/11) **Page 137, proof of Lemma 5.13, first paragraph:** In line 1, change “discrete” to “closed and discrete”; and in line 2, change “discrete subset” to “closed discrete subset.”
- (1/20/11) **Page 137, proof of Theorem 5.14, second paragraph:** Change “infinite discrete subset” to “infinite closed discrete subset.”
- (7/17/19) **Page 140, displayed formulas:** In both displayed formulas, change  $\mathbb{R}$  to  $[0, 1]$ .
- (4/12/20) **Page 141, just above the displayed equation:** In the line above the display and in the display itself, change  $A$  to  $B$  (four times), to avoid conflict with the use of  $A$  as the index set for the open cover.
- (4/12/20) **Page 141, displayed equation:** Change  $D_\gamma^{n+1}$  to  $D_\gamma^{n+1} \setminus \{0\}$ .
- (9/16/11) **Page 141, line 5 from the bottom:** Change  $\tilde{U}_\alpha^{n+1}$  to  $\tilde{U}_{\alpha_i}^{n+1}$  (twice).
- (2/5/13) **Page 141, line 4 from the bottom:** Change “the minimum” to “one-half the minimum.”
- (2/5/13) **Page 141, line 3 from the bottom:** Change “supported in  $\partial D_\gamma^{n+1}(\varepsilon/2)$ ” to “supported in  $D_\gamma^{n+1} \setminus \partial D_\gamma^{n+1}(\varepsilon/2)$ ”
- (7/17/19) **Page 143, proof of Proposition 5.24, last paragraph:** Change  $U \cap e_0$  to  $U \cap \bar{e}_0$ .
- (10/16/20) **Page 144, three lines above Lemma 5.26:** Change “the finite subcomplex  $\mathcal{E}_n$ ” to “the finite subcomplex  $M_n$ .”
- (7/24/19) **Page 145, second paragraph:** Change  $e_n$  to  $e_k$  twice (once in the first line, and once in (5.1)).
- (7/22/19) **Page 146, Case 1, second paragraph:** Change  $Y_n$  to  $Y_{v_n}$  (twice).
- (4/12/20) **Page 152, sentence after the proof of Prop. 5.38:** Change  $i = 1, \dots, k$  to  $i = 0, \dots, k$ .
- (3/24/11) **Page 156, Problem 5-4:** add the hypothesis that  $\dim M > 1$ .
- (5/27/17) **Page 158, second sentence:** Replace this sentence by “More generally, suppose  $K$  is a finite Euclidean simplicial complex and  $w$  is a point in  $\mathbb{R}^n$  such that each ray starting at  $w$  intersects  $|K|$  in at most one point.”
- (4/12/20) **Page 158, Problem 5-18(b):** In the hint, change “simplex” to “cell.”
- (11/7/19) **Page 165, Example 6.7:** After the second sentence, add “(The disks should be chosen so that their closures are disjoint.)”
- (4/12/20) **Page 167, line 5 from the bottom:** Insert “the” before “sum.”
- (9/16/11) **Page 172, first paragraph, next-to-last line:** Change  $P'_1 \amalg Q$  to  $P_1 \amalg Q$ .
- (9/16/11) **Page 176, Fig. 6.21:** The label  $b$  near the lower right should be  $c$ , and the label  $w$  near the middle of the right-hand side should be  $x$ .
- (5/20/18) **Page 180, Proposition 6.20:** In the statement of the proposition, change “compact surface” to “connected compact surface.” Then in the second sentence of the proof, change both occurrences of “surface” to “connected compact surface.”
- (11/5/17) **Page 181, first full paragraph:** Replace the sentence starting with “However” by “However, we will prove in Chapter 10 that a compact surface cannot have both an oriented presentation and a nonoriented one.”
- (2/26/18) **Page 181, Problem 6-4:** Replace the first sentence by “Suppose  $M$  is a compact 2-manifold that contains a subset  $B \subseteq M$  that is homeomorphic to the Möbius band, and whose interior is homeomorphic to the Möbius band minus its boundary.”
- (9/16/11) **Page 190, line 3 from the bottom:** Change  $\Phi_g(f)$  to  $\Phi_g[f]$ .
- (1/20/11) **Page 193, proof of Proposition 7.16, second paragraph, line 2:** Change “ $H_1 = f$ ” to “ $H_1 = \tilde{f}$ .”
- (12/8/21) **Page 195, displayed equation:** Replace the last “ $<$ ” sign by “ $\leq$ .”
- (8/3/18) **Page 201, Corollary 7.38:** This corollary should be moved after the statement of Theorem 7.40.
- (11/25/12) **Page 211, line 6:** Delete redundant “each.”

- (7/9/15) **Page 215, Problem 7-9:** Change “connected” to “path-connected.”
- (5/31/16) **Page 221, Theorem 8.4:** Remark: This theorem is true without the assumption that  $B$  is locally connected, and the proof is not really any more difficult; see, for example, the proof of Theorem 1.7 in [Hat02].
- (7/22/19) **Page 222, first paragraph:** Change  $\{J_1, \dots, J_k\}$  to  $\{J_1, \dots, J_m\}$ .
- (1/20/11) **Page 224, two lines above the subheading:** Change  $\tilde{f}_0(1)$  to  $\tilde{f}_1(0)$ .
- (1/20/11) **Page 228, displayed equations (8.4):** Replace these equations by

$$\begin{aligned}\deg \varphi &= \deg(\rho_\varphi \circ \varphi)_*, \\ \deg \psi &= \deg(\rho_\psi \circ \psi)_*.\end{aligned}\tag{8.4}$$

- (7/13/15) **Page 230, Problem 8-5:** Replace the last sentence of the hint by the following: “Prove that  $p_\varepsilon|_{\mathbb{S}^1}$  and  $p_n(z) = z^n$  are homotopic as maps from  $\mathbb{S}^1$  to  $\mathbb{C} \setminus \{0\}$ . If  $p$  has no zeros, use degree theory to derive a contradiction.”
- (7/13/15) **Page 231, Problem 8-10(c):** Change “index of  $V$  around the loop  $\omega$ ” to “winding number of  $V$  around the loop  $\omega$ .”
- (9/16/11) **Page 239, fourth line below the section heading:** Change “generated by  $G$ ” to “generated by  $S$ .”
- (11/29/19) **Page 241, middle of the page:** Change the definition of *group presentation* as follows: “We define a *group presentation* to be an ordered pair, denoted by  $\langle S | R \rangle$ , where  $S$  is an arbitrary set and  $R$  is a set of words formed from the elements of  $S$ .”
- (11/29/19) **Page 241, just below the last displayed equation:** Replace “where  $\bar{R}$  is the *normal closure of  $R$  in  $F(S)$* ” by “where now we interpret  $R$  as a set of elements of the free group  $F(S)$ , and  $\bar{R}$  is the *normal closure of  $R$  in  $F(S)$* .”
- (7/28/16) **Page 244, fourth line below the section heading:** Change  $n \in \mathbb{Z}$  to  $n \in \mathbb{N}$ .
- (7/28/16) **Page 247, Example 9.22, last line:** The formula for  $G_{\text{tor}}$  should be  $G_{\text{tor}} = \{0\} \times \mathbb{Z}/k_1 \times \dots \times \mathbb{Z}/k_m$ .
- (12/3/19) **Page 249, Problem 9-4(b):** Change “a subset of the free group  $F(S_i)$ ” to “a set of words in the elements of  $S_i$ .”
- (12/3/19) **Page 249, Problem 9-5:** Change “subsets of the free group  $F(S)$ ” to “sets of words in the elements of  $S$ .”
- (11/28/17) **Page 252, just above diagram (10.2):** Change “the following diagram commutes” to “the right half of the following diagram commutes.”
- (7/29/19) **Page 256, statement of Theorem 10.7:** Change “spaces” to “path-connected spaces.”
- (12/1/20) **Page 257, Example 10.8:** In the first line of the example, and in the three lines immediately above it, change “proposition” to “theorem” (three times).
- (7/29/19) **Page 257, last paragraph, second sentence:** Change that sentence to read “If two or more edges are incident with the same two vertices, or if two or more self-loops are incident with the same vertex, they are called *multiple edges*.”
- (12/7/20) **Page 260, proof of Theorem 10.12, second paragraph, first line:** Change  $\Gamma$  to  $\pi_1(\Gamma, v)$ .
- (12/7/20) **Page 261, last paragraph of the proof, fourth line from the bottom:** Change two equalities to isomorphisms: “ $\pi_1(V, v) \cong F([f_1], \dots, [f_n])$  and  $\pi_1(U, v) \cong F([f_{n+1}])$ .”
- (9/16/11) **Page 263, line 2:** Change  $\tilde{U} \cap \tilde{V}$  to  $q(D \setminus \{z\})$ .
- (8/2/13) **Page 268, lines 2 & 3:** Change “preceding corollary” to “preceding theorem.”
- (11/5/17) **Page 268, statement of Corollary 10.24:** Change the statement to “A compact surface cannot have both an oriented presentation and a nonoriented one.”
- (5/23/11) **Page 269, line below equation (10.7):** Insert missing comma after “surjective.”
- (10/3/20) **Page 271, line 3:** Replace the phrase “the endpoints of the paths  $a_i$  in this product are of the form  $i/n$ ” by “the paths  $a_i$  in this product are defined on subintervals whose endpoints are integral multiples of  $1/n$ .”
- (7/8/14) **Page 275, Problem 10-21(c):** Delete “with nonempty intersection.”
- (8/27/18) **Page 278, second line below the heading:** Before “disjoint union,” insert “nonempty.”
- (7/29/19) **Page 279, second line:** Change “Theorem 4.15” to “Proposition 4.13.”

- (5/31/16) **Page 282, Theorem 11.13:** Remark: This theorem, like Theorem 8.4, is true without the assumption that  $B$  is locally connected.
- (5/17/12) **Page 302, Problem 11-5, first line:** Change “dimension  $n$ ” to “dimension  $n \geq 2$ .”
- (11/6/19) **Page 303, Problem 11-9:** Add the hypothesis that the spaces are nonempty.
- (12/10/15) **Page 303, Problem 11-12(c):** Change “ $(1, 0)$  or  $(-1, 0)$ ” to “ $1$  or  $-1$ ” [to be consistent with the complex notation used elsewhere for  $\mathbb{S}^1$ ].
- (5/17/12) **Page 305, Problem 11-20:** At the end of the problem, add: “For the counterexample, you may use without proof the fact that  $\mathbb{S}^2$  is not contractible. (This follows, for example, from Corollary 13.11 and Theorem 13.23.)”
- (2/25/18) **Page 312, last sentence of the paragraph after Exercise 12.13:** Allen Hatcher’s name is misspelled.
- (7/8/14) **Page 315, paragraph above the displayed diagram:** After “ $Q$  is a normal covering map,” insert “and  $\hat{H} = \text{Aut}_Q(E)$ .”
- (7/8/14) **Page 315, just below the displayed diagram:** Replace the last two paragraphs on page 315 and the first (partial) paragraph on page 316 with the following:

We have to show that  $\hat{q}$  is a covering map. Let  $x \in X$  be arbitrary, and let  $U$  be a neighborhood of  $x$  that is evenly covered by  $q$ . We will show that  $U$  is also evenly covered by  $\hat{q}$ . Given a component  $U_i$  of  $q^{-1}(U)$ , let  $\hat{U}_i = Q(U_i) \subseteq \hat{E}$ ; then  $\hat{U}_i$  is connected, and it is open in  $\hat{E}$  because  $Q$  is an open map (Proposition 11.1). Suppose  $\hat{U}_i = Q(U_i)$  and  $\hat{U}_j = Q(U_j)$  are any two such sets. If they have a point  $\hat{e}$  in common, then  $\hat{e} = Q(e_i) = Q(e_j)$  for some  $e_i \in U_i$  and  $e_j \in U_j$ . Since  $Q$  identifies points of  $E$  if and only if they are in the same  $\hat{H}$ -orbit, there is some  $\varphi \in \hat{H}$  such that  $e_j = \varphi(e_i)$ . Then  $\varphi(U_i) = U_j$  by Proposition 12.1(c), so  $Q(U_i) = Q \circ \varphi(U_i) = Q(U_j)$ . This shows that any such sets  $\hat{U}_i, \hat{U}_j$  are either disjoint or equal. Since  $Q$  is surjective,  $\hat{q}^{-1}(U)$  is equal to the disjoint union of the connected open sets  $\hat{U}_i$  as  $U_i$  ranges over the components of  $q^{-1}(U)$ .

It remains only to show that for any such set  $\hat{U}_i$ , the restricted map  $\hat{q}: \hat{U}_i \rightarrow U$  is a homeomorphism. The following diagram commutes:

$$\begin{array}{ccc} U_i & & \\ \downarrow q & \searrow Q & \\ & \hat{U}_i & \\ & \swarrow \hat{q} & \\ & U & \end{array} \quad (12.3)$$

Since  $q = \hat{q} \circ Q$  is injective on  $U_i$ , so is  $Q$ ; and  $Q: U_i \rightarrow \hat{U}_i$  is surjective by definition. Because  $Q$  is an open map, it follows that  $Q: U_i \rightarrow \hat{U}_i$  is a homeomorphism. Since  $q$  and  $Q$  are homeomorphisms in (12.3), so is  $\hat{q}$ .

- (9/27/11) **Page 318, statement of Proposition 12.21, second line:** Insert “on” after “acting.”
- (12/9/19) **Page 320, paragraph after the proof of Prop. 12.24, first line:** Before “locally,” insert “nonempty.”
- (9/23/14) **Page 321, line 4:** Change  $E \times E$  to  $E$ .
- (9/27/11) **Page 329, paragraph just below the diagram:** Change every occurrence of  $\tilde{p}$  to  $\tilde{q}$  (five times).
- (6/26/22) **Page 329, last paragraph, third sentence:** Change “The map  $G \times P \rightarrow \mathbb{B}^2$ ” to “The map  $\hat{\delta}: G \times P \rightarrow \mathbb{B}^2$ .”
- (9/27/11) **Page 330, just below the bulleted list:** Change  $\tilde{p}$  to  $\tilde{q}$ .
- (9/27/11) **Page 332, first full paragraph, second line:** Change  $\tilde{p}$  to  $\tilde{q}$ .
- (9/27/11) **Page 332, second full paragraph, lines 6 and 7:** Change  $\tilde{p}$  to  $\tilde{q}$  (twice).
- (9/16/14) **Page 335, Problem 12-10:** Interchange the definitions of  $G$  and  $H$  in the sixth and seventh lines. (Otherwise, part (c) is false as stated.)

- (10/12/14) **Page 337, Problem 12-19:** Replace the first sentence of the problem with the following: “Suppose we are given a continuous action of a metrizable topological group (e.g., a discrete group)  $G$  on a first countable Hausdorff space  $E$ .”
- (7/22/19) **Page 349, line 3:** Change  $\Delta_p$  to  $\Delta_{p+1}$ .
- (9/27/11) **Page 352, lines 3 and 4:** Change  $c_p$  to  $c_q$  (twice), and change  $p$  to  $q$  (twice).
- (7/22/19) **Page 352, next-to-last line:** Change  $c_p$  to  $c_q$  (twice), and change  $p$  to  $q$  (once).
- (12/15/17) **Page 354, paragraph above the last display:** Insert “of some reparametrization” after “extension of the circle representative.”
- (3/16/21) **Page 355, commutative diagram near the bottom of the page:** Change the period after  $X$  to a comma.
- (7/22/19) **Page 360, proof of Lemma 13.20:** In the second line of the displayed equation, change  $F_{i,p}$  to  $F_{i,p+1}$ .
- (7/22/19) **Page 361, first line of text:** Change “ $\in \mathbb{R}^n$ ” to “ $\subseteq \mathbb{R}^n$ .”
- (4/1/21) **Page 369, line above Proposition 13.33:** Delete spurious “and.”
- (10/8/15) **Page 370, line 5 from the bottom:** Change “It follows . . .” to “Assuming  $X$  is path-connected, it follows . . .”
- (10/8/15) **Page 371, at the end of the first (partial) paragraph:** Insert “If  $X$  is not path-connected, just apply this argument to the path component containing the image of  $\varphi$ , and use Proposition 13.5.”
- (9/26/17) **Page 371, statement of Theorem 13.34(e):** Change “dimension  $n$ ” to “dimension  $n \geq 2$ ,” and change “the zero map” to “not injective.”
- (4/1/21) **Pages 371–372, proof of Theorem 13.34:** Change “Theorem 13.33” to “Proposition 3.33” (five times).
- (9/26/17) **Page 372, proof of Theorem 13.34, last paragraph:** Change “if  $\varphi_* = 0$ ” to “if  $\varphi_*$  is injective.”
- (9/26/17) **Page 372, Example 13.35(b), last line:** Change “the zero map” to “noninjective.”
- (9/29/17) **Page 372, Example 13.35(c):** Replace the last sentence by “The image of  $\varphi_*$  is the infinite cyclic group generated by  $\gamma(\alpha_1^2 \dots \alpha_n^2)$ , so  $\varphi_*$  is injective and  $H_2(M) = 0$ .”
- (9/26/19) **Page 399, next-to-last line:** Change  $x \in X$  to  $x \in M_1$ .
- (12/26/18) **Page 401, line 4 from the bottom:** Change “subset” to “nonempty subset.”
- (10/7/19) **Page 402, Exercise C.1:** Change “any subset” to “any nonempty subset.”
- (6/6/18) **Page 411, near the middle of the page:** The index entry for  $\bar{R}$  should read “(normal closure of a subset).”
- (2/25/18) **Page 422:** The index entry for “Hatcher, Allen” is misspelled.