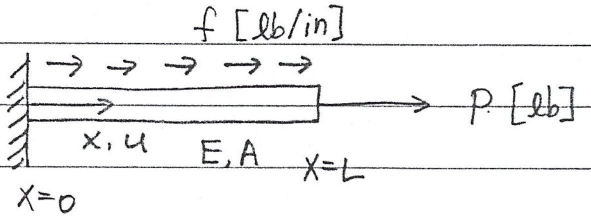
(pMTPE 증명.) Uniaxial bar



$$\Pi(u) = \int_0^L \frac{1}{2} EA\left(\frac{\partial u}{\partial x}\right)^2 dx - \int_0^L fu\, dx - (Pu)_{x=L}$$

Set $u = u_0 + \Delta u$

u_0 = actual displacement

Δu = actual displacement로 부터 (deviation or perturbation)

$$\Pi(u_0+\Delta u) = \int_0^L \frac{1}{2} EA\left[\frac{\partial u_0}{\partial x} + \frac{\partial \Delta u}{\partial x}\right]^2 dx$$

$$- \int_0^L f(u_0+\Delta u)\, dx - \left[P(u_0+\Delta u)\right]_{x=L}$$

$$= \underbrace{\int_0^L \frac{1}{2} EA\left(\frac{\partial u_0}{\partial x}\right)^2 dx - \int_0^L f u_0\, dx - (P u_0)_{x=L}}_{= \Pi(u_0)}$$

$$+ \underbrace{\int_0^L EA \frac{\partial u_0}{\partial x}\frac{\partial \Delta u}{\partial x} dx - \int_0^L f \Delta u\, dx - (P\Delta u)_{x=L}}_{= 0}$$

$\because$ Δu를 δu로 생각하면 $\delta \Pi = 0$

$$+ \int_0^L \frac{1}{2} EA\left(\frac{\partial \Delta u}{\partial x}\right)^2 dx$$

$$\Pi(u_0 + \Delta u) - \Pi(u_0) = \int_0^L \frac{1}{2} EA \left(\frac{\partial \Delta u}{\partial x} \right)^2 dx \geq 0$$

$\therefore$ Π는 actual disp (u_0)에서 최소.