Solve differential Equation using Galerkin Method.

written 5.4 years ago by • modified 3 months ago

teamques10 (/u/1/teamques1) ★ 32k

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Solve the following differential Equation using Galerkin Method.

$\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y = 0 \quad 0 < x < 1$

Boundary conditions are : y (0) = 1 , y'(1) = 0.1

Find y (0.2) and compare with exact solution.

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1 Answer

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Galerkin Method:

order of d.e = 2

Degree of polynomial = 3

Let approximate solution be y

$y = C_0 + C_1x + C_2x^2 + C_3x^3$
$y' = C_1x + 2C_2x + 3C_3x^2$

From boundary condition 1: y(0) = 1

$C_0 = 1$

From boundary condition 2 : y'(1) = 0.1

$0.1 = C_1 + 2C_2 + 3C_3$
$\therefore C_1 = 0.1 - 2C_2 - 3C_3$

Approximate solution be,

$y = 1 + (0.1 - 2C_2 - 3C_3x) + C_2x^2 + C_3x^3$
$\therefore y = 1 + 0.1x + C_2(x^2 - 2x) + C_3(x^3 - 3x)$
$\frac{dy}{dx} = 0.1 + C_2(2x - 2) + C_3(3x^2 - 3)$
$\frac{d^2y}{dx^2} = 2C_2 + 6xC_3$

Residue,

$R = \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 6y$
$= 2C_2 + 6xC_3 + 3x[0.1 + C_2(2x + 2) + C_3(3x^2 - 3)]$
$- 6[1 + 0.1x + C_2(x^2 - 2x) + C_3(x^3 - 3x)]$
$= C_2[2 + 3x(2x - 2) + 12x - 6x^2] + C_3[6x + 3x(3x^2 - 3) + 18x - 6x^3] + 0.3x - 6 -$
$-.6x$
$C_2(6x + 2) + C_3(3x^2 + 15x) - 0.3x - 6$

Weighted Integral form

$\int_0^1 WiRdx = 0 \quad ------(1)$

For Galerkin method,

wi = coefficients of C_i in y

$W_1 = (x^2 - 2x)$ and $w_2 = (x^3 - 3x)$

Equation 1, i=1

$\int_0^1 (x^2 - 2x)[C_2(6x + 2) + C_3(3x^3 + 15x) - 0.3x - 6]dx = 0$
$- 3.883C_2 - 6.95C_3 + 4.125 = 0$
$3.883C_2 + 6.95C_3 = 4.125 \quad ------(2)$

Equation 1: i = 2

$\int_0^1 (x^3 - 3x)[C_2(6x + 2) + C_3(3x^3 + 15x) - 0.3x - 6]dx = 0$
$- 7.3C_2 - 13.371C_3 + 7.74 = 0$
$7.3C_2 + 13.371C_3 = 7.74 \quad ------(3)$

Solving equations (2) and (3),

$C_2 = 2.64$ and $C_3 = -0.862$
$\therefore y = 1 + 0.1x + 2.64(x^2 - 2x) - 0.862(x^3 - 3x)$
$y(0.2) = 0.58$

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