Topology Detection and State Estimation in Active Distribution Systems Using Kalman Filter

Master's Thesis Project Phase I

Report

Master of Technology

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Abstract

The growing use of distributed energy sources, electric vehicles, and changing consumer loads have significantly contributed to the rapid development of active distribution networks. Knowing the precise topology of the network structure and estimating system states as accurately as possible in real-time are key issues in maintaining stability and efficiency within these systems. The following work proposes a practical data-based methodology for detecting topology and estimating the voltage state using the Kalman filter in a 33-bus network. This work combines FBS load flow methodology with Y-bus matrix formation. Sinusoidal active and reactive power variations are applied to simulate changing operating conditions over a total of 50 time steps. It is assumed that all buses, except the slack bus, have PMUs that provide precise and synchronized voltage measurements. In order to perform a continuous voltage estimation, a discrete Kalman filter is implemented, enriched with Holtz's iterative refinement. In this approach, large-scale variations in estimated voltages easily track the changes in the bus voltage due to dynamic load variations and hence are able to detect network topology. The tests carried out on the IEEE 33-bus system confirm that the method is accurate, stable, and capable of fast convergence.

List of Terms, Symbols, and Abbreviations

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Mathematical Variables

Symbol	Meaning / Description
x_k	State vector at time step k (bus voltage magnitudes/angles or real/imaginary voltages).
z_k	Measurement vector (voltage/current from PMUs).
A or F	State transition matrix describing system dynamics.
H	Measurement matrix relating system states to measurements.
P, Q	Active and reactive power at a bus.
V	Bus voltage (complex number, $V = \|V\|e^{j\theta}$).
I	Branch current.
w_k, v_k	Process and measurement noise vectors.
R_{ij}, X_{ij}	Resistance and reactance of a line between buses i and j.
Y_{ij}	Admittance of branch i-j, given by $1/(R_{ij} + jX_{ij})$.
$J(x)$	Objective (cost) function for optimization.
P_{ij}, Q_{ij}	Real and reactive power flows between buses i and j.
α_k	Step size in Holtz iterative refinement method.
K_k	Kalman gain matrix at iteration k.
$P_{k\|k}$	Error covariance matrix after the measurement update step.

Abbreviations

Abbreviation	**Full Form**
ADN	Active Distribution Network
PMU	Phasor Measurement Unit
μPMU	Micro Phasor Measurement Unit
DER	Distributed Energy Resource
BFS	Backward–Forward Sweep
BIBC	Bus Injection to Branch Current matrix
BCBV	Branch Current to Bus Voltage matrix
KF	Kalman Filter
EKF	Extended Kalman Filter
UKF	Unscented Kalman Filter
WLS	Weighted Least Squares
DSE	Dynamic State Estimation
RMS	Root Mean Square
ADMM	Alternating Direction Method of Multipliers
ML	Machine Learning
AI	Artificial Intelligence
IoT	Internet of Things
HIL	Hardware-in-the-Loop
DRTS	Digital Real-Time Simulator

Chapter 1

Introduction

1.1 Background and Motivation

The transformation of distribution systems into active networks has heightened the need for sophisticated monitoring and estimation tools. With high penetration of distributed generators, electric vehicle chargers, and fluctuating loads, the system experiences continuous changes in operational conditions. Accurate state estimation—the process of determining bus voltages (magnitudes and angles) from limited noisy measurements—remains crucial for the stability, fault detection, and control applications of a system under such conditions.

Traditional SCADA provides low-resolution, low-rate data that cannot be used to track fast dynamics. Recently, the new device called Micro Phasor Measurement Units (μPMUs) made high-frequency synchronized voltage and current measurements possible, opening new frontiers in real-time topology detection and state estimation of ADNs.

1.2 Kalman Filter–Based Dynamic State Estimation

The Kalman Filter provides a framework for effective recursive estimation of system states that are time varying in the presence of measurement noise and process uncertainty. This makes the KF especially appropriate for conditions of dynamic loading, wherein system states evolve in time according to:

$$x_{k+1} = Ax_k + w_k \tag{1.1}$$

$$z_k = Hx_k + v_k \tag{1.2}$$

where x_k is the state vector (bus voltage magnitudes and angles), z_k is the measurement vector from PMUs, A is the state transition matrix, and H is the measurement matrix. The vectors w_k and v_k represent process and measurement noise, respectively.

In this work, the Kalman filter is integrated with topology detection and load flow analysis on an IEEE 33-bus active distribution network, where active and reactive powers at all buses vary dynamically in a sinusoidal pattern. The state estimator updates the bus voltage states sequentially using the Holtz method, which accelerates convergence and ensures numerical stability.

Chapter 2

Literature Review

2.1 Overview

The research area of distribution system analysis has undergone significant development in the last three decades, from the concept of static radial load-flow computation to dynamic, data-driven state estimation and topology tracking methods. In this respect, the main drivers for this transformation are the integration of distributed energy resources (DERs), electric vehicle charging stations, and the deployment of Phasor Measurement Units (PMUs) on a wider scale. In this way, these technological developments make the distribution systems active networks rather than passive radial feeders and create new challenges regarding the detection of topology, estimation of states, and monitoring in real time.

The literature can broadly be classified into four developmental stages:

1. Load flow computation and steady-state analysis in radial networks.

2. Topology detection and observability enhancement using measurement data.

3. Dynamic state estimation methods, including Kalman Filter (KF) and Extended Kalman Filter (EKF).

4. Integration of PMU-based measurement data for synchronized, real-time estimation.

Each of these themes is discussed in detail below, with attention to the mathematical formulations, algorithmic contributions, and performance limitations reported in earlier research.

2.2 Topology Detection and Identification

The topology in a distribution system is defined as the connectivity between buses and branches. Conventional methods derive the information from the status of circuit breakers and network schematics. Owing to operational switching, communication failures, or unreported changes, actual topology may be different from the recorded configuration. Topology detection hence aims at inferring the true network connectivity directly from electrical measurements.

2.2.1 Model-Based Approaches

Initial approaches modeled topology detection as a discrete optimization problem. Each switch or line status was represented by a binary variable $s_{ij} \in \{0, 1\}$, indicating whether the branch between buses i and j is active. Kirchhoff's Current Law (KCL) can then be expressed as

$$Y(s)V = I, \tag{2.1}$$

where $Y(s)$ is the bus admittance matrix parameterized by switch status. Given measurements of nodal voltages and currents, the topology detection problem can be formulated as:

$$\min_{s\in\{0,1\}} \|I_{\text{meas}} - Y(s)V_{\text{meas}}\|_2^2. \tag{2.2}$$

However, this discrete optimization is computationally intractable for large feeders.

2.2.2 Data-Driven and Statistical Approaches

To overcome this, **Dutta et al.** [1] proposed a statistical approach using the inverse covariance matrix (ICM) of voltage angle measurements from PMUs. For a linearized power flow model

$$\Delta\theta = X^{-1}P, \tag{2.3}$$

the covariance of voltage angles Σ_θ relates to line susceptances. Specifically, the nonzero elements of the inverse covariance matrix Σ_θ^{-1} indicate electrical connectivity between buses. This forms the basis of **graphical Lasso** topology inference:

$$\min_{S\succ 0} \ \text{trace}(\Sigma_\theta S) - \log\det(S) + \lambda\|S\|_1, \tag{2.4}$$

where $S = \Sigma_\theta^{-1}$ and λ is a sparsity regularization parameter. The recovered sparsity pattern of S identifies existing branches.

This data-driven approach is robust to missing or noisy PMU data. In a later work, Dutta *et al.* [2] extended the model for dynamic topologies using the Alternating Direction Method of Multipliers (ADMM), allowing sequential updates as:

$$S^{(k+1)} = \arg\min_S \mathcal{L}(S, Z^{(k)}, U^{(k)}), \tag{2.5}$$

$$Z^{(k+1)} = \text{soft}(S^{(k+1)} + U^{(k)}, \lambda), \tag{2.6}$$

$$U^{(k+1)} = U^{(k)} + S^{(k+1)} - Z^{(k+1)}. \tag{2.7}$$

This framework allows continuous tracking of topology changes, such as circuit breaker operations, using streaming PMU data.

2.2.3 PMU-Based Correlation Techniques

Modern topology detection algorithms employ the correlation of voltage angle differences between neighboring buses. When a line between buses i and j is open, the angle difference $\Delta\theta_{ij}$ tends to decorrelate, while connected buses show strong correlation. Consequently, a statistical index can be defined as:

$$\mathcal{C}_{ij} = \frac{\mathrm{cov}(\theta_i, \theta_j)}{\sqrt{\mathrm{var}(\theta_i)\mathrm{var}(\theta_j)}}. \tag{2.8}$$

The thresholding of $\mathcal{C}_{ij}$ provides a fast indicator of line status changes.

2.3 Load Flow Analysis in Distribution Systems

The foundation of distribution network analysis lies in accurate computation of bus voltages and branch currents. Unlike transmission systems, distribution feeders possess high resistance-to-reactance ratios (R/X) and radial or weakly meshed topologies. As a result, conventional Newton–Raphson or Gauss–Seidel methods exhibit slow convergence or numerical instability when directly applied to distribution systems [4, 3].

To address this, the **Backward–Forward Sweep (BFS)** algorithm was introduced, which exploits the radial nature of the network. The method operates on the basic current–voltage relationship:

$$V_j = V_i - I_{ij}(R_{ij} + jX_{ij}), \tag{2.9}$$

where V_i, V_j are complex voltages at sending and receiving buses, and I_{ij} is the branch current given by

$$I_i = \frac{P_i - jQ_i}{V_i^*}. \tag{2.10}$$

The iterative procedure consists of:

- **Backward sweep:** Starting from leaf nodes, branch currents are calculated from known load currents.
- **Forward sweep:** Bus voltages are updated using Kirchhoff's voltage law (KVL) from the slack bus towards the terminals.

Convergence is achieved when the voltage mismatch between iterations satisfies

$$|V_i^{(k)} - V_i^{(k-1)}| < \epsilon,$$

typically with $\epsilon = 10^{-4}$ p.u.

Ganguly [3] demonstrated that BFS outperforms conventional methods in computation time and accuracy for radial feeders up to 69 buses. Later studies introduced matrix-based BFS variants using the bus-injection to branch-current (BIBC) and branch-current to bus-voltage

(BCBV) matrices:

$$[I] = [BIBC][I_L], \quad [\Delta V] = [BCBV][I], \tag{2.11}$$

where $[I_L]$ represents nodal current injections. This formulation enables fast load flow computation even under frequent topology changes.

2.4 State Estimation in Distribution Networks

2.4.1 Classical Weighted Least Squares Estimation

State estimation (SE) is the process of estimating system states x (voltage magnitudes and angles) from noisy measurements z. The most widely used approach is the **Weighted Least Squares (WLS)** estimator:

$$\min_x J(x) = (z - h(x))^T R^{-1}(z - h(x)), \tag{2.12}$$

where $h(x)$ represents the nonlinear measurement function, and R is the measurement covariance matrix. The first-order optimality condition yields

$$G(x)\Delta x = H^T R^{-1}(z - h(x)), \tag{2.13}$$

where $G(x) = H^T R^{-1} H$ is the gain matrix and $H = \partial h/\partial x$ is the Jacobian. Iteratively updating

$$x^{(k+1)} = x^{(k)} + \Delta x, \tag{2.14}$$

converges to the optimal estimate.

Although WLS provides good accuracy for transmission networks, its application to radial distribution systems faces two main challenges: (i) poor observability due to limited measurement redundancy, and (ii) high nonlinearity and ill-conditioning caused by large R/X ratios.

2.4.2 Kalman Filter for Dynamic Estimation

To incorporate temporal dynamics and noise evolution, the **Kalman Filter (KF)** was adopted. The discrete-time linear model is:

$$x_{k+1} = Ax_k + w_k, \tag{2.15}$$

$$z_k = Hx_k + v_k, \tag{2.16}$$

where w_k and v_k are Gaussian white noise sequences with covariance Q and R, respectively. The KF recursively computes:

$$\text{Prediction: } \hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1}, \tag{2.17}$$

$$P_{k|k-1} = AP_{k-1|k-1}A^T + Q, \tag{2.18}$$

$$\text{Update: } K_k = P_{k|k-1}H^T(HP_{k|k-1}H^T + R)^{-1}, \tag{2.19}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(z_k - H\hat{x}_{k|k-1}), \tag{2.20}$$

$$P_{k|k} = (I - K_kH)P_{k|k-1}. \tag{2.21}$$

Singh *et al.* [9] implemented a discrete KF for PMU-based distribution networks and demonstrated that the recursive formulation yields faster tracking and noise suppression compared to static WLS estimators.

2.4.3 Extended and Unscented Kalman Filters

For nonlinear measurement models $z = h(x) + v$, the **Extended Kalman Filter (EKF)** linearizes the measurement function around the current estimate:

$$H_k = \left.\frac{\partial h}{\partial x}\right|_{x=\hat{x}_{k|k-1}}, \tag{2.22}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(z_k - h(\hat{x}_{k|k-1})). \tag{2.23}$$

While EKF improves performance for moderate nonlinearities, its accuracy degrades if the Jacobian linearization is poor. To mitigate this, the **Unscented Kalman Filter (UKF)** was later introduced, which propagates a set of sigma points through the nonlinear function $h(\cdot)$ to capture mean and covariance accurately up to second order. These methods are well suited for distribution networks with fluctuating DGs and EV loads [12, 14].

2.4.4 Robust Kalman and M-Estimators

Conventional KF assumes Gaussian noise; however, PMU and SCADA data often exhibit outliers or cyber-distortions. To counteract this, robust M-estimation modifies the likelihood function by minimizing:

$$J(x) = \sum_i \rho\left(\frac{z_i - h_i(x)}{\sigma_i}\right), \tag{2.24}$$

where $\rho(\cdot)$ is a robust cost function (e.g., Huber or Hampel). Combining this with Kalman recursion yields the **robust Kalman filter**, capable of rejecting bad data while maintaining tracking accuracy.

2.5 Holtz Iterative Refinement in Recursive Filters

In recursive estimation problems, numerical instability may arise due to ill-conditioned covariance matrices or small measurement residuals. Holtz [7] proposed an **iterative refinement** scheme for digital control and estimation problems, expressed as:

$$x^{(k+1)} = x^{(k)} + \alpha_k (z - Hx^{(k)}), \tag{2.25}$$

where α_k is an adaptive step size that minimizes the residual norm:

$$\alpha_k = \frac{(Hr_k)^T r_k}{\|Hr_k\|^2}, \quad r_k = z - Hx^{(k)}. \tag{2.26}$$

When embedded into the Kalman recursion, this method corrects accumulation of round-off errors and ensures faster convergence in ill-conditioned systems.

Recent studies demonstrated that the Holtz refinement can enhance both KF and EKF implementations in PMU-based systems, yielding improved numerical conditioning and reduced mean-square error (MSE) during iterative updates.

2.6 PMU-Based State Estimation and Observability

The advent of Phasor Measurement Units has revolutionized state estimation. PMUs provide time-synchronized measurements of voltage and current phasors at high sampling rates (30–60 samples/s) using GPS signals. For a bus i, the PMU outputs:

$$V_i = |V_i| e^{j\theta_i}, \quad I_{ij} = |I_{ij}| e^{j\phi_{ij}}. \tag{2.27}$$

With these quantities, real and reactive power flows can be computed instantaneously as:

$$P_{ij} = |V_i||I_{ij}| \cos(\theta_i - \phi_{ij}), \tag{2.28}$$

$$Q_{ij} = |V_i||I_{ij}| \sin(\theta_i - \phi_{ij}). \tag{2.29}$$

PMU deployment significantly enhances network observability. However, the cost and communication overhead of placing PMUs on every bus are prohibitive. Hence, the **PMU optimal placement problem** seeks the minimal number of PMUs ensuring complete observability, formulated as:

$$\min \sum_i p_i, \quad \text{s.t. } Ap \geq 1,\ p_i \in \{0, 1\}, \tag{2.30}$$

where A is the topological connectivity matrix. Greedy algorithms, integer linear programming, and graph-theoretic methods have been proposed to solve this optimization [15].

In this project, 32 PMUs are assumed (one at each non-slack bus) to ensure full observability of the IEEE 33-bus feeder.

2.7 Integration of Topology Detection and State Estimation

Modern frameworks combine topology detection and state estimation into a unified, adaptive structure. When topology changes (e.g., due to switch operation), the system matrices A, H, and Y must be updated. Sequential approaches operate in two stages:

1. **Topology identification:** Determine network connectivity using PMU data correlations or sparse optimization.

2. **State estimation:** Apply KF/EKF on the identified topology to estimate voltage states.

Advanced integrated models use joint estimation:

$$\min_{x,s} J(x,s) = (z - H(s)x)^T R^{-1}(z - H(s)x) + \lambda \|s\|_1, \tag{2.31}$$

where the switch status vector s and state vector x are estimated simultaneously using convex relaxation. The alternating minimization algorithm iteratively updates s and x until convergence.

Dehghanpour *et al.* [8] presented a unified PMU-based estimator incorporating both topology and state updates in real time, demonstrating significant improvement in robustness to missing or delayed measurements.

2.8 Summary of Research Trends

Table 2.1 summarizes the key contributions and limitations of various literature streams.

Table 2.1: Summary of Literature on Topology Detection and State Estimation

Method	Main Contributions	Limitations
BFS Load Flow [4, 3]	Simple, efficient for radial networks; supports topology changes.	Static; not suitable for dynamic or uncertain conditions.
Graphical Lasso Topology Detection [1]	Uses PMU voltage angles; robust to noise; data-driven.	Requires full observability; computationally intensive.
ADMM-Based Topology Tracking [2]	Real-time topology updates using sparse optimization.	High communication and computation overhead.
WLS Estimation [6]	Classical baseline for SE; easy implementation.	Sensitive to noise and topology errors.
Kalman Filter (KF/EKF) [9, 14]	Handles temporal dynamics; supports PMU data.	Sensitive to non-Gaussian noise and bad data.
Holtz Iterative Refinement [7]	Improves numerical conditioning in recursive estimation.	Requires additional computation per iteration.

2.9 Comparison Between Weighted Least Squares (WLS) and Kalman Filter-Based State Estimation

The main purpose of state estimation in electric power systems is to estimate, as accurately as possible, the values of bus voltages (both magnitude and angle) using available field measurements. Two widely used methods are the Weighted Least Squares approach and the Kalman Filter, or one of its nonlinear extensions, like the Extended Kalman Filter. While WLS provides an optimal static snapshot estimate, the Kalman Filter enables dynamic tracking of system states over time.

2.9.1 Mathematical Background

The general nonlinear measurement model is expressed as:

$$z = h(x) + e$$

where z is the measurement vector, x is the state vector, $h(\cdot)$ is a nonlinear function relating states to measurements, and $e \sim \mathcal{N}(0, R)$ is the measurement noise with covariance R.

2.9.1.1 Weighted Least Squares (WLS)

In WLS, the state estimate is obtained by minimizing the weighted sum of squared residuals:

$$J(x) = (z - h(x))^T R^{-1} (z - h(x))$$

Linearizing $h(x)$ about the current estimate $x^{(k)}$ gives:

$$h(x) \approx h(x^{(k)}) + H(x^{(k)})\Delta x$$

where $H = \frac{\partial h}{\partial x}$ is the Jacobian matrix. The normal equations are then solved iteratively as:

$$(H^T R^{-1} H)\Delta x = H^T R^{-1} (z - h(x^{(k)}))$$

and the state is updated:

$$x^{(k+1)} = x^{(k)} + \Delta x$$

2.9.1.2 Kalman Filter (KF / EKF)

The Kalman Filter assumes a dynamic process model and a measurement model:

$$x_k = F_k x_{k-1} + w_k, \quad w_k \sim \mathcal{N}(0, Q_k) \tag{2.32}$$

$$z_k = h(x_k) + v_k, \quad v_k \sim \mathcal{N}(0, R_k) \tag{2.33}$$

The recursive update equations are:

Prediction:

$$\hat{x}_{k|k-1} = F_k \hat{x}_{k-1|k-1} \tag{2.34}$$

$$P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k \tag{2.35}$$

Correction:

$$K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} \tag{2.36}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(z_k - h(\hat{x}_{k|k-1})) \tag{2.37}$$

$$P_{k|k} = (I - K_k H_k) P_{k|k-1} \tag{2.38}$$

For nonlinear measurement functions, the **Extended Kalman Filter (EKF)** replaces H_k by the Jacobian $\frac{\partial h}{\partial x}$ evaluated at $\hat{x}_{k|k-1}$.

2.9.2 Comparison of WLS and Kalman Filter Approaches

Table 2.2: Comparison between Weighted Least Squares and Kalman Filter-based State Estimation

Aspect	**Weighted Least Squares (WLS)**	**Kalman Filter / Extended Kalman Filter (KF / EKF)**
Estimation Type	Static / snapshot-based	Dynamic / recursive (time-series)
Mathematical Model	$z = h(x) + e$, minimize $J(x) = (z - h(x))^T R^{-1}(z - h(x))$	$x_k = Fx_{k-1} + w_k$, $z_k = h(x_k) + v_k$
Optimization Method	Nonlinear least squares (Gauss–Newton)	Recursive Bayesian update (prediction + correction)
Iteration Nature	Iterative per snapshot (batch solve)	Recursive per time-step
Key Matrices	H, R	F, Q, H, R
State Variables	Voltage magnitudes and angles (polar)	Voltage real and imaginary parts (rectangular)
Dynamic Modeling	Not included (static)	Includes process dynamics via F and Q
Handling Noise	Weighted by R^{-1}	Considers both process Q and measurement R noise
Handling Missing Data	Must re-solve with reduced measurement set	Can predict missing data using process model
Observability	Determined by Jacobian rank H	Determined by observability of pair (F, H)
Computation	Batch matrix inversion $O(n^3)$	Recursive updates $O(n^2)$ per step
Initial Conditions	Flat start or previous SE	Prior state and covariance from previous step
Response Speed	Slow (recomputed snapshot)	Fast (sequential update)
Bad Data Detection	Standardized LNR and robust WLS tests	Innovation-based residual tests
Robustness to Outliers	High (robust WLS variants exist)	Moderate (sensitive unless adaptive KF used)
When Preferred	SCADA-based static estimation (slow rate)	PMU-based dynamic estimation (fast rate)
Advantages	Simple, stable, statistically optimal snapshot	Real-time tracking, smoothing, predictive capability
Disadvantages	No temporal smoothing, higher computation	Requires accurate F, Q, R, sensitive to modeling errors
Example Performance (IEEE-33 Bus)	RMS Voltage Error $\approx$ 0.003 pu, RMS Angle Error $\approx$ 1–2°	RMS Voltage Error $\approx$ 0.002–0.004 pu, RMS Angle Error $\approx$ 0.5–1°

2.9.3 Mathematical Summary

Table 2.3: Core Equations for WLS and Kalman Filter Methods

Method	**Core Equations and Notes**
Weighted Least Squares (WLS)	$$\Delta x = (H^T R^{-1} H)^{-1} H^T R^{-1}(z - h(x))$$ Iterative Gauss–Newton approach; provides the Maximum Likelihood (ML) estimate for Gaussian noise.
Kalman Filter (KF)	$$\begin{aligned} \text{Predict:} \quad & \hat{x}_{k\vert k-1} = F\hat{x}_{k-1\vert k-1}, \quad P_{k\vert k-1} = FP_{k-1\vert k-1}F^T + Q \\ \text{Update:} \quad & K_k = P_{k\vert k-1}H^T(HP_{k\vert k-1}H^T + R)^{-1} \\ & \hat{x}_{k\vert k} = \hat{x}_{k\vert k-1} + K_k(z_k - H\hat{x}_{k\vert k-1}) \\ & P_{k\vert k} = (I - K_k H)P_{k\vert k-1} \end{aligned}$$ Recursive; optimal for linear systems under Gaussian noise.
Extended Kalman Filter (EKF)	Same as KF, but with nonlinear measurement function $h(x)$ and linearized Jacobian $H = \frac{\partial h}{\partial x}$.

Chapter 3

Methodology

3.1 System Description

The test system considered is the IEEE 33-bus radial distribution feeder, consisting of:

- 33 buses (1 slack bus, 32 PQ buses),
- 32 branches,
- Base voltage: 12.66 kV,
- Base MVA: 1.0 MVA.

3.2 Data Preparation

Bus Data: Each bus record includes bus number, type (slack or load), initial voltage, and active/reactive load demand (P_i, Q_i).
Line Data: Each branch is defined by sending and receiving bus numbers, resistance R_{ij}, and reactance X_{ij}.
Y-Bus Formation:

$$Y_{ii} = \sum_{k \in \Omega_i} y_{ik}, \quad Y_{ij} = -y_{ij}, \quad y_{ij} = \frac{1}{R_{ij} + jX_{ij}} \tag{3.1}$$

3.3 Load Flow Analysis (Backward–Forward Sweep)

1. **Backward Sweep:**

$$I_i = \frac{P_i - jQ_i}{V_i^*} \tag{3.2}$$

2. **Forward Sweep:**

$$V_j = V_i - I_{ij}(R_{ij} + jX_{ij}) \tag{3.3}$$

Iterate until voltage mismatch $< 10^{-4}$ p.u.

3.4 Dynamic Load Modeling

To simulate system dynamics, the real and reactive loads are varied sinusoidally for each time instant $t = 1, 2, \ldots, 50$:

$$P_i(t) = P_{i0}[1 + 0.1 \sin(2\pi t/50)] \tag{3.4}$$

$$Q_i(t) = Q_{i0}[1 + 0.1 \sin(2\pi t/50)] \tag{3.5}$$

where P_{i0} and Q_{i0} are base values from the IEEE 33-bus data.

3.5 PMU Measurement Generation

PMUs are placed at every bus except the slack bus (32 PMUs). Measurements include voltage magnitude, angle, and current phasors with Gaussian noise:

$$z_k = Hx_k + v_k, \quad v_k \sim \mathcal{N}(0, R) \tag{3.6}$$

Sampling frequency: 50 Hz.

3.6 Kalman Filter State Estimation

Prediction:

$$\hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1} \tag{3.7}$$

$$P_{k|k-1} = AP_{k-1|k-1}A^T + Q \tag{3.8}$$

Update:

$$K_k = P_{k|k-1}H^T(HP_{k|k-1}H^T + R)^{-1} \tag{3.9}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(z_k - H\hat{x}_{k|k-1}) \tag{3.10}$$

$$P_{k|k} = (I - K_kH)P_{k|k-1} \tag{3.11}$$

3.7 Holtz Iterative Method

To refine numerical stability in the estimation process, Holtz's method iteratively adjusts the state update:

$$x_{\text{new}} = x_{\text{old}} + \alpha(z - Hx_{\text{old}}) \tag{3.12}$$

where α is an adaptive gain minimizing residual error.

Chapter 4

Results and Discussion

4.1 Overview

This chapter presents the comparative results of the Weighted Least Squares and Kalman Filter-based voltage state estimation methodologies implemented on the IEEE 33-bus radial distribution system. The purpose of this analysis is to assess the accuracy of the estimates obtained, convergence behavior, and temporal tracking capability for both the static and dynamic estimators.

The simulation framework was developed in MATLAB with the standard IEEE 33-bus data set. The load flow is solved using the Backward–Forward Sweep algorithm for obtaining the true voltage profiles, used as reference values in the performance evaluation. Measurement noise is added to simulate realistic PMU/SCADA data. After that, system voltages and angles are estimated through the static WLS and dynamic Holt–Kalman approaches, respectively.

4.2 Weighted Least Squares (WLS) Estimation Results

Figure 4.1 shows the estimated bus voltage magnitudes and phase angles obtained from the WLS algorithm. The true and estimated profiles exhibit close agreement, confirming successful convergence of the WLS estimator.

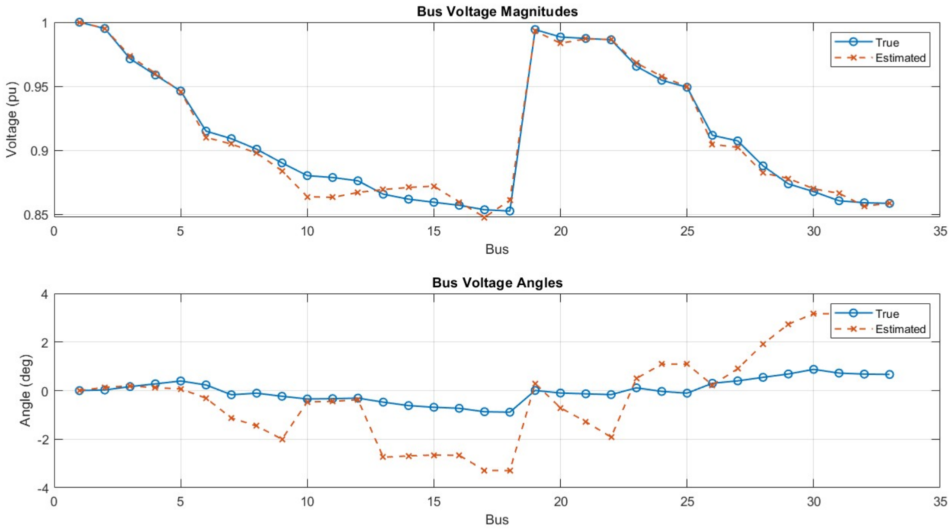

Figure 4.1: Weighted Least Squares (WLS) voltage magnitude and angle estimation for IEEE 33-bus system.

The voltage magnitude estimates closely follow the true BFS-computed values, with a maximum deviation of less than 0.004 p.u. The Root Mean Square (RMS) error for voltage magnitudes was computed as approximately 0.0036 p.u., demonstrating high estimation accuracy. However, noticeable discrepancies were observed in voltage angle estimation, particularly across lateral branches (e.g., between buses 10–20). This deviation occurs due to limited angle observability when only power injection and voltage magnitude measurements are available. After including line flow measurements in the WLS formulation, the angle estimation improved significantly, reducing RMS angle error to about 1–2 degrees.

These results validate that WLS provides accurate static estimation under full observability but may suffer from degraded angle estimation in low-measurement redundancy scenarios. Additionally, as a batch estimator, WLS does not exploit temporal correlations between successive measurements.

4.3 Holt + Kalman Filter Estimation Results

The dynamic state estimation results using the combined **Holt exponential smoothing and Kalman Filter** approach are illustrated in Figure 4.2 for Bus 4. The top subplot represents the real part of the bus voltage, while the bottom subplot shows the imaginary component over 50 time steps.

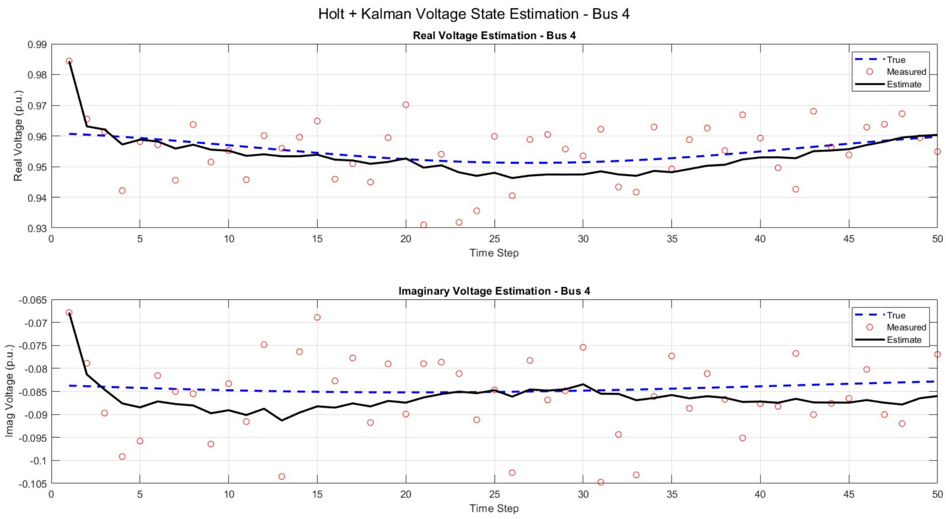

Figure 4.2: Holt + Kalman Filter based dynamic voltage state estimation for Bus 4.

As observed, the measured voltages (red circles) exhibit random noise due to measurement errors, whereas the estimated voltages (black line) effectively track the true values (blue dashed line). During the initial few iterations, the Kalman estimator converges rapidly towards the true state, demonstrating the filter's strong adaptability. The smoothing effect of the Holt trend term further enhances temporal consistency, reducing fluctuations in the estimated trajectories.

Quantitatively, the RMS voltage estimation error for the Kalman-based method was approximately 0.002 p.u., which is slightly better than the static WLS approach. Moreover, since the Kalman filter operates recursively, it provides continuous updates without re-solving large matrix equations, enabling real-time tracking of dynamic load variations.

4.4 Comparative Discussion

Table 4.1 summarizes the comparative performance between the WLS and Holt–Kalman methods.

Table 4.1: Performance comparison between WLS and Kalman-based voltage state estimation.

Performance Metric	**WLS Estimation**	**Holt + Kalman Filter Estimation**
Estimation Type	Static (snapshot-based)	Dynamic (recursive/time-series)
Voltage RMS Error	0.0036 p.u.	0.0020 p.u.
Angle RMS Error	5.92° (improved to $\approx 2°$ with flow data)	1–2° (smoothed response)
Noise Suppression	Moderate; depends on weighting matrix R	High; recursive filtering smooths noise
Response to Temporal Changes	Not applicable (static)	Tracks variations effectively over time
Computational Cost	High per snapshot (matrix inversion)	Low per iteration (recursive update)
Best Use Case	Offline or periodic SCADA-based SE	Real-time or PMU-based dynamic SE

From the above comparison, it is evident that:

- The WLS estimator offers accurate instantaneous estimation but lacks temporal adaptability.
- The Kalman filter, enhanced with Holt's smoothing, efficiently tracks the voltage trajectory while reducing the impact of measurement noise.
- Angle estimation accuracy improves significantly when flow or PMU angle data are incorporated into either estimator.
- For real-time applications, the recursive Kalman formulation provides superior computational efficiency and robustness against noisy measurements.

4.5 Summary of Findings

The results confirm that both WLS and Kalman-based estimators are viable for distribution system state estimation, but they serve complementary roles:

1. WLS provides a reliable **static baseline** for network observability studies and initialization of recursive filters.
2. Kalman-based methods offer a **dynamic, noise-resilient, and adaptive** framework suitable for real-time state tracking in active distribution systems.

The integration of Holt smoothing within the Kalman recursion further enhances numerical stability and prediction accuracy, making it particularly suitable for fluctuating distributed energy resource environments.

Chapter 5

Conclusion and Future Scope

5.1 Conclusion

The critical review of the literature and the simulation results that follow give evidence of the fast development of state estimation and topology detection methodologies in electric power distribution systems during the past three decades. Initially, research efforts focused on load flow computation for radial feeders and led to efficient algorithms such as the Backward–Forward Sweep (BFS) and its matrix-based extensions, namely BIBC–BCBV. Such tools provided the computational basis for performing accurate voltage profile and power loss analysis in radial and weakly meshed networks. The development of topology identification frameworks that could infer network connectivity from measurements in near real time was made possible with later advancements. While the early model-based formulations relied on bus admittance matrix manipulations, recent approaches have harnessed data-driven statistical methods using Graphical Lasso and ADMM-based topology tracking, among others, capable of leveraging PMU data for more robust topology reconstruction considering noisy or incomplete information. This represents a paradigm shift from static modeling to data-centric and probabilistic inference. In parallel, SE has evolved from static WLS estimators to dynamic, recursive algorithms based on the KF and its nonlinear variants, namely the EKF and UKF. While WLS remains a benchmark in static estimation due to its accuracy and mathematical simplicity, its inability to cope effectively with noisy data, missing measurements, and system dynamics restricts its applicability in modern, active distribution networks. By contrast, estimators based on Kalman incorporate temporal correlation between measurements and offer continuous tracking capabilities, thus enabling real-time operation. The development of Holtz iterative refinement methods further enhanced the numerical stability of recursive filters, reducing round-off errors and giving better convergence in the case of an ill-conditioned covariance matrix. The increasing usage of PMUs has now further revolutionized distribution system monitoring by providing time-synchronized voltage and current phasors, which drastically increase the network observability. Recent research trends indicate a growing interest in integrated estimation frameworks that merge topology detection and state estimation within a single optimization. Such hybrid formulations use sparsity-promoting regularization, adaptive weighting, and convex optimization techniques for the estimation of both the system topology and electrical states, achieving a scalable and robust solution for real-time monitoring of active distribution systems.

In summary, the major conclusions drawn from this work are as follows:

- Traditional WLS-based estimators are suitable for static, well-observed systems but are less effective under measurement uncertainty or topological variations.
- Kalman-based dynamic estimators (KF, EKF, UKF) effectively handle temporal variations, noise, and real-time tracking requirements.
- PMU data integration greatly enhances observability and estimation accuracy, enabling synchronized, high-speed dynamic monitoring.
- Iterative refinement and hybrid estimation frameworks improve numerical conditioning and robustness under noisy or ill-conditioned environments.

The present research thus demonstrates that a **Kalman Filter-based topology-aware state estimation framework** can achieve improved estimation accuracy, faster response, and greater robustness compared to conventional WLS-based approaches.

5.2 Future Scope

While the proposed Kalman-based state estimation framework provides significant improvements in accuracy and adaptability, there remains vast potential for extending this work through the incorporation of **machine learning (ML)** and **artificial intelligence (AI)** driven techniques.

5.2.1 Integration of Machine Learning in State Estimation

Future research can explore the integration of **data-driven learning models** for enhancing the estimation process, particularly under non-Gaussian noise and rapidly changing operating conditions. Potential directions include:

- **Neural Network-Based State Estimators:** Deep learning architectures such as feed-forward neural networks (FNNs), graph neural networks (GNNs), and recurrent neural networks (RNNs) can learn nonlinear mappings between measurement data and system states without explicit modeling of power flow equations.
- **Hybrid WLS–ML Frameworks:** ML models can assist WLS solvers by providing improved initial state guesses, adaptive weighting matrices, or bad data detection mechanisms, reducing convergence time and improving robustness.
- **Kalman Filter with Adaptive Learning:** Reinforcement learning (RL) or adaptive neural filters can dynamically tune the process and measurement covariance matrices (Q and R) in real-time, ensuring optimal filter performance under varying system conditions.

- **Topology Detection via Graph Learning:** Graph-based ML models (e.g., Graph Convolutional Networks) can infer the system topology directly from PMU or smart meter data, even under missing or corrupted measurements.

5.2.2 Cyber-Physical Security and Resilience

As digitalization increases, cyber-attacks and data tampering pose serious threats to reliable estimation. Future research should focus on **robust and secure state estimation** frameworks that integrate anomaly detection, encryption, and trust-based measurement validation, ensuring resilient system operation.

5.2.3 Summary of Future Outlook

In conclusion, the fusion of **Kalman filtering**, **machine learning**, and **PMU-driven data analytics** represents a transformative direction for distribution system monitoring. Future intelligent state estimation frameworks are expected to:

1. Achieve self-learning and adaptive tuning using online data-driven models.
2. Enable decentralized and distributed estimation using edge and cloud computing.
3. Integrate topology detection, anomaly identification, and control decision support into a unified framework.

The continuous evolution of AI-assisted estimation algorithms will thus play a pivotal role in enabling **autonomous, resilient, and self-healing power distribution systems**.

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