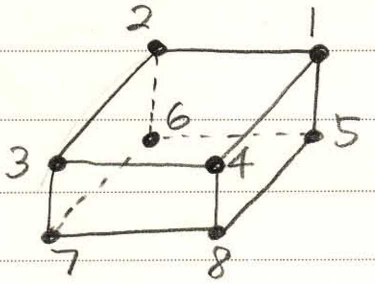
8 - node, 3-d solid element

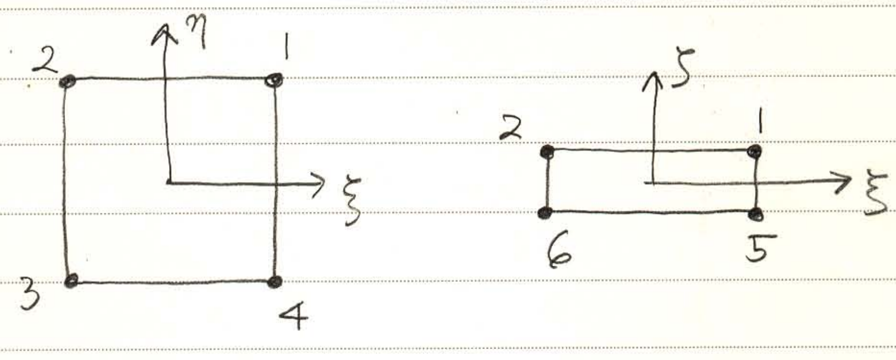


element node numbering.

(1) Displacement Formulation.

The geometry of 3-d element

$$\underset{\sim}{X} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \sum_{i=1}^{8} N_i(\xi, \eta, \zeta)\, \underset{\sim}{X_i}$$



$$-1 \le \xi, \eta, \zeta \le 1$$

$$N_1 = \frac{1}{8}(\xi+1)(\eta+1)(\zeta+1)$$

$$N_2 = -\frac{1}{8}(\xi-1)(\eta+1)(\zeta+1)$$

$$N_3 = \frac{1}{8}(\xi-1)(\eta-1)(\zeta+1)$$

$$N_4 = -\frac{1}{8}(\xi+1)(\eta-1)(\zeta+1)$$

$$N_5 = -\frac{1}{8}(\xi+1)(\eta+1)(\zeta-1)$$

$$N_6 = \frac{1}{8}(\xi-1)(\eta+1)(\zeta-1)$$

$$N_7 = -\frac{1}{8}(\xi-1)(\eta-1)(\zeta-1)$$

$$N_8 = \frac{1}{8}(\xi+1)(\eta-1)(\zeta-1)$$

displacement vector

$$\underset{\sim}{U} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{i=1}^{8} N_i(\xi, \eta, \zeta)\, \underset{\sim}{U_i}$$

$$u = \sum_{i=1}^{8} N_i u_i \ , \quad v = \sum_{i=1}^{8} N_i v_i \ , \quad w = \sum_{i=1}^{8} N_i w_i$$

In matrix form.

$$\begin{Bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial u}{\partial \zeta} \end{Bmatrix} = \underset{3\times 24}{\begin{bmatrix} \frac{\partial N_i}{\partial \xi} & \underset{\sim}{0} & \underset{\sim}{0} \\ \frac{\partial N_i}{\partial \eta} & \underset{\sim}{0} & \underset{\sim}{0} \\ \frac{\partial N_i}{\partial \zeta} & \underset{\sim}{0} & \underset{\sim}{0} \end{bmatrix}} \cdot \underset{24\times 1}{\underset{\sim}{q_e}} = \underset{\sim}{B_u}\, \underset{\sim}{q_e}$$

$$\begin{Bmatrix} \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \\ \frac{\partial v}{\partial \zeta} \end{Bmatrix} = \begin{bmatrix} \underset{\sim}{0} & \frac{\partial N_i}{\partial \xi} & \underset{\sim}{0} \\ \underset{\sim}{0} & \frac{\partial N_i}{\partial \eta} & \underset{\sim}{0} \\ \underset{\sim}{0} & \frac{\partial N_i}{\partial \zeta} & \underset{\sim}{0} \end{bmatrix} \cdot \underset{\sim}{q_e} = \underset{\sim}{R_v} \, \underset{\sim}{q_e}$$

$$\begin{Bmatrix} \frac{\partial w}{\partial \xi} \\ \frac{\partial w}{\partial \eta} \\ \frac{\partial w}{\partial \zeta} \end{Bmatrix} = \begin{bmatrix} \underset{\sim}{0} & \underset{\sim}{0} & \frac{\partial N_i}{\partial \xi} \\ \underset{\sim}{0} & \underset{\sim}{0} & \frac{\partial N_i}{\partial \eta} \\ \underset{\sim}{0} & \underset{\sim}{0} & \frac{\partial N_i}{\partial \zeta} \end{bmatrix} \cdot \underset{\sim}{q_e} = R_w \, q_e.$$

$$\underset{24\times 1}{\underset{\sim}{q_e}} = \lfloor u_1 \; u_2 \cdots u_8 \; v_1 \; v_2 \cdots v_8 \; w_1 \; w_2 \cdots w_8 \rfloor^T$$

Jacobian $\underset{\sim}{J}$

$$\underset{\sim}{J} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta} \end{bmatrix}$$

$$= \begin{bmatrix} \sum \frac{\partial N_i}{\partial \xi} x_i & \sum \frac{\partial N_i}{\partial \xi} y_i & \sum \frac{\partial N_i}{\partial \xi} z_i \\ \sum \frac{\partial N_i}{\partial \eta} x_i & \sum \frac{\partial N_i}{\partial \eta} y_i & \sum \frac{\partial N_i}{\partial \eta} z_i \\ \sum \frac{\partial N_i}{\partial \zeta} x_i & \sum \frac{\partial N_i}{\partial \zeta} y_i & \sum \frac{\partial N_i}{\partial \zeta} z_i \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial z} \end{bmatrix} = \underset{\sim}{J}^{-1} \begin{bmatrix} \frac{\partial u}{\partial \xi} & \frac{\partial v}{\partial \xi} & \frac{\partial w}{\partial \xi} \\ \frac{\partial u}{\partial \eta} & \frac{\partial v}{\partial \eta} & \frac{\partial w}{\partial \eta} \\ \frac{\partial u}{\partial \zeta} & \frac{\partial v}{\partial \zeta} & \frac{\partial w}{\partial \zeta} \end{bmatrix}$$

$$\begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial z} \end{Bmatrix} = \begin{Bmatrix} \underset{\sim}{J}^{-1}_{1} \\ \underset{\sim}{J}^{-1}_{2} \\ \underset{\sim}{J}^{-1}_{3} \end{Bmatrix} \underset{\sim}{R}_u \cdot \underset{\sim}{q}_{e}$$

$\underset{\sim}{J}^{-1}_{i}$ ≒ ith row of $\underset{\sim}{J}^{-1}$

$$\begin{Bmatrix} \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial v}{\partial z} \end{Bmatrix} = \begin{Bmatrix} \underset{\sim}{J}^{-1}_{1} \\ \underset{\sim}{J}^{-1}_{2} \\ \underset{\sim}{J}^{-1}_{3} \end{Bmatrix} \underset{\sim}{R}_v \cdot \underset{\sim}{q}_{e}$$

$$\begin{Bmatrix} \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial z} \end{Bmatrix} = \begin{Bmatrix} \underset{\sim}{J}^{-1}_{1} \\ \underset{\sim}{J}^{-1}_{2} \\ \underset{\sim}{J}^{-1}_{3} \end{Bmatrix} \underset{\sim}{R}_w \cdot \underset{\sim}{q}_{e}$$

linear strain

$$\underset{\sim}{E} = \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{Bmatrix} = \begin{Bmatrix} \partial u/\partial x \\ \partial v/\partial y \\ \partial w/\partial z \\ \partial u/\partial y + \partial v/\partial x \\ \partial v/\partial z + \partial w/\partial y \\ \partial w/\partial x + \partial u/\partial z \end{Bmatrix} = \begin{Bmatrix} \underset{\sim}{J}_1^{-1} \underset{\sim}{R}_u \\ \underset{\sim}{J}_2^{-1} \underset{\sim}{R}_v \\ \underset{\sim}{J}_3^{-1} \underset{\sim}{R}_w \\ \underset{\sim}{J}_2^{-1} \underset{\sim}{R}_u + \underset{\sim}{J}_1^{-1} \underset{\sim}{R}_v \\ \underset{\sim}{J}_3^{-1} \underset{\sim}{R}_v + \underset{\sim}{J}_2^{-1} \underset{\sim}{R}_w \\ \underset{\sim}{J}_1^{-1} \underset{\sim}{R}_w + \underset{\sim}{J}_3^{-1} \underset{\sim}{R}_u \end{Bmatrix} \underset{\sim}{q}_e$$

$$= \underset{\sim}{B}\, \underset{\sim}{q}_e \qquad (1)$$

유한 요소 지배 방정식 적용.

$$\int_{V_i} \delta \underset{\sim}{E}^T \underset{\sim}{C}\, \underset{\sim}{E}\, dV = \delta \underset{\sim}{q}_i^T \left[\int_{V_i} \underset{\sim}{B}^T \underset{\sim}{C}\, \underset{\sim}{B}\, dV \right] \underset{\sim}{q}_i$$

$$= \delta \underset{\sim}{q}_i^T \left[\iiint \underset{\sim}{B}^T \underset{\sim}{C}\, \underset{\sim}{B}\, |J|\, d\xi\, d\eta\, d\zeta \right] \underset{\sim}{q}_i$$

$$= \delta \underset{\sim}{q}_i^T\, \underset{\sim}{K}^i\, \underset{\sim}{q}_i$$

$|J|$ = det of $\underset{\sim}{J}$

2x2x2 point 수치적분

strain. $\underset{\sim}{E}$ 를 분해

$$\underset{\sim}{E} = \underset{\sim}{E}_0 + \zeta \underset{\sim}{E}_\zeta$$

여기서 $\underset{\sim}{E}_0 = \frac{1}{2}(\underset{\sim}{E}_a + \underset{\sim}{E}_{-a})$

$$E_\zeta = \frac{1}{2a}(\underset{\sim}{E}_a - \underset{\sim}{E}_{-a})$$

첨자 a & $-a$ 는 $\zeta = a$ & $\zeta = -a$

같은 방법으로

$$\underset{\sim}{B} = \underset{\sim}{B}_0 + \zeta \underset{\sim}{B}_\zeta$$

$$\underset{\sim}{B}_0 = \frac{1}{2}(\underset{\sim}{B}_a + \underset{\sim}{B}_{-a})$$

$$\underset{\sim}{B}_\zeta = \frac{1}{2a}(\underset{\sim}{B}_a - \underset{\sim}{B}_{-a})$$

$$\underset{\sim}{J} = \underset{\sim}{J}_0 + \zeta \underset{\sim}{J}_\zeta$$

$$= (1 + r\zeta)\underset{\sim}{J}_0$$

$$\underset{\sim}{J}_0 = \frac{1}{2}(\underset{\sim}{J}_a + \underset{\sim}{J}_{-a})$$

$$\underset{\sim}{J}_\zeta = \frac{1}{2a}(\underset{\sim}{J}_a - \underset{\sim}{J}_{-a})$$

$$r(\xi, \eta) = \frac{J_\zeta}{J_0}$$

If 두께가 일정하면 $\underset{\sim}{J} = \underset{\sim}{J}_0$

$$\int_{V_i} \delta \underset{\sim}{E}^T \underset{\sim}{C} \underset{\sim}{E} \, dv = \int_{V_i} (\delta \underset{\sim}{E_0}^T + \zeta \delta \underset{\sim}{E_\zeta}) \underset{\sim}{C} (\underset{\sim}{E_0} + \zeta \underset{\sim}{E_\zeta}) dv$$

$$\underset{\sim}{E} = \underset{\sim}{B} \underset{\sim}{q_i} = B_0 \underset{\sim}{q_i} + \zeta \underset{\sim}{B_\zeta} \underset{\sim}{q_i}$$

$$= \underset{\sim}{E_0} + \zeta \underset{\sim}{E_\zeta}$$

If 일정 두께

$$\int \delta \underset{\sim}{E}^T \underset{\sim}{C} \underset{\sim}{E} \, dv = \int \delta \underset{\sim}{E}^T \underset{\sim}{C} \underset{\sim}{E} |J| d\xi d\eta d\zeta$$

$$= \delta \underset{\sim}{q_i}^T \int (\underset{\sim}{B_0}^T + \zeta \underset{\sim}{B_\zeta}^T) \underset{\sim}{C} (\underset{\sim}{B_0} + \zeta \underset{\sim}{B_\zeta}) |J| d\xi d\eta d\zeta \, \underset{\sim}{q_i}$$

$$= \delta \underset{\sim}{q_i}^T \int \left[\underset{\sim}{B_0}^T \underset{\sim}{C} \underset{\sim}{B_0} + \zeta \underset{\sim}{B_0}^T \underset{\sim}{C} \underset{\sim}{B_\zeta} + \zeta \underset{\sim}{B_\zeta}^T \underset{\sim}{C} \underset{\sim}{B_0} + \zeta^2 \underset{\sim}{B_\zeta}^T \underset{\sim}{C} \underset{\sim}{B_\zeta} \right] |J_0| d\xi d\eta d\zeta$$

$$= \delta \underset{\sim}{q_i}^T \int \left(2 \underset{\sim}{B_0}^T \underset{\sim}{C} \underset{\sim}{B_0} + \frac{2}{3} \underset{\sim}{B_\zeta}^T \underset{\sim}{C} \underset{\sim}{B_\zeta} \right) |J_0| d\xi d\eta$$